MA 180 Professor Fred Katiraie Test I Form B (Fall 2007)

Name: Solution **Total Possible Points = 150 Plus 10 pts Extra Credits**

1) City B is located at 75 miles west and 25 miles north of city A. City C is located at 100 miles east and 125 miles south of city A. Find the distance between city B and city C. You can choose city A as the origin of the rectangular coordinate system. Write your answer rounded to two decimal places, if necessary. (5 Points)

$B=(-75,25)$ $C=(100,-125)$

$BC=\sqrt{(100--75)^2+(-125-25)^2}=230.49$ miles

2) Find the standard form of equation of a circle with endpoints of a diameter at (5 Points)

(5 , 9) and (-1, 3)

Center $=\left(\frac{5+-1}{2},\frac{9+3}{2}\right)=(2,6)$

r-dius $=\sqrt{(5-2)^2+(9-6)^2}=\sqrt{9+9}=\sqrt{18}$

$(x-2)^2+(y-6)^2=18$

3) If (a, 2a) is a point on the graph of $3x-2y=17$, what is a? (5 Points)

$3(a)-2(2a)=17$

$3a-4a=17$

$-a=17 \Longrightarrow a=-17$

4) Find the x and y intercepts of the following $5x^2+6x-8-y=0$ (5 Points)

To find x intercept let $y=0$

$5x^2+6x-8=0$

$(5x-4)(x+2)$

$x=\frac{4}{5}$, $x=-2$

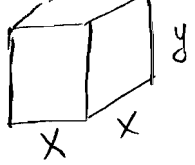
$\left(\frac{4}{5},0\right)$ $(-2,0)$ x intercepts

To find y intercept let $x=0$

$5(0)^2+6(0)-8-y=0$

$y=8$

y intercept $=(0,8)$



5) An open box with a square base is required to have a volume of 50 cubic feet. Express the amount A of material used to make such a box as a function of the length x of a side of the base. (5 Points)

$x^2y = 50 \Rightarrow y = \frac{50}{x^2}$

$A = x^2 + 4xy = x^2 + 4x\left(\frac{50}{x^2}\right)$

$A = x^2 + \frac{200}{x} \qquad x > 0$

6) Find the center and radius of the circle with the given equation

$4x^2 + 4y^2 - 24x + 16y - 20 = 0$ Divide by 4 (7 points)

$x^2 + y^2 - 6x + 4y - 5 = 0$

$x^2 - 6x + 9 + y^2 + 4y + 4 = 5 + 9 + 4$

$(x-3)^2 + (y+2)^2 = 18$

Center $= (3, -2)$

radius $= \sqrt{18} = 3\sqrt{2}$

Solve

7)

$1 - \frac{9}{5x} = \frac{7}{3}$ Multiply by $15x$ (7 points)

$15x(1) - 15x\frac{9}{5x} = \cancel{15}^{5}x\frac{7}{\cancel{3}}$

$15x - 27 = 35x$

$-15 \qquad -15$

$-27 = 20x$

$x = \frac{-27}{20}$

8) Find the average rate of change for the function $f(x) = 4x^3 - 5x + 2$ between -4 to X

x	y
-4	-234
x	$4x^3 - 5x + 2$

Avg Rate of Change $= \frac{4x^3 - 5x + 2 - (-234)}{x - (-4)}$ (7 points)

$= \frac{4x^3 - 5x + 236}{x + 4}$

Algebraically Solve:

9) $\sqrt{2x+3} - x + 10 = 10$ (6 points)

$\sqrt{2x+3} = x$

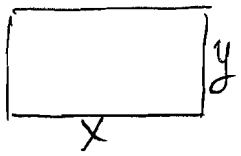
$2x + 3 = x^2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ $\quad$ ~~$x = -1$~~

Extraneous solution



10) David has available 900 yards of fencing and wishes to enclose a rectangular area. (5 points Each)

a) Express the area A of the rectangle as a function of the width x of the rectangle.

$2x+2y=900 \Rightarrow x+y=450$

$y=450-x$

$A=xy=x(450-x)=450x-x^2$

b) What is the domain of A?

$0<x<450$

11) Write an equation of the line passing through the point (6 , 5) and perpendicular to the line $y = 3x - 5$. (10 points)

$m=-\frac{1}{3}$

$5=\frac{-1}{3}(6)+b$

$5=-2+b$

$7=b$

$y=-\frac{1}{3}x+7$

12) Use long division method and perform $3x^3+2x^2-x+3$ divided by $x+3$ (10 points)

$$\begin{array}{r} 3x^2-7x+20 \\ x+3 \;\overline{)\; 3x^3+2x^2-x+3} \\ \ominus 3x^3 \oplus 9x^2 \\ \hline -7x^2-x \\ \oplus -7x^2 \ominus 21x \\ \hline 20x+3 \\ \ominus 20x \oplus 60 \\ \hline -57 \end{array}$$

Quotient $=3x^2-7x+20$

Remainder $=-57$

13) Each month a gas station sells x gallons of gas at $2.99 per gallon. The cost to the owner of the gas station for each gallon of gas is $1.99, and the monthly fixed cost for running the gas station is $27000. (10 points)

a) Find the ~~cost function. (Hint: Cost = Variable Cost~~ + Fixed Cost)

$Cost = 1.99x + 27000$

b) Find the ~~revenue function. (Hint: Revenue~~ = Price * Quantity)

$R = 2.99x$

c) Write an equation that relates the monthly profit, in dollars, to the number of gallons of gasoline sold. (Hint: Profit = Revenue – Cost)

$P = 2.99x - (1.99x + 27000) = 1x - 27000$

d) If the monthly profit is $113000, find the number of gallons of gas that are sold in that month.

$113000 = 1x - 27000$

$140000 = x$

Gallons

14) A wire of length 10x is bent into the shape of a circle. (10 points)

a) Express the circumference of the circle as a function of x.

$C(x) = 10x$

$2\pi r = 10x \Rightarrow r = \frac{10x}{2\pi} = \frac{5x}{\pi}$

b) Express the area of the circle as a function of x.

$A = \pi r^2 = \pi\left(\frac{5x}{\pi}\right)^2 = \frac{25\pi x^2}{\pi^2} = \frac{25x^2}{\pi}$

15) Find the value of $\frac{f(x+h)-f(x)}{h}$ assuming h is not zero for the function $f(x) = 4x^2 - 5$

(Clearly state each of the steps of the process.) (10 points)

$f(x+h) = 4(x+h)^2 - 5 = 4(x^2+2xh+h^2) - 5$

$f(x+h) = 4x^2 + 8xh + 4h^2 - 5$

$f(x+h) - f(x) = 4x^2 + 8xh + 4h^2 - 5 - (4x^2 - 5) = h(8x+4h)$

$\frac{f(x+h)-f(x)}{h} = \frac{h(8x+4h)}{h} = 8x+4h$

16) Given $f(x) = -4x^2 + 5x + 35$. Find x such that $f(x) = 15$ (5 points)

$-4x^2 + 5x + 35 = 15$

$-4x^2 + 5x + 20 = 0$

$4x^2 - 5x - 20 = 0$

$x = \frac{-(-5) \pm \sqrt{25 - 4(4)(-20)}}{8}$

$x \Rightarrow 2.95$

$\searrow -1.70$

17) Give the domain of the function. (10 points)

a) $f(x) = 3x^2 + \frac{2}{x-7} + 5$

Domain is all Reals except 7

b) $f(x) = \sqrt{-5x+10}$

$-5x + 10 \geq 0$

$-5x \geq -10$

$x \leq 2$

c) $f(x) = \frac{x+7}{x^2+x-42} = \frac{x+7}{(x+7)(x-6)}$

Domain is all Reals except -7 and 6

d) $g(x) = \frac{x}{\sqrt{5-2x}}$

$5 - 2x > 0$

$-2x > -5$

$x < \frac{5}{2}$

18) Use Quadratic formula to solve the following: $4x^2 + 12x = -2$ (6 points)

$4x^2 + 12x + 2 = 0$

$x = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(2)}}{8} = \frac{-12 \pm \sqrt{112}}{8}$

$= \frac{-12 \pm 4\sqrt{7}}{8} = \frac{-3}{2} \pm \frac{\sqrt{7}}{2}$

$x \rightarrow -0.177$

$\searrow -2.823$

(0, 0)

19) Let P = (x , y) be a point on the graph of $y = 2x^2 - 8$ $(x, 2x^2-8)$ (10 points)

a) Express the distance d from P to the origin as a function of x.

$$d = \sqrt{(x-0)^2 + (2x^2-8-0)^2} = \sqrt{x^2 + (2x^2-8)^2}$$

b) What is d if x = 0? $d(0) = \sqrt{0^2 + (-8)^2} = 8$

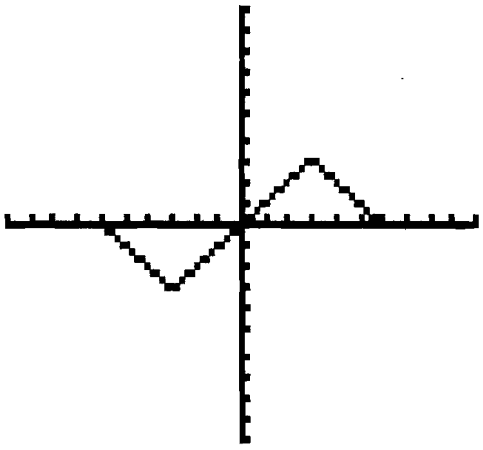
c) What is d if x = 1? $d(1) = \sqrt{1 + (-6)^2} = \sqrt{37}$

d) For what values of x is d smallest?

$$x = \pm 1.97$$

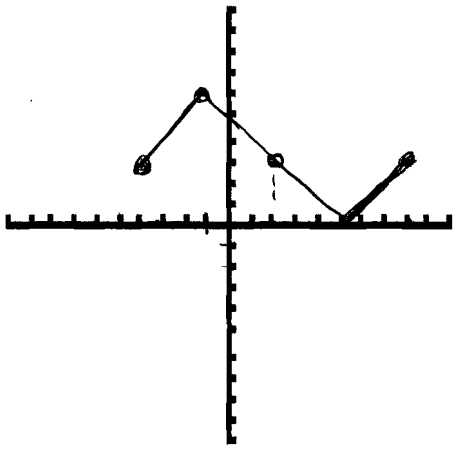
20) The graph of $y = f(x)$ is given below; Sketch a graph of $y = -[f(x-2)-3]$ * Shift Right 2 * Down 3 * Reflect About X-axis (7 points)

X	Y
-6	0
-3	-3
	0
3	3
6	0



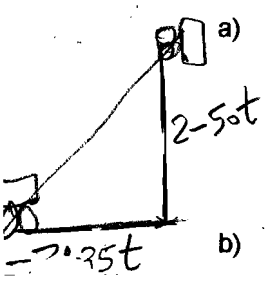
New

X	Y
-4	3
-1	6
+2	3
5	0
8	3



21) **Extra Credit (10 points)**

Two cars are approaching an intersection. One is 2 miles north of the intersection and is moving at a constant speed of 50 miles per hour. At the same time, the other car is 3 miles west of the intersection and is moving at a constant speed of 35 miles per hour.



a) Express the distance d between the cars as a function of time t.

$$d = \sqrt{(2-50t)^2 + (-3+35t)^2}$$

b) At time t = 1 Hour, what is the distance between the cars?

$$d = \sqrt{(2-50)^2 + (-3+35)^2} = 57.69 \text{ miles}$$