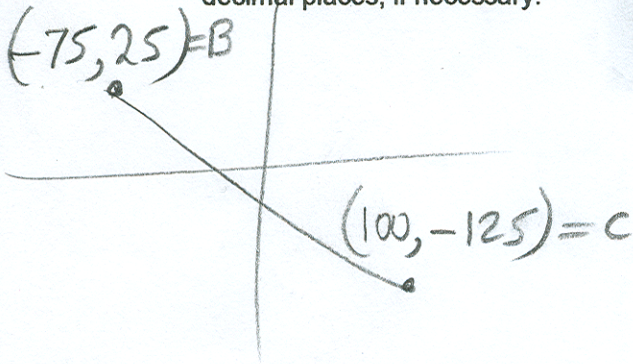


Name: Solution

Total Possible Points = 150 Plus 10 pts Extra Credits

- 1) City B is located at 75 miles west and 25 miles north of city A. City C is located at 100 miles east and 125 miles south of city A. Find the distance between city B and city C. You can choose city A as the origin of the rectangular coordinate system. Write your answer rounded to two decimal places, if necessary. (5 Points)



$$BC = \sqrt{(100 - (-75))^2 + (-125 - 25)^2}$$

$$= \sqrt{175^2 + (-150)^2} = \underline{230.49 \text{ miles}}$$

- 2) Find the standard form of equation of a circle with endpoints of a diameter at (5, 9) and (-1, 3) (5 Points)

(5, 9) and (-1, 3)

$$\text{center} = \left(\frac{5 + (-1)}{2}, \frac{9 + 3}{2} \right) = (2, 6)$$

$$\text{radius} = \sqrt{(5 - 2)^2 + (9 - 6)^2} = \sqrt{9 + 9} = \sqrt{18}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\underline{(x - 2)^2 + (y - 6)^2 = 18}$$

- 3) If (a, 10) is a point on the graph of $3x - 2y = 17$, what is a? (5 Points)

$$3a - 20 = 17$$

$$3a = 37$$

$$\underline{a = \frac{37}{3}}$$

- 4) Find the x and y intercepts of the following $5x^2 + 6x - 8 - y = 0$ (5 Points)

x-intercepts let $y = 0$

$$5x^2 + 6x - 8 = 0$$

$$(5x - 4)(x + 2) = 0$$

$$\underline{x = \frac{4}{5}}$$

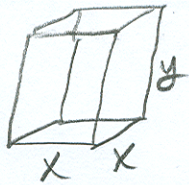
$$\underline{x = -2}$$

$$\underline{\left(\frac{4}{5}, 0 \right) \text{ and } (-2, 0)}$$

To find y-intercept let $x = 0$

$$y = -8$$

$$\underline{\text{y-intercept} = (0, -8)}$$



5) An open box with a square base is required to have a volume of 27 cubic feet. Express the amount A of material used to make such a box as a function of the length x of a side of the base.

(5 Points)

$$V = x^2 y \implies 27 = x^2 y \implies y = \frac{27}{x^2}$$

Area = Bottom + 4 sides

$$A = x^2 + 4\left(x \cdot \frac{27}{x^2}\right) = x^2 + \frac{108}{x}; x > 0$$

6) Find the center and radius of the circle with the given equation

$$3x^2 + 3y^2 - 24x + 36y - 21 = 0 \quad \text{Divide by 3}$$

(7 points)

$$x^2 + y^2 - 8x + 12y = 7$$

$$(x^2 - 8x + 16) + (y^2 + 12y + 36) = 7 + 16 + 36$$

$$(x-4)^2 + (y+6)^2 = 59$$

Center = $(4, -6)$
 Radius = $\sqrt{59}$

Solve

7)

$$4 - \frac{9}{7} = \frac{7}{3x}$$

Multiply both sides by $21x$

(7 points)

$$21x(4) - 21x \cdot \frac{9}{7} = 21x \cdot \frac{7}{3x}$$

$$84x - 27x = 49$$

$$57x = 49$$

$x = \frac{49}{57}$

8) Find the average rate of change for the function $f(x) = 4x^3 - 5x + 2$

between -4 to X

x	$f(x)$
-4	-234
x	$4x^3 - 5x + 2$

$$\text{Avg Rate of Change} = \frac{4x^3 - 5x + 236}{x - (-4)}$$

$$= \frac{4x^3 - 5x + 236}{x + 4}$$

Algebraically Solve:

9) $\sqrt{2x+3} - x + 1 = 1$

(6 points)

$$\sqrt{2x+3} = x$$

$$2x+3 = x^2$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$x = 3$ ~~$x = -1$~~ Extraneous solution



10) David has available 400 yards of fencing and wishes to enclose a rectangular area. (5 points Each)

a) Express the area A of the rectangle as a function of the width x of the rectangle.

$$2x + 2y = 400 \implies x + y = 200$$

$$y = 200 - x$$

$$A = x \cdot y = x(200 - x) = \boxed{200x - x^2}$$

b) What is the domain of A ?

$$\boxed{0 < x < 200}$$

11) Write an equation of the line passing through the point $(6, 5)$ and perpendicular to the line $y = 3x - 5$. (10 points)

$$m_{\perp} = -\frac{1}{3}$$

$$5 = -\frac{1}{3}(6) + b$$

$$5 = -2 + b$$

$$\boxed{7 = b}$$

$$\boxed{y = -\frac{1}{3}x + 7}$$

12) Use long division method and perform $3x^3 + 2x^2 - x + 3$ divided by $x - 3$ (10 points)

$$\begin{array}{r}
 3x^2 + 11x + 32 \\
 \hline
 x - 3 \overline{) 3x^3 + 2x^2 - x + 3} \\
 \underline{\ominus 3x^3 + 9x^2} \\
 11x^2 - x + 3 \\
 \underline{\ominus 11x^2 + 33x} \\
 32x + 3 \\
 \underline{\ominus 32x + 96} \\
 99
 \end{array}$$

Quotient $3x^2 + 11x + 32$

Remainder 99

13) Each month a gas station sells x gallons of gas at \$2.19 per gallon. The cost to the owner of the gas station for each gallon of gas is \$1.99, and the monthly fixed cost for running the gas station is \$37000. (10 points)

a) Find the cost function. (Hint: Cost = Variable Cost + Fixed Cost)

$$C(x) = 1.99x + 37000$$

b) Find the revenue function. (Hint: Revenue = Price * Quantity)

$$R(x) = 2.19x$$

c) Write an equation that relates the monthly profit, in dollars, to the number of gallons of gasoline sold. (Hint: Profit = Revenue - Cost)

$$P = 2.19x - (1.99x + 37000) = 0.2x - 37000$$

d) If the monthly profit is \$113000, find the number of gallons of gas that are sold in that month.

$$113000 = 0.2(x) - 37000$$

$$x = 750,000 \text{ Gallons of Gas}$$

14) A wire of length x is bent into the shape of a circle. (10 points)

a) Express the circumference of the circle as a function of x .

$$C(x) = x \quad \text{but} \quad x = 2\pi r$$

$$r = \frac{x}{2\pi}$$

b) Express the area of the circle as a function of x .

$$A = \pi r^2 = \pi \left(\frac{x}{2\pi}\right)^2 = \pi \frac{x^2}{4\pi^2} = \frac{x^2}{4\pi}$$

15) Find the value of $\frac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = x^2 - 2x$

(Clearly state each of the steps of the process.)

$$f(x+h) = (x+h)^2 - 2(x+h) = x^2 + 2xh + h^2 - 2x - 2h \quad (10 \text{ points})$$

$$f(x+h) - f(x) = x^2 + 2xh + h^2 - 2x - 2h - (x^2 - 2x) = 2xh + h^2 - 2h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(2x+h-2)}{h} = 2x+h-2$$

16) Given $f(x) = -4x^2 + 3x + 15$. Find x such that $f(x) = 15$

(5 points)

$$\begin{aligned} -4x^2 + 3x + 15 &= 15 \\ -4x^2 + 3x &= 0 \\ x(-4x + 3) &= 0 \end{aligned}$$

→ $x = 0$

→ $x = \frac{3}{4}$

17) Give the domain of the function.

(10 points)

a) $f(x) = 3x^2 + \frac{2}{x} + 5$

Domain is all Reals except $x = 0$

b) $f(x) = \sqrt{-x+10}$

$$-x + 10 \geq 0$$

$$10 \geq x$$

c) $f(x) = \frac{x+7}{x^2+13x+42}$

$$f(x) = \frac{x+7}{(x+7)(x+6)}$$

Domain is all Reals except $x = 6$ and $x = -7$

d) $g(x) = \frac{x}{\sqrt{5-x}}$

$$5 - x > 0$$

$$5 > x$$

18) Find the average rate of change for the function over the given interval.

$f(x) = 2x^2 + 3x$ between $x = 6$ and $x = 8$

(6 points)

x	y
6	90
8	152

$$\text{Avg Rate of change} = \frac{152 - 90}{8 - 6} = 31$$

$$(0,0) \text{ to } (x, x^2 - 8)$$

19) Let $P = (x, y)$ be a point on the graph of $y = x^2 - 8$

(10 points)

a) Express the distance d from P to the origin as a function of x .

$$d = \sqrt{(x-0)^2 + (x^2-8-0)^2} = \sqrt{x^2 + (x^2-8)^2}$$

b) What is d if $x = 0$?

$$d = \sqrt{0 + (0-8)^2} = 8$$

c) What is d if $x = 1$?

$$d = \sqrt{1 + (1-8)^2} = \sqrt{50}$$

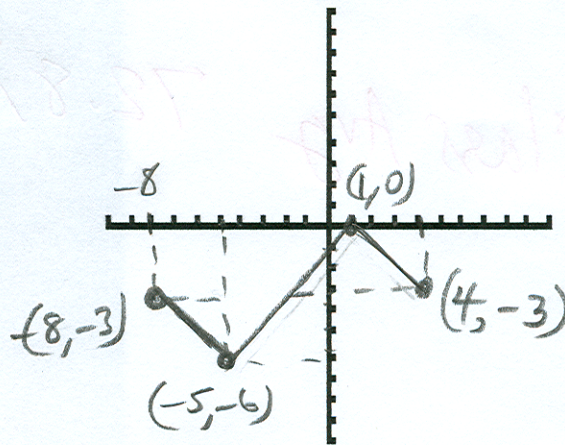
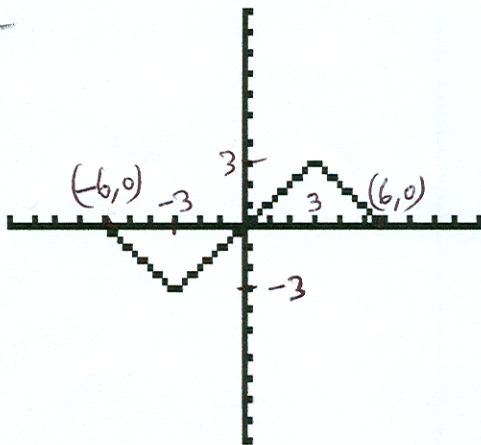
d) For what values of x is d smallest?

$$x \approx \pm 2.77$$

20) The graph of $y = f(x)$ is given below;
(7 points)

Sketch a graph of $y = f(x+2) - 3$

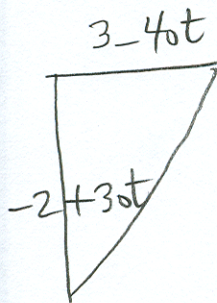
x	y
-6	0
-3	-3
3	3
6	0



x	f(x+2) - 3
-8	-3
-5	-6
1	0
4	-3

21) **Extra Credit (10 points)**

Two cars are approaching an intersection. One is 2 miles south of the intersection and is moving at a constant speed of 30 miles per hour. At the same time, the other car is 3 miles east of the intersection and is moving at a constant speed of 40 miles per hour.



a) Express the distance d between the cars as a function of time t .

$$d = \sqrt{(3-40t)^2 + (-2+30t)^2}$$

b) At time $t = 1$ Hour, what is the distance between the cars?

$$d = \sqrt{(3-40)^2 + (-2+30)^2} = 46.4 \text{ miles}$$