

Name:

Solution

Total Possible Points = 150 plus 10 pts extra credit

- 1) If
- $(a, 10)$
- is a point on the graph of
- $3x - 2y = 17$
- , what is
- a
- ?

(5 Points)

$$3a - 20 = 17$$

$$3a = 37$$

$$a = \frac{37}{3}$$

- 2) Find the
- x
- and
- y
- intercepts of the following
- $5x^2 + 6x - 8 - y = 0$

(5 Points)

To find x intercept let $y=0$

$$5x^2 + 6x - 8 = 0$$

$$(5x - 4)(x + 2) = 0$$

$$x = \frac{4}{5}$$

$$x = -2$$

$$\left(\frac{4}{5}, 0\right)$$

$$(-2, 0)$$

To find y intercept let $x=0$

$$y = -8$$

$$(0, -8) = y \text{ intercept}$$

- 3) An open box with a square base is required to have a volume of 37 cubic feet. Express the amount
- A
- of material used to make such a box as a function of the length
- x
- of a side of the base.

(7 Points)

$$37 = x^2 y \implies y = \frac{37}{x^2}$$

$$A = x^2 + 4xy = x^2 + 4x\left(\frac{37}{x^2}\right) = x^2 + \frac{148}{x}; x > 0$$

Solve

4)

$$4 - \frac{9}{7} = \frac{7}{3x}$$

Multiply both sides by $21x$

(5 points)

$$21x \cdot 4 - 21x \cdot \frac{9}{7} = 21x \cdot \frac{7}{3x}$$

$$84x - 27x = 49$$

$$57x = 49$$

$$x = \frac{49}{57}$$

- 5) City B is located at 75 miles west and 25 miles north of city A. City C is located at 100 miles east and 125 miles south of city A. Find the distance between city B and city C. You can choose city A as the origin of the rectangular coordinate system. Write your answer rounded to two decimal places, if necessary. (5 Points)

$$B = (-75, 25)$$

$$C = (100, -125)$$

$$BC = \sqrt{(100 - (-75))^2 + (-125 - 25)^2}$$

$$= 230.49 \text{ miles}$$

- 6) Find the standard form of equation of a circle with endpoints of a diameter at (5, 9) and (-3, -3) (5 Points)

(5, 9) and (-3, -3)

$$\text{Center} = \left(\frac{5 + (-3)}{2}, \frac{9 + (-3)}{2} \right) = (1, 3) \quad (-3, -3)$$

$$\text{radius} = \sqrt{(-3 - 1)^2 + (-3 - 3)^2} = \sqrt{16 + 36} = \sqrt{52}$$

$$(x - 1)^2 + (y - 3)^2 = (\sqrt{52})^2 \Rightarrow (x - 1)^2 + (y - 3)^2 = 52$$

- 7) Find the center and radius of the circle with the given equation

$$3x^2 + 3y^2 - 24x + 36y - 21 = 0$$

$$x^2 + y^2 - 8x + 12y - 7 = 0$$

(7 points)

$$x^2 - 8x + y^2 + 12y = 7$$

$$(x^2 - 8x + 16) + (y^2 + 12y + 36) = 7 + 16 + 36$$

$$(x - 4)^2 + (y + 6)^2 = 59$$

$$\text{Center} = (4, -6)$$

$$\text{Radius} = \sqrt{59}$$

- 8) Find the average rate of change for the function $f(x) = 4x^3 - 5x + 2$ between -4 to X

(7 points)

X	f(x)
-4	-234
X	$4X^3 - 5X + 2$

$$\text{Avg rate of change} = \frac{4X^3 - 5X + 2 - (-234)}{X - (-4)}$$

$$= \frac{4X^3 - 5X + 236}{X + 4}$$

Algebraically Solve:

9) $\sqrt{2x+3} - x + 1 = 1$ (6 points)

$$\frac{\sqrt{2x+3} - x + 1}{+x - 1 - 1 + x} = 1$$

$\sqrt{2x+3} = x$ Also Square Both sides

$2x+3 = x^2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x=3$ ~~$x=-1$~~ Extraneous solution

10) David has available 600 yards of fencing and wishes to enclose a rectangular area. (5 points Each)

a) Express the area A of the rectangle as a function of the width x of the rectangle.

$2x + 2y = 600 \Rightarrow x + y = 300 \Rightarrow y = 300 - x$

$A(x) = x \cdot y = x(300 - x) \Rightarrow A(x) = 300x - x^2$

$A(x) = 300x - x^2$

b) What is the domain of A ?

$0 < x < 300$

11) Each month a gas station sells x gallons of gas at \$2.19 per gallon. The cost to the owner of the gas station for each gallon of gas is \$1.99, and the monthly fixed cost for running the gas station is \$37000. (10 points)

a) Find the cost function. (Hint: Cost = Variable Cost + Fixed Cost)

$C(x) = 1.99x + 37000$

b) Find the revenue function. (Hint: Revenue = Price * Quantity)

$R(x) = 2.19x$

c) Write an equation that relates the monthly profit, in dollars, to the number of gallons of gasoline sold. (Hint: Profit = Revenue - Cost)

$P(x) = 2.19x - (1.99x + 37000) = 0.2x - 37000$

d) If the monthly profit is \$113000, find the number of gallons of gas that are sold in that month.

$113000 = 0.2x - 37000$

$x = 750,000$ Gallons of Gas

12) A wire of length $4X$ is bent into the shape of a circle.

(10 points)

a) Express the circumference of the circle as a function of x .

$$C(x) = 4X$$

b) Express the area of the circle as a function of x .

$$4X = 2\pi r \Rightarrow r = \frac{4X}{2\pi} = \frac{2X}{\pi}$$

$$A = \pi r^2 = \pi \left(\frac{2X}{\pi}\right)^2 = \frac{4X^2\pi}{\pi^2}$$

$$A(x) = \frac{4X^2}{\pi}$$

13) Write an equation of the line passing through the point $(6, 5)$ and perpendicular to the line $y = 3x - 5$.

(10 points)

$$m \perp = -\frac{1}{3}$$

$$5 = -\frac{1}{3}(6) + b$$

$$5 = -2 + b$$

$$\begin{array}{r} +2 \\ 7 = b \end{array}$$

$$y = \frac{1}{3}x + 7$$

14) Use long division method and perform

$3x^3 + 2x^2 - x + 3$ divided by $x - 3$

(10 points)

$$\begin{array}{r} 3x^2 + 11x + 32 \\ x-3 \overline{) 3x^3 + 2x^2 - x + 3} \\ \underline{\ominus 3x^3 + 9x^2} \\ 11x^2 - x + 3 \\ \underline{\ominus 11x^2 + 33x} \\ 32x + 3 \\ \underline{\ominus 32x + 96} \\ 99 \end{array}$$

$$\text{Quotient} = 3x^2 + 11x + 32$$

$$\text{Remainder} = 99$$

15) Find the average rate of change for the function over the given interval.

$f(x) = 2x^2 + 3x$ between $x = 6$ and $x = 8$

(6 points)

x	$f(x)$
6	90
8	152

$$\text{Avg Rate of Change} = \frac{152 - 90}{8 - 6} = 31$$

- 16) Find the value of $\frac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = x^2 - 2x$
(Clearly state each of the steps of the process.)

$$f(x+h) = (x+h)^2 - 2(x+h) = x^2 + 2xh + h^2 - 2x - 2h \quad (10 \text{ points})$$

$$\begin{aligned} f(x+h) - f(x) &= x^2 + 2xh + h^2 - 2x - 2h - (x^2 - 2x) \\ &= x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x = h(2x + h - 2) \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(2x + h - 2)}{h} = \boxed{2x + h - 2}$$

- 17) Given $f(x) = -4x^2 + 3x + 15$. Find x such that $f(x) = 15$ (5 points)

$$15 = -4x^2 + 3x + 15$$

$$x(-4x + 3) = 0$$

$$x = 0 \quad x = \frac{3}{4}$$

- 18) Give the domain of the function. (10 points)

a) $f(x) = 3x^2 + \frac{2}{x} + 5$

Domain is all Reals except $x = 0$

b) $f(x) = \sqrt{-x + 10}$

$$-x + 10 \geq 0$$

$$\boxed{10 \geq x}$$

c) $f(x) = \frac{x+7}{x^2 + 13x + 42}$

$$= \frac{x+7}{(x+7)(x+6)}$$

$$(x+7)(x+6)$$

Domain is all Reals except $x = 7$
and $x = 6$

d) $g(x) = \frac{x}{\sqrt{5-x}}$

$$5 - x > 0$$

$$\boxed{5 > x}$$

$$(0, 0) \quad (x, x^2 - 1)$$

19) Let $P = (x, y)$ be a point on the graph of $y = x^2 - 1$

(10 points)

a) Express the distance d from P to the origin as a function of x .

$$d = \sqrt{(x-0)^2 + (x^2-1-0)^2} = \sqrt{x^2 + (x^2-1)^2}$$

b) What is d if $x = 0$?

$$d = 1$$

c) What is d if $x = 1$?

$$d = 1$$

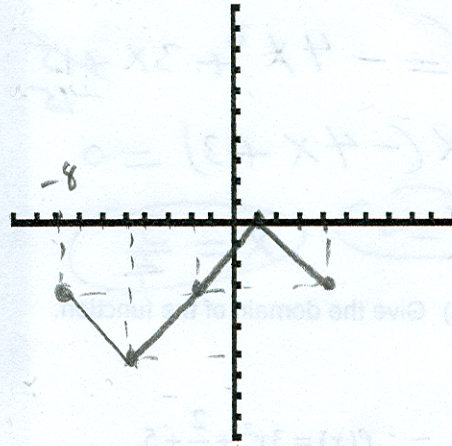
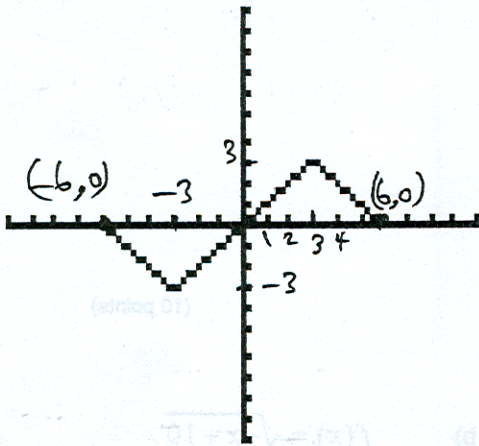
d) For what values of x is d smallest?

$$x = \pm 0.705$$

20) The graph of $y = f(x)$ is given below; (7 points)

Sketch a graph of $y = f(x+2) - 3$

x	$f(x)$
-6	0
-3	-3
0	0
3	3
6	0



x	$f(x+2) - 3$
-8	-3
-5	-6
-2	-3
-1	0
2	-3

21) **Extra Credit (10 points)**

Two cars are approaching an intersection. One is 3 miles south of the intersection and is moving at a constant speed of 20 miles per hour. At the same time, the other car is 2 miles east of the intersection and is moving at a constant speed of 30 miles per hour.

a) Express the distance d between the cars as a function of time t .

$$d = \sqrt{(3+20t)^2 + (2-30t)^2}$$

b) At time $t = 1$ hour, what is the distance between the cars?

$$d = \sqrt{(3+20)^2 + (2-30)^2} = 32.76 \text{ miles}$$

