Name: _Solution_   **Total Possible Points = 150 Plus 10 pts Extra Credits**

1) City B is located at 75 miles west and 25 miles north of city A. City C is located at 100 miles east and 125 miles south of city A. Find the distance between city B and city C. You can choose city A as the origin of the rectangular coordinate system. Write your answer rounded to two decimal places, if necessary.   (5 Points)

$$B = (-75, 25) \qquad C = (100, -125)$$

$$BC = \sqrt{(100 - -75)^2 + (-125 - 25)^2} = 230.49 \text{ miles}$$

2) Find the standard form of equation of a circle with endpoints of a diameter at   (5 Points)

(5 , 9) and (-1, 3)

$$Center = \left(\frac{5 + -1}{2}, \frac{9 + 3}{2}\right) = (2, 6)$$

$$radius = \sqrt{(5-2)^2 + (9-6)^2} = \sqrt{9+9} = \sqrt{18}$$

$$(x - 2)^2 + (y - 6)^2 = 18$$

3) If (a, 2a) is a point on the graph of $3x - 2y = 17$, what is a?   (5 Points)

$$3(a) - 2(2a) = 17$$
$$3a - 4a = 17$$
$$-a = 17 \implies a = -17$$

4) Find the x and y intercepts of the following $5x^2 + 6x - 8 - y = 0$ (5 Points)

To find x intercept let y = 0
$$5x^2 + 6x - 8 = 0$$
$$(5x - 4)(x + 2)$$
$$x = \frac{4}{5}, \quad x = -2$$

$$\left(\frac{4}{5}, 0\right) \quad x \text{ intercepts}$$
$$(-2, 0)$$

to find y intercept let x = 0
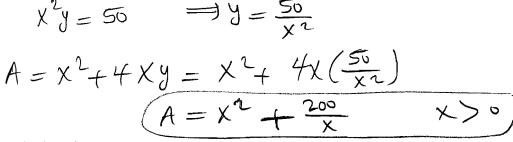$$5(0)^2 + 6(0) - 8 - y = 0$$
$$y = -8$$
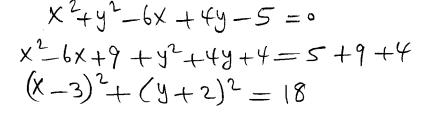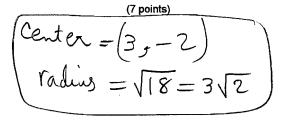
$$y \text{ intercept} = (0, 8)$$

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5) An ⟨open box⟩ with a square base is required to have a volume of 50 cubic feet. Express the amount A of material used to make such a box as a function of the length x of a side of the base.
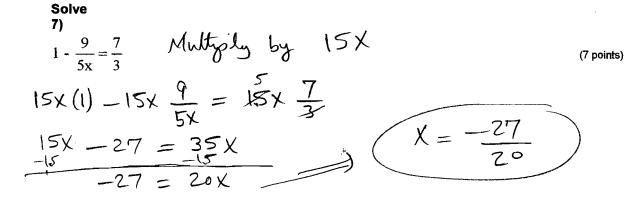
(5 Points)

$$x^2 y = 50 \implies y = \frac{50}{x^2}$$

$$A = x^2 + 4xy = x^2 + 4x\left(\frac{50}{x^2}\right)$$

$$\boxed{A = x^2 + \frac{200}{x} \qquad x > 0}$$

6) Find the center and radius of the circle with the given equation
$$4x^2 + 4y^2 - 24x + 16y - 20 = 0 \qquad \text{Divide by } 4$$

(7 points)

$$x^2 + y^2 - 6x + 4y - 5 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 5 + 9 + 4$$

$$(x-3)^2 + (y+2)^2 = 18$$

$$\boxed{\text{Center} = (3, -2)}$$
$$\boxed{\text{radius} = \sqrt{18} = 3\sqrt{2}}$$

Solve

7)

$$1 - \frac{9}{5x} = \frac{7}{3} \qquad \text{Multiply by } 15X$$

(7 points)

$$15x(1) - 15x\frac{9}{5x} = \overset{5}{\cancel{15}}x\frac{7}{\cancel{3}}$$

$$15x - 27 = 35x$$
$$\underline{-15 \qquad \quad -15}$$
$$-27 = 20x$$

$$\boxed{X = \frac{-27}{20}}$$

8) Find the average rate of change for the function $f(x) = 4x^3 - 5x + 2$
between   - 4 to X

x	y
-4	-234
x	$4x^3 - 5x + 2$

$$\text{Avg Rate of Change} = \frac{4x^3 - 5x + 2 - (-234)}{X - (-4)}$$

(7 points)

$$= \boxed{\frac{4x^3 - 5x + 236}{X + 4}}$$

Algebraically Solve:

9) $\sqrt{2x+3} - x + 10 = 10$

(6 points)

$$\sqrt{2x+3} = x$$

$$2x + 3 = x^2$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\boxed{X = 3} \qquad \cancel{\boxed{X = -1}}$$

Extraneous solution

10) David has available 900 yards of fencing and wishes to enclose a rectangular area.
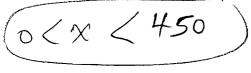
(5 points Each)

a) Express the area A of the rectangle as a function of the width x of the rectangle.
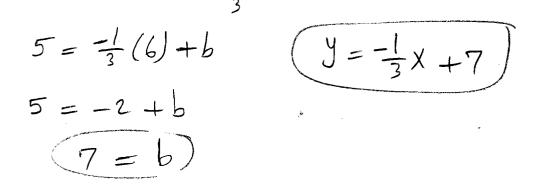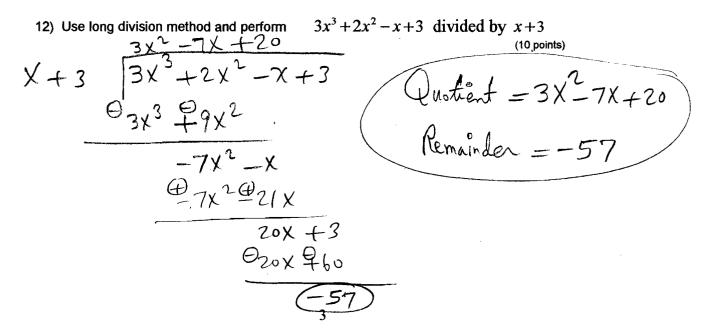
$$2x + 2y = 900 \implies x + y = 450$$

$$\boxed{y = 450 - x}$$

$$A = xy = x(450 - x) = \boxed{450x - x^2}$$

b) What is the domain of A?

$$\boxed{0 < x < 450}$$

11) Write an equation of the line passing through the point (6 , 5) and perpendicular to the line y = 3x – 5. (10 points)

$$m = -\frac{1}{3}$$

$$5 = -\frac{1}{3}(6) + b$$

$$\boxed{y = -\frac{1}{3}x + 7}$$

$$5 = -2 + b$$

$$\boxed{7 = b}$$

12) Use long division method and perform $3x^3 + 2x^2 - x + 3$ divided by $x + 3$

(10 points)

$$
\begin{array}{r}
3x^2 - 7x + 20 \\
x+3 \overline{\smash{\big)}\ 3x^3 + 2x^2 - x + 3} \\
\ominus 3x^3 \oplus 9x^2 \\
\hline
-7x^2 - x \\
\oplus 7x^2 \oplus 21x \\
\hline
20x + 3 \\
\ominus 20x \oplus 60 \\
\hline
\boxed{-57}
\end{array}
$$

$$\boxed{\text{Quotient} = 3x^2 - 7x + 20}$$
$$\boxed{\text{Remainder} = -57}$$

13) Each month a gas station sells x gallons of gas at $2.99 per gallon. The cost to the owner of the gas station for each gallon of gas is $1.99, and the monthly fixed cost for running the gas station is $27000. (10 points)

a) Find the cost function. (Hint: Cost = Variable Cost + Fixed Cost)

$$\text{Cost} = 1.99X + 27000$$

b) Find the revenue function. (Hint: Revenue = Price * Quantity)

$$R = 2.99X$$

c) Write an equation that relates the monthly profit, in dollars, to the number of gallons of gasoline sold. (Hint: Profit = Revenue − Cost)

$$P = 2.99X - (1.99X + 27000) = 1X - 27000$$

d) If the monthly profit is $113000, find the number of gallons of gas that are sold in that month.

$$113000 = 1X - 27000$$
$$140000 = X$$
$$\text{Gallons}$$

14) A wire of length 10x is bent into the shape of a circle. (10 points)

a) Express the circumference of the circle as a function of x.

$$C(X) = 10X$$

$$2\pi r = 10X \implies r = \frac{10X}{2\pi} = \frac{5X}{\pi}$$

b) Express the area of the circle as a function of x.

$$A = \pi r^2 = \pi \left(\frac{5X}{\pi}\right)^2 = \frac{25\pi X^2}{\pi^2} = \frac{25X^2}{\pi}$$

15) Find the value of $\dfrac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = 4x^2 - 5$ (Clearly state each of the steps of the process.) (10 points)

$$f(x+h) = 4(x+h)^2 - 5 = 4(x^2 + 2xh + h^2) - 5$$

$$f(x+h) = 4x^2 + 8xh + 4h^2 - 5$$

$$f(x+h) - f(x) = 4x^2 + 8xh + 4h^2 - 5 - (4x^2 - 5) = h(8x+4)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(8x+4h)}{h} = 8x+4h$$

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16) Given $f(x) = -4x^2 + 5x + 35$. Find x such that $f(x) = 15$ (5 points)

$-4x^2 + 5x + 35 = 15$

$-4x^2 + 5x + 20 = 0$

$4x^2 - 5x - 20 = 0$

$x = \dfrac{-(-5) \pm \sqrt{25 - 4(4)(-20)}}{8}$

$x = 2.95$
$x = -1.70$

17) Give the domain of the function. (10 points)

a) $f(x) = 3x^2 + \dfrac{2}{x-7} + 5$

Domain is all Reals except 7

b) $f(x) = \sqrt{-5x + 10}$

$-5x + 10 \geq 0$

$-5x \geq -10$

$x \leq 2$

c) $f(x) = \dfrac{x+7}{x^2 + x - 42} = \dfrac{x+7}{(x+7)(x-6)}$

Domain is all Reals except -7 and 6

d) $g(x) = \dfrac{x}{\sqrt{5 - 2x}}$

$5 - 2x > 0$

$-2x > -5$

$x < \dfrac{5}{2}$

18) Use Quadratic formula to solve the following: $4x^2 + 12x = -2$ (6 points)

$4x^2 + 12x + 2 = 0$

$x = \dfrac{-12 \pm \sqrt{(12)^2 - 4(4)(2)}}{8} = \dfrac{-12 \pm \sqrt{112}}{8}$

$= \dfrac{-12 \pm 4\sqrt{7}}{8} = \dfrac{-3}{2} \pm \dfrac{\sqrt{7}}{2}$

$x = -0.177$
$x = -2.823$

19) Let P = (x , y) be a point on the graph of $y = 2x^2 - 8$ $(0,0)$ $(x, 2x^2 - 8)$ (10 points)

a) Express the distance d from P to the origin as a function of x.

$$d = \sqrt{(x-0)^2 + (2x^2 - 8 - 0)^2} = \sqrt{x^2 + (2x^2 - 8)^2}$$

b) What is d if x = 0? $d(0) = \sqrt{0^2 + (-8)^2} = 8$

c) What is d if x = 1?

$$d(1) = \sqrt{1 + (-6)^2} = \sqrt{37}$$

d) For what values of x is d smallest?

$$x = \pm 1.97$$

20) The graph of $y = f(x)$ is given below; Sketch a graph of $y = -[f(x-2) - 3]$ (7 points)

* Shift Right 2
* Down 3
* Reflect About X-axis

X	y
-6	0
-3	-3
0	0
3	3
6	0

New:

X	y
-4	3
-1	6
+2	3
5	0
8	3

21)   **Extra Credit (10 points)**
Two cars are approaching an intersection. One is 2 miles north of the intersection and is moving at a constant speed of 50 miles per hour. At the same time, the other car is 3 miles west of the intersection and is moving at a constant speed of 35 miles per hour.

a)   Express the distance d between the cars as a function of time t.

$2 - 50t$

$-3 + 35t$

$$d = \sqrt{(2 - 50t)^2 + (-3 + 35t)^2}$$

b)   At time t = 1 Hour, what is the distance between the cars?

$$d = \sqrt{(2 - 50)^2 + (-3 + 35)^2} = 57.69 \text{ miles}$$

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