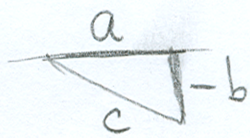
180 Professor Katiraie Quiz Six

Name Solution

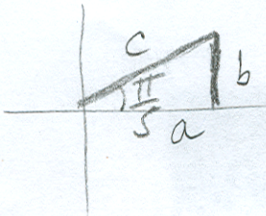
Find the exact value of the expression. Do not use a calculator.

1) $\tan^{-1}\left[\tan\left(-\frac{\pi}{8}\right)\right]$



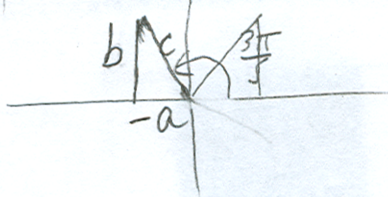
1) $\frac{-\pi}{8}$

2) $\sin^{-1}\left[\sin\left(\frac{\pi}{5}\right)\right]$



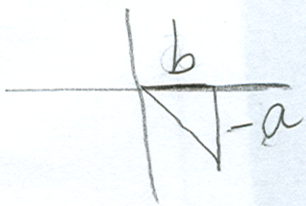
2) $\frac{\pi}{5}$

3) $\sin^{-1}\left[\sin\left(\frac{3\pi}{5}\right)\right] = \frac{2\pi}{5}$



3) $\frac{2\pi}{5}$

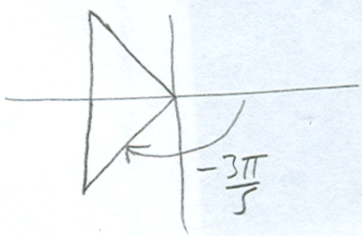
4) $\tan^{-1}\left[\tan\left(-\frac{a}{b}\right)\right]$



$= -\frac{a}{b}$

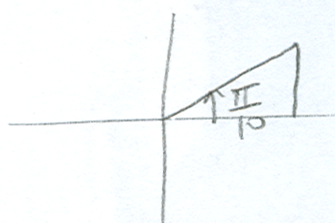
4) $\frac{-a}{b}$

5) $\cos^{-1}\left[\cos\left(-\frac{3\pi}{5}\right)\right] =$



5) $\frac{3\pi}{5}$

6) $\cos^{-1}\left[\cos\left(\frac{\pi}{10}\right)\right]$



6) $\frac{\pi}{10}$

Write the equation of a sine function that has the given characteristics.

7) Amplitude: 4
Period: π
Phase Shift: -2

7) $y = 4\sin(2x+4$

$$T = \frac{2\pi}{w} \Rightarrow \pi = \frac{2\pi}{w} \Rightarrow w\pi = 2\pi \Rightarrow \boxed{w = 2}$$

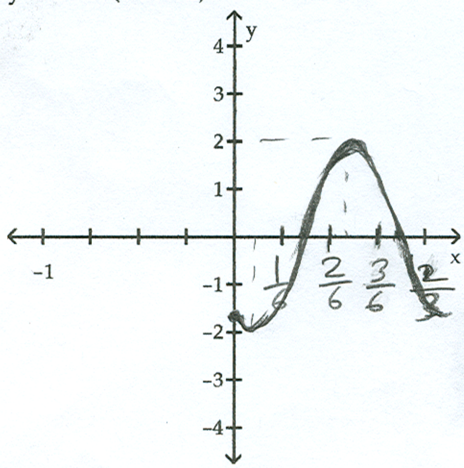
$$\frac{\phi}{w} = -2 \Rightarrow \phi = -2w = -2(2) = -4$$

Graph the function. Show at least one period.

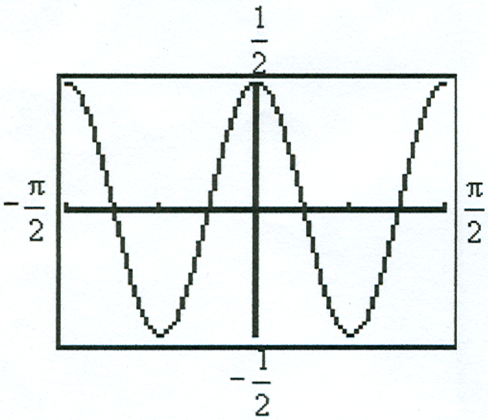
8) $y = 2\sin(3\pi x - 2)$

$\boxed{A=2}$ $\boxed{\text{Period} = \frac{2\pi}{3\pi} = \frac{2}{3}}$ 8)

phase shift $\frac{2}{3\pi}$ Right



Find an equation for the graph.



9)

$T = \frac{\pi}{2}$

$\frac{2\pi}{\omega} = \frac{\pi}{2}$

$\omega\pi = 4\pi$

$\omega = 4$

$A = \frac{1}{2}$

$y = A\cos(\omega x)$

$\boxed{A = \frac{1}{2}\cos(4x)}$

9) ______

Use transformations to graph the function.

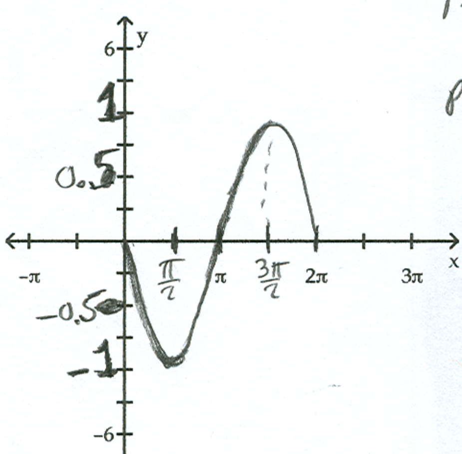
10) $y = \cos\left(x + \frac{\pi}{2}\right)$

$A = 1$

$\text{Period} = \frac{2\pi}{\omega} = 2\pi$

$\text{phase shift} = -\frac{\pi/2}{1} = -\frac{\pi}{2}$

10)



MA180 Professor Katiraie Spring 2007 Quiz Seven

Name *Solution*

Complete the identity.

1) $\sin^2\theta + \sin^2\theta\cot^2\theta = ?$

$$\sin^2\theta + \cancel{\sin^2\theta}\cdot\frac{\cos^2\theta}{\cancel{\sin^2\theta}} = \sin^2\theta + \cos^2\theta \boxed{=1}$$

2) $\tan(\pi - \theta) = ?$

$$= \frac{\tan\pi - \tan\theta}{1+\tan\pi\tan\theta} = \boxed{\frac{0-\tan\theta}{1} = -\tan\theta} = -\tan\theta$$

3) $\cos\left(\frac{\pi}{2}+\theta\right) = ?$

$$= \cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta$$
$$= (0)\cos\theta - 1\sin\theta = \boxed{-\sin\theta}$$

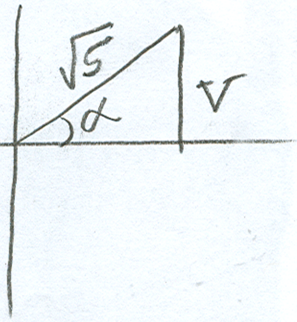
4) $\frac{1}{\cos^2\theta} - \frac{1}{\cot^2\theta} = ?$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} = \boxed{\frac{\cos^2\theta}{\cos^2\theta} = 1}$$

Use a right triangle to write the expression as an algebraic expression. Assume that v is positive and in the domain of the given inverse trigonometric function.

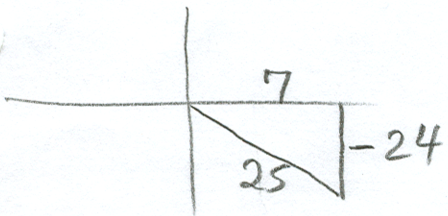
5) $\sin\left(\sin^{-1}\frac{v}{\sqrt{5}}\right)$

$$= \sin\alpha = \boxed{\frac{v}{\sqrt{5}} = \frac{v\sqrt{5}}{5}}$$



Use the information given about the angle θ, 0 ≤ θ ≤ 2π, to find the exact value of the indicated trigonometric function.

6) $\cos\theta = \frac{7}{25}$, $\frac{3\pi}{2} < \theta < 2\pi$ Find $\sin(2\theta)$.



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-24}{25}\right)\left(\frac{7}{25}\right)$$
$$= \boxed{\frac{-336}{625}}$$

$7^2 + y^2 = 25^2$

$y = -24$

Find the exact value of the trigonometric function.

7) $\cos \frac{5\pi}{12}$ $= \cos(75^\circ) = \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

8) $\sin\left(-\frac{11\pi}{12}\right)$

$\sin(-165^\circ) = \sin(-135^\circ + -30^\circ)$

$= \sin(-135^\circ)\cos(-30^\circ) + \cos(-135^\circ)\sin(-30^\circ)$

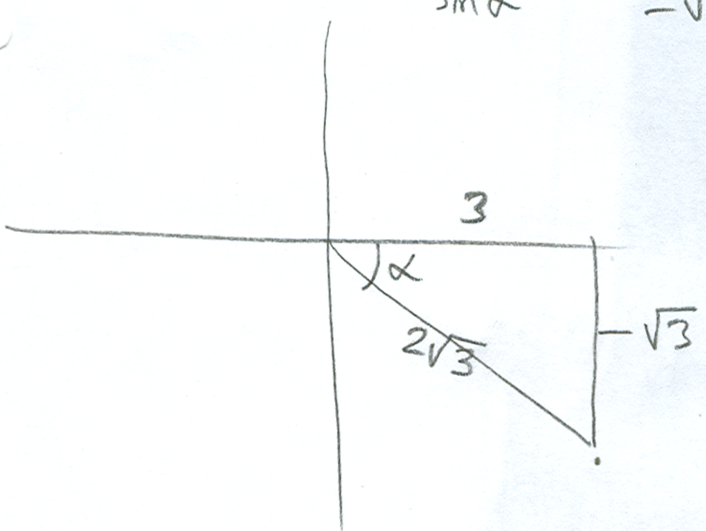
$= -\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2} \cdot -\frac{1}{2} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

Find the exact value of the expression.

9) $\sec^{-1}(-2)$

$\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3} \text{ OR } 120^\circ}$

10) $\csc\left(\tan^{-1} \frac{-\sqrt{3}}{3}\right)$ $= \frac{1}{\sin \alpha} = \frac{2\sqrt{3}}{-\sqrt{3}} = \boxed{-2}$



$c^2 = 3^2 + (-\sqrt{3})^2$

$= 9 + 3 = 12$

$c^2 = 12$

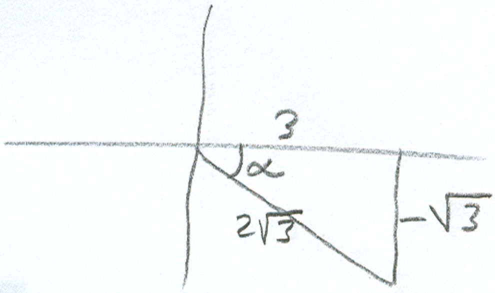
$c = 2\sqrt{3}$

MA180 Professor Katiraie Spring 2007 Quiz Seven

Name ~~Solution~~

Find the exact value of the expression.

1) $\sec\left(\tan^{-1}\frac{-\sqrt{3}}{3}\right)$ $= \dfrac{1}{\cos\alpha} = \boxed{\dfrac{2\sqrt{3}}{3}}$

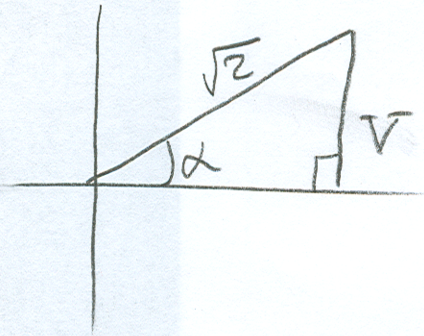


2) $\sec^{-1}(-2)$ $= \cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3} \text{ Radians OR } 120^\circ}$

Use a right triangle to write the expression as an algebraic expression. Assume that v is positive and in the domain of the given inverse trigonometric function.

3) $\sin\left(\sin^{-1}\frac{v}{\sqrt{2}}\right)$

$\sin\alpha = \boxed{\dfrac{V}{\sqrt{2}}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \boxed{\dfrac{V\sqrt{2}}{2}}$



Complete the identity.

4) $\sin^2\theta + \sin^2\theta\cot^2\theta = ?$

$\sin^2\theta + \cancel{\sin^2\theta} \cdot \dfrac{\cos^2\theta}{\cancel{\sin^2\theta}} = \sin^2\theta + \cos^2\theta = \boxed{1}$

5) $\dfrac{1}{\cos^2\theta} - \dfrac{1}{\cot^2\theta} = ?$ $= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1-\sin^2\theta}{\cos^2\theta} = \boxed{\dfrac{\cos^2\theta}{\cos^2\theta} = 1}$

Find the exact value of the trigonometric function.

6) $\sin\left(-\frac{11\pi}{12}\right) = \sin(-165^\circ) = \sin(-135^\circ + -30^\circ)$

$= \sin(-135^\circ)\cos(-30^\circ) + \cos(-135^\circ)\sin(-30^\circ)$

$= \frac{-\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{-\sqrt{2}}{2}\frac{-1}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

7) $\cos\frac{5\pi}{12} = \cos 75^\circ$

$= \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Complete the identity.

8) $\cos\left(\frac{\pi}{2} + \theta\right) = ?$ $= \cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta$

$= 0 - \sin\theta = \boxed{-\sin\theta}$

9) $\tan(\pi - \theta) = ?$ $= \frac{\tan\pi - \tan\theta}{1 + \tan\pi\tan\theta} = \boxed{\frac{-\tan\theta}{1}}$

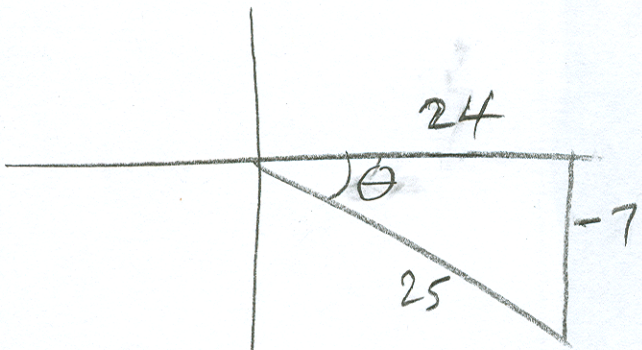
Use the information given about the angle θ, 0 ≤ θ ≤ 2π, to find the exact value of the indicated trigonometric function.

10) $\cos\theta = \frac{24}{25}$, $\frac{3\pi}{2} < \theta < 2\pi$ Find $\sin(2\theta)$.

$24^2 + y^2 = 25^2$

$y^2 = 49$

$y = \pm 7$



$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{-7}{25}\right)\left(\frac{24}{25}\right) = \boxed{\frac{-336}{625}}$

Name Solution

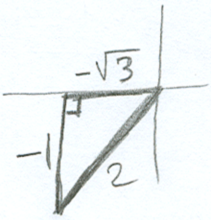
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6} = 30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3} = 60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	1	0	undefined

Find the exact value of the expression.

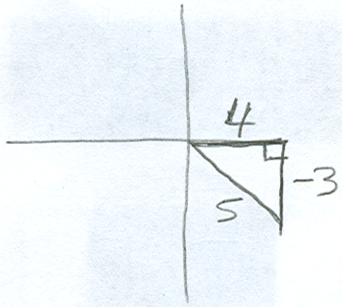
1) $\cos^{-1}\left(\sin \frac{7\pi}{6}\right)$

$\cos^{-1}\left(\frac{-1}{2}\right) = \boxed{\frac{5\pi}{6}}$

$= \boxed{150°}$



2) $\cos\left(\sin^{-1} \frac{-3}{5}\right) = \boxed{\frac{4}{5}}$

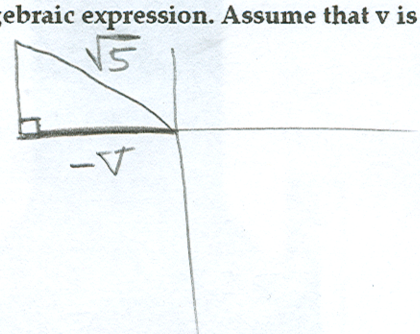


3) $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \cos^{-1}\left(\frac{-3}{2\sqrt{3}}\right) = \cos^{-1}\left(\frac{-3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}\right) = \cos^{-1}\left(\frac{-\cancel{3}\sqrt{3}}{2 \cdot \cancel{3}}\right)$

$= \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \boxed{\frac{5\pi}{6} \text{ OR } 150°}$

Use a right triangle to write the expression as an algebraic expression. Assume that v is positive and in the domain of the given inverse trigonometric function.

4) $\cos\left(\cos^{-1} \frac{-v}{\sqrt{5}}\right) = \frac{-v}{\sqrt{5}}$



Simplify the expression as far as possible.

5) $\frac{\cos\theta}{1 + \sin\theta} + \tan\theta = \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta + \sin\theta(1+\sin\theta)}{(\cos\theta)(1+\sin\theta)}$

$= \frac{\cos^2\theta + \sin\theta + \sin^2\theta}{(\cos\theta)(1+\sin\theta)} = \frac{(1+\sin\theta)}{(\cos\theta)(1+\sin\theta)} = \frac{1}{\cos\theta} = \boxed{\sec\theta}$

Complete the identity.

6) $\frac{1}{\sin^2 \theta} - \frac{1}{\tan^2 \theta} = ?$ $= \csc^2\theta - \cot^2\theta \boxed{=1}$

OR $\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} = \frac{1-\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta} \boxed{=1}$

Find the exact value of the trigonometric function.

7) $\sin 165°$ $= \sin(135° + 30°) =$

$= \sin 135° \cos 30° + \cos 135° \sin 30°$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2}\frac{1}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

8) $\cos \frac{\pi}{12}$ $= \cos(15°) = \cos(45° - 30°)$

$= \cos 45° \cos 30° + \sin 45° \sin 30° =$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

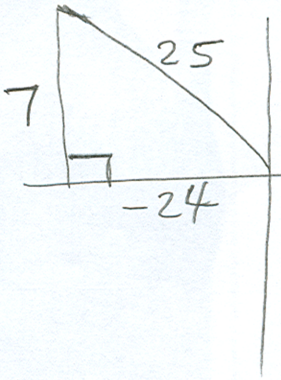
Complete the identity.

9) $\sin\left(\frac{\pi}{2} - \theta\right) = ?$

$\sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta = \cos\theta - 0 = \boxed{\cos\theta}$

Use the information given about the angle θ, 0 ≤ θ ≤ 2π, to find the exact value of the indicated trigonometric function.

10) $\sin \theta = \frac{7}{25}$, $\frac{\pi}{2} < \theta < \pi$ Find $\sin(2\theta)$.



$7^2 + x^2 = 25^2$

$x^2 = 576$

$x = \pm 24$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(\frac{7}{25}\right)\left(\frac{-24}{25}\right)$

$= \boxed{\frac{-336}{625}}$

Name Solution

Solve the equation on the interval $0 \le \theta < 2\pi$.

1) $\sqrt{2}\cos(2\theta) = 1$

$\cos(2\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$2\theta = \frac{\pi}{4} + 2K\pi$ $\quad 2\theta = \frac{7\pi}{4} + 2K\pi$

$\theta = \frac{\pi}{8} + K\pi$ $\quad \theta = \frac{7\pi}{8} + K\pi$

$\theta = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$

2) $2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

Solve the equation. Give a general formula for all the solutions.

3) $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3} + 2K\pi$ and $\theta = \frac{2\pi}{3} + 2K\pi$

Solve the equation on the interval [0, 2π).

4) Suppose f(x) = cos θ −1. Solve f(x) = 0.

$\cos\theta - 1 = 0$

$\cos\theta = 1$

$\theta = 0$

Solve the equation on the interval $0 \le \theta < 2\pi$.

5) $2\sin^2\theta = \sin\theta$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ $\quad \sin\theta = \frac{1}{2}$

$\theta = 0$ $\quad \theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

$\theta = \pi$

6) $\cos\theta = \sin\theta$

$\Downarrow$

$\theta = \frac{\pi}{4}$ and $\frac{5\pi}{4}$

7) $\tan(2\theta) - \tan\theta = 0$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{\tan\theta}{1}$$

$$2\tan\theta = \tan\theta - \tan^3\theta \Rightarrow \tan\theta + \tan^3\theta = 0$$

$$\tan\theta(1+\tan^2\theta) = 0 \Rightarrow \tan\theta = 0 \qquad \theta = 0 \text{ and } \theta = \pi$$

8) $\cos(2\theta) = \sqrt{2} - \cos(2\theta)$

$$2\cos(2\theta) = \sqrt{2}$$

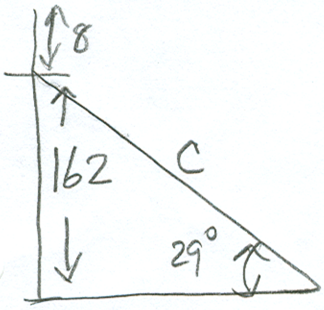
$$\cos(2\theta) = \frac{\sqrt{2}}{2} \rightarrow 2\theta = \frac{\pi}{4} + 2k\pi, \quad 2\theta = \frac{7\pi}{4} + 2k\pi$$

$$\theta = \frac{\pi}{8} + k\pi \; ; \quad \theta = \frac{7\pi}{8} + k\pi$$

$$\theta = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$$

Solve the problem.

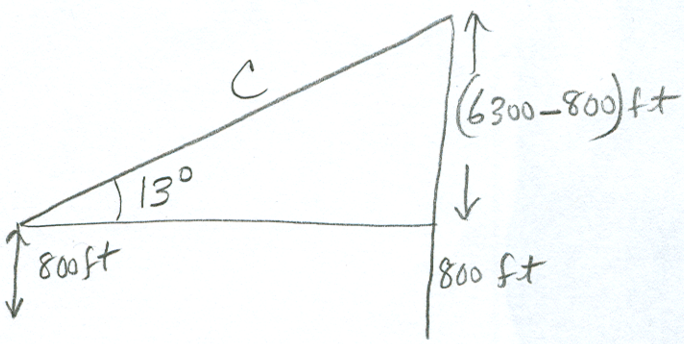
9) A radio transmission tower is 170 feet tall. How long should a guy wire be if it is to be attached 8 feet from the top and is to make an angle of 29° with the ground? Give your answer to the nearest tenth of a foot.



$$\sin 29^\circ = \frac{162}{C}$$

$$C = \frac{162}{\sin 29^\circ} = 334.15 \text{ feet}$$

10) A straight trail with a uniform inclination of 13° leads from a lodge at an elevation of 800 feet to a mountain lake at an elevation of 6300 feet. What is the length of the trail (to the nearest foot)?



$$\sin 13^\circ = \frac{6300-800}{C}$$

$$C = \frac{5500}{\sin 13^\circ} = 24449.76 \text{ feet}$$

MA180 Professor Katiraie Quiz Eight Form B Spring 2007

Name Prof. Katiraie

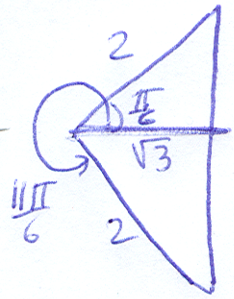
Solve the equation on the interval $0 \le \theta < 2\pi$.

1) $4 \csc \theta - 3 = 1$

$4\csc\theta = 4 \Longrightarrow \csc\theta = 1 \Longrightarrow \sin\theta = 1$

$\theta = \frac{\pi}{2}$

2) $2 \cos (2\theta) = \sqrt{3}$ $\cos(2\theta) = \frac{\sqrt{3}}{2}$



$2\theta = \frac{\pi}{6} + 2K\pi$ and $2\theta = \frac{11\pi}{6} + 2K\pi$

$\theta = \frac{\pi}{12} + K\pi$ and $\theta = \frac{11\pi}{12} + K\pi$

$\theta = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

Solve the equation. Give a general formula for all the solutions.

3) $\tan \theta = -1$

$\theta = \frac{3\pi}{4} + K\pi$, and $\frac{7\pi}{4} + K\pi$

Solve the equation on the interval $[0, 2\pi)$.

4) Suppose $f(x) = \cos \theta - 1$. Solve $f(x) = 0$.

$\cos\theta - 1 = 0 \Longrightarrow \cos\theta = 1$

$\theta = 0$

Solve the equation on the interval $0 \le \theta < 2\pi$.

5) $2 \sin^2 \theta = \sin \theta$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0 \longrightarrow \sin\theta = 0$ $\sin\theta = \frac{1}{2}$

$\theta = 0, \pi$ & $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

6) $\sin^2 \theta + \sin \theta = 0$

$\sin\theta(\sin\theta + 1) = 0$

$\sin\theta = 0$ $\sin\theta = -1$

$\theta = 0, \pi$ and $\theta = \frac{3\pi}{2}$

7) $\cos\theta = -\sin\theta$

$\frac{\cos\theta}{\cos\theta}$ $\frac{}{\cos\theta}$

$\boxed{\theta = \frac{3\pi}{4}}$ and $\boxed{\frac{7\pi}{4}}$

$\tan\theta = -1$

8) $\sec^2\theta - 2 = \tan^2\theta$

Recall $\tan^2\theta + 1 = \sec^2\theta$

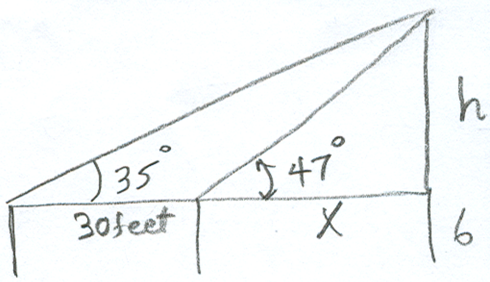
$\Downarrow$

$\tan^2\theta + 1 - 2 = \tan^2\theta$

$-1 = 0$ $\boxed{\text{No Solution}}$

Solve the problem.

9) John (whose line of sight is 6 ft above horizontal) is trying to estimate the height of a tall oak tree. He first measures the angle of elevation from where he is standing as 35°. He walks 30 feet closer to the tree and finds that the angle of elevation has increased by 12°. Estimate the height of the tree rounded to the nearest whole number.



$\tan 35° = \frac{h}{X+30}$ $\qquad$ $\tan 47° = \frac{h}{X}$

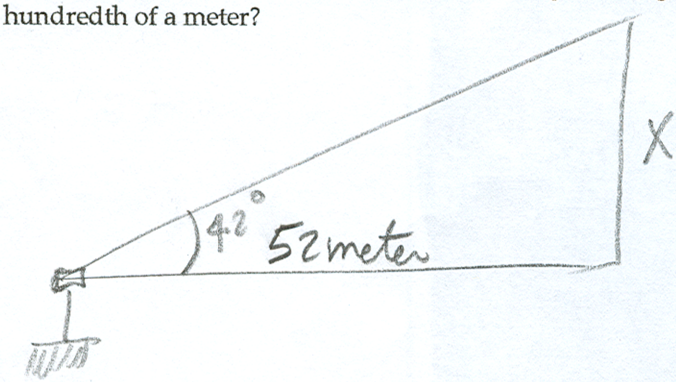
and $X + 30 = \frac{h}{\tan 35°}$ $\qquad$ $h = X \tan 47°$

$X = \frac{h}{\tan 35°} - 30$ $\Longleftrightarrow$ These are equal $\qquad$ $X = \frac{h}{\tan 47°}$

$\frac{h}{\tan 35°} - 30 = \frac{h}{\tan 47°}$ $\Rightarrow$ By Graphing $Y_1 = \frac{X}{\tan 35°} - 30$, $Y_2 = \frac{X}{\tan 47°}$ $\Longrightarrow X = 60.5$

height $= 60.5 + 6$ $\boxed{= 66.5 \text{ feet} \approx 67\text{ft}}$

10) A photographer points a camera at a window in a nearby building forming an angle of 42° with the camera platform. If the camera is 52 m from the building, how high above the platform is the window, to the nearest hundredth of a meter?



$\tan 42° = \frac{X}{52}$

$X = 52 \tan 42°$

$\boxed{X = 46.82 \text{ meters}}$