MA 180 Professor Fred Katiraie Test IV Form A (Spring 2007)

Name: Total Possible Points = 150 Points

$$A(t) = A_0 e^{kt} \qquad u(t) = T + (u_0 - T)e^{kt} \qquad P(t) = \frac{c}{1 + ae^{-bt}}$$

1) Find the value of $\dfrac{f(x+h) - f(x)}{h}$ for the function $f(x) = 2x^2 - 2x + 3$

(Assume h is not zero, Clearly state each of the steps of the process.)

(10 points)

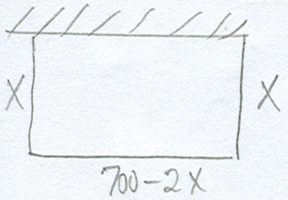
$f(x+h) = 2(x+h)^2 - 2(x+h) + 3 = 2(x^2 + 2xh + h^2) - 2x - 2h + 3$

$= 2x^2 + 4xh + 2h^2 - 2x - 2h + 3$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{2x} - 2h + \cancel{3} - \cancel{2x^2} + \cancel{2x} - \cancel{3}}{h}$

$= \dfrac{h(4x + 2h - 2)}{h} = \boxed{4x + 2h - 2}$

2) A developer wants to enclose a rectangular grassy lot that borders a city – street for parking. If the developer has 700 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed? (10 points)



$A = X(700 - 2X) = 700X - 2X^2 = -2X^2 + 700X$

$X = \dfrac{-b}{2a} = \dfrac{-700}{2(-2)} = 175$ feet

$\boxed{A = (175)(350) = 61250 \text{ ft}^2}$

3) A thermometer reading 80 degrees F is placed inside a cold storage room with a constant temperature of 32 degrees F. If the thermometer reads 75 degrees F in 15 minutes, how long will it take for the thermometer to reach 60 degrees F? Assume the cooling follows Newton's Law of Cooling (and Round your answer to the nearest whole minute) (10 points)

$75 = 32 + (80 - 32)e^{15K}$

$43 = 48e^{15K}$

$K = -0.0073$

$60 = 32 + (80 - 32)e^{-0.0073t}$

$28 = 48e^{-0.0073t}$

$\boxed{t = 73.84 \approx 74 \text{ minutes}}$

4) Solve the following equations algebraically. (10 points)
(Must Show All the Appropriate Steps)

a) $\log x + \log(x+15) = 2$

$\log_{10} x(x+15) = 2 \Longrightarrow x^2 + 15x = 100$

$x^2 + 15x - 100 = 0$

$(x+20)(x-5) = 0$

$\cancel{x = -20}$ $\boxed{x = 5}$

b) $\ln(3+x) - \ln(x-4) = \ln(2)$

$\ln\left(\frac{3+x}{x-4}\right) = \ln 2$

$\frac{3+x}{x-4} = \frac{2}{1} \Longrightarrow 2x - 8 = 3 + x \Longrightarrow \boxed{x = 11}$

c) $\log_3(x) - \log_3(x-6) = 4$

$\log_3\left(\frac{x}{x-6}\right) = 4 \Longrightarrow \frac{x}{x-6} = \frac{81}{1} \Longrightarrow 81x - 486 = x$

$80x = 486$

$\boxed{x = 6.075}$

5) A fossilized leaf contains 20% of its normal amount of carbon-14. How old is the fossil (to the nearest year)? (Use 5600 years as the half – life of carbon 14)

(10 points)

$\frac{1}{2} = 1e^{5600K}$

$K = \frac{\ln(\frac{1}{2})}{5600} = -1.238 \times 10^{-4}$

$A = A_0 e^{Kt}$

$\downarrow$

$0.20 = 1.00\, e^{-1.238 \times 10^{-4} t} \Longrightarrow \ln(0.20) = -1.238 \times 10^{-4} t$

$t = \frac{\ln(0.20)}{-1.238 \times 10^{-4}} = 13002.8$

$\approx$ 13003 years

$v = r\omega = r\left(\frac{\theta}{t}\right)$

6) An object is traveling around a circle with a radius of 20 meters. If in 10 seconds a central angle of $\frac{1}{5}$ radian is swept out, what is the linear speed of the object? (5 points)

$v = (20)\left(\frac{\frac{1}{5}}{10}\right) = 20\left(\frac{1}{50}\right) = \boxed{\frac{2}{5}\ \frac{\text{meters}}{\text{sec}}}$

7) An irrigation sprinkler in a field of lettuce sprays water over a distance of 40 feet as it rotates through an angle of 155^0. What area of the field receives water? Round the answer to two decimal places. (5 points)

$155^\circ * \frac{\pi}{180^\circ} = \frac{155\pi}{180}$ radian

$A = \frac{1}{2} r^2\theta = \frac{1}{2}(40)^2\left(\frac{155\pi}{180}\right)$

$= \boxed{2164.21\ ft^2}$

8) Salt Lake City, Utah, is due north of Flagstaff, Arizona. Find the distance between Salt Lake City ($40^0 45'$ north latitude) and Flagstaff ($35^0 16'$ north latitude). Assume that the radius of the Earth is 3960 miles. Round to nearest whole mile. (5 points)

$S = r\theta$ and $\theta = 40^\circ 45' - 35^\circ 16' = 5.48^\circ * \frac{\pi}{180^\circ} = 0.03\pi$ radians

$S = (3960 \text{ miles})(0.030\pi) = 120.6\pi = \boxed{379 \text{ miles}}$

9) TRUE/FALSE. Circle one. (2.5 pts each)

a) True False $\sin(\sin^{-1}\pi) = \pi$

b) True False The period of $y = \cos\left(\frac{\pi}{3}x\right)$ is greater than the period of $y = \tan\left(\frac{\pi}{3}x\right)$

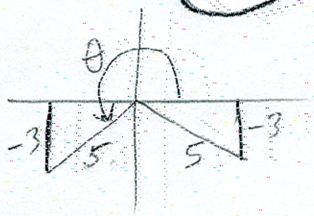
$\frac{2\pi}{\frac{\pi}{3}} = \frac{2\pi}{1}\cdot\frac{3}{\pi} = 6$

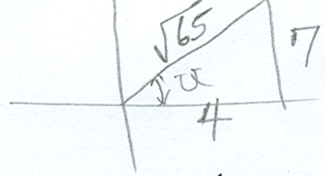
$\frac{\pi}{\frac{\pi}{3}} = \frac{\pi}{1}\cdot\frac{3}{\pi} =$

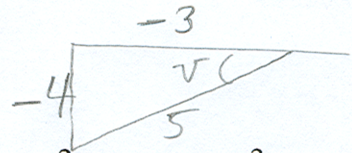
$\cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta$

c) True False If $0 < \theta < \frac{\pi}{2}$, *then* $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\pi$

d) True False If $\sin\theta = -\frac{3}{5}$, *then the* $\csc(-\theta) = -\frac{5}{3}$







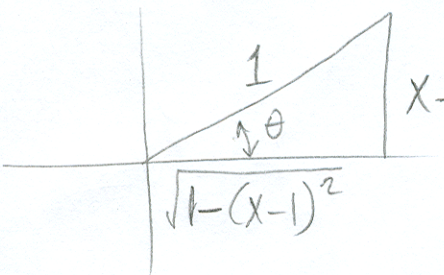
10) Given: $\cot u = \frac{4}{7}$ for $0 < u < \frac{\pi}{2}$ and $\cos v = -\frac{3}{5}$ for $\pi < v < \frac{3\pi}{2}$ (9 pts)

Find the **exact** value of $\tan(u+v)$ (Hint: Draw triangles in appropriate quadrants)

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\frac{7}{4} + \frac{4}{3}}{1 - \frac{7}{\cancel{4}} \cdot \frac{\cancel{-4}}{-3}} = \boxed{\frac{-37}{16}}$$

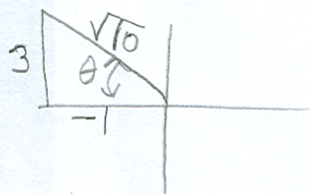
11) Find an algebraic expression for: $\sec(\sin^{-1}(x-1))$

(Hint: Draw a triangle in the first quadrant) (8 pts)



$$\sec(\sin^{-1}(X-1)) = \frac{1}{\sqrt{1-(X-1)^2}} = \frac{\sqrt{1-(X-1)^2}}{1-(X-1)^2}$$

12) Given: $\tan\theta = -3$ *and* $\cos\theta < 0$



Find the **exact value** for $\sin 2\theta$. (8 pts)

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{-1}{\sqrt{10}}\right) = \frac{-6}{10} = \boxed{\frac{-3}{5}}$$

13) Algebraically solve $2\cos(2x) + 1 = 0$ over the interval $[0, 2\pi)$: (8 pts)

$$\cos(2X) = -\frac{1}{2}$$

$$2X = \frac{2\pi}{3} + 2K\pi \qquad 2X = \frac{4\pi}{3} + 2K\pi$$

$$X = \frac{\pi}{3} + K\pi \qquad X = \frac{2\pi}{3} + K\pi$$

$$X = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

14) Solve algebraically over the interval $[0,2\pi)$: $4\sin^2(x)-3=0$ (10 pts)

$$\sin^2(x) = \frac{3}{4} \Longrightarrow \sin(x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$$

$\sin x = \frac{\sqrt{3}}{2}$ OR $\sin x = -\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ OR $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

15) Verify the identity (change the left hand side to look like the right side.). (8 pts)

$$\cos\left(\frac{\pi}{2}-x\right)\csc x = 2$$

$$\left(\cos\left(\frac{\pi}{2}\right)\cos x + \sin\frac{\pi}{2}\sin x\right)\frac{1}{\sin x} \stackrel{?}{=} 2$$

0

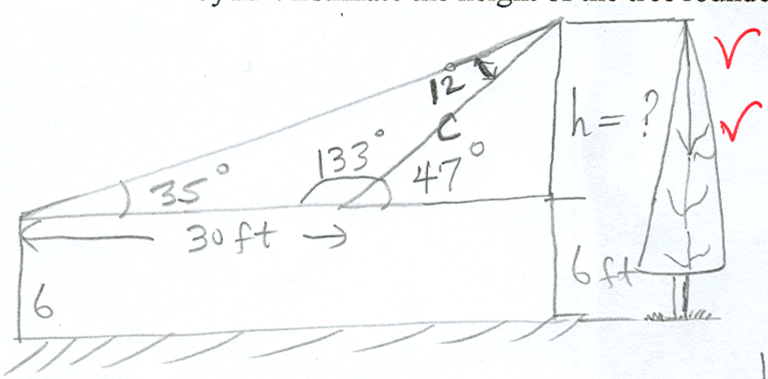
$$\sin x\left(\frac{1}{\sin x}\right) \neq 2$$

$$1 \neq 2$$

~~Identity~~ Does Not Exist

Not Equal

16) John (whose line of sight is 6 ft above horizontal) is trying to estimate the height of a tall oak tree. He first measures the angle of elevation from where he is standing as 35^0. He walks 30 feet closer to the tree and finds that the angle of elevation has increased by 12^0. Estimate the height of the tree rounded to the nearest whole number. (8 pts)



$$\frac{\sin 12^\circ}{30} = \frac{\sin 35^\circ}{c} \Rightarrow c = 82.76$$

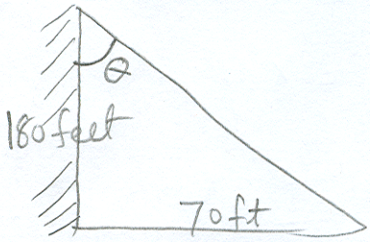
$$\sin 47^\circ = \frac{h}{c}$$

$$h = c\sin 47^\circ = 82.76\sin 47^\circ$$
$$= 60.5 \text{ feet}$$

Height of tree $= 60.5 + 6 = 66.5$

$\boxed{\approx 67 \text{ feet}}$

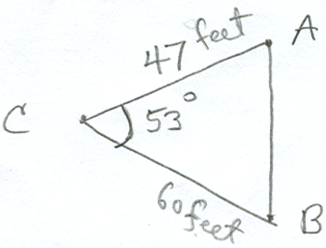
17) A building 180 feet tall casts a 70 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.) (8 pts)



$$\tan\theta = \frac{70}{180}$$

$$\theta = \tan^{-1}\left(\frac{70}{180}\right) = 21.25^\circ$$

18) Two points A and B are on opposite sides of a building.
A surveyor selects a third point C to place a transit. Point C is 47 feet from point A and 60 feet from point B. The angle ACB is 53^0. How far apart are points A and B? (8 pts)

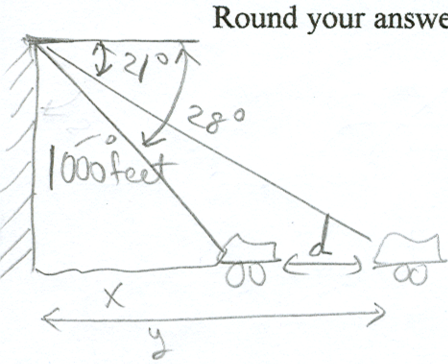


$$(AB)^2 = 47^2 + 60^2 - 2(47)(60)\cos 53^\circ$$

$$AB = 49.14 \text{ feet}$$

(8 POINTS EXTRA CREDIT)

From the edge of a 1000-foot cliff, the angles of depression to two cars in the valley below are 21^0 and 28^0. How far apart are the cars? (8 pts)
Round your answers to the nearest 0.1 ft.



$$\tan 62^\circ = \frac{x}{1000} \quad \text{and} \quad \tan 69^\circ = \frac{y}{1000}$$

$$x = 1000\tan 62^\circ = 1880.73 \text{ feet}$$

$$y = 1000\tan 69^\circ = 2605.09 \text{ feet}$$

$$\text{Distance} = 2605.09 - 1880.73 = 724.4 \text{ feet}$$