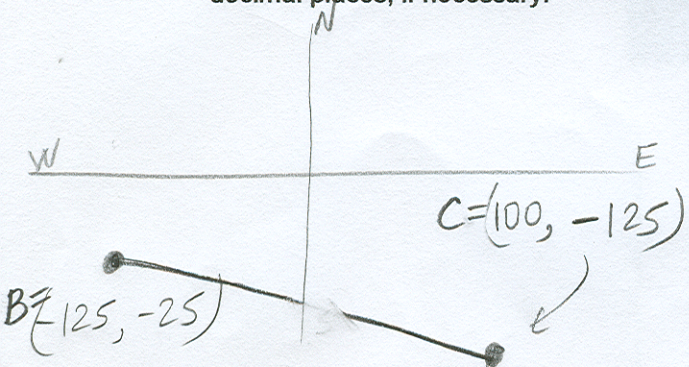


Name: **SOLUTION**

Total Possible Points = 150 Points

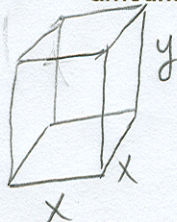
- 1) City B is located at 125 miles west and 25 miles south of city A. City C is located at 100 miles east and 125 miles south of city A. Find the distance between city B and city C. You can choose city A as the origin of the rectangular coordinate system. Write your answer rounded to two decimal places, if necessary. (5 Points)



$$d = \sqrt{(100+125)^2 + (-125+25)^2}$$

$$= 246.22 \text{ miles}$$

- 2) An open box with a square base is required to have a volume of 40 cubic feet. Express the amount A of material used to make such a box as a function of the length x of a side of the base. (5 Points)



$$x^2 y = 40 \Rightarrow y = \frac{40}{x^2}$$

$$A = x^2 + 4xy = x^2 + 4(x)\left(\frac{40}{x^2}\right) = x^2 + \frac{160}{x}$$

Algebraically Solve:

3) $\sqrt{5x+6} - x + 100 = 100$ (6 points)

$$(\sqrt{5x+6})^2 = x^2$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6$$

$$x = -1$$

Extraneous solution

- 4) Carlos has 600 yards of fencing available and wishes to enclose a rectangular area. (5 points)

Express the area A of the rectangle as a function of the width x of the rectangle.

$$2x + 2y = 600 \Rightarrow x + y = 300 \Rightarrow y = 300 - x$$

$$A = xy = x(300 - x) = 300x - x^2$$

5) A wire of length $8x$ is bent into the shape of a circle.

(10 points)

a) Express the circumference of the circle as a function of x .

$$C(x) = 8x$$

$$2\pi r = 8x$$

$$r = \frac{8x}{2\pi} = \frac{4x}{\pi}$$

b) Express the area of the circle as a function of x .

$$A = \pi r^2 = \pi \left(\frac{4x}{\pi} \right)^2 = \frac{16x^2 \pi}{\pi^2} = \frac{16x^2}{\pi}$$

6) Find the value of $\frac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = x^2 - 2x$

(Clearly state each of the steps of the process.)

$$f(x+h) = (x+h)^2 - 2(x+h) = x^2 + 2xh + h^2 - 2x - 2h \quad (10 \text{ points})$$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - 2x - 2h - (x^2 - 2x)}{h} = \frac{h(2x + h - 2)}{h} = 2x + h - 2$$

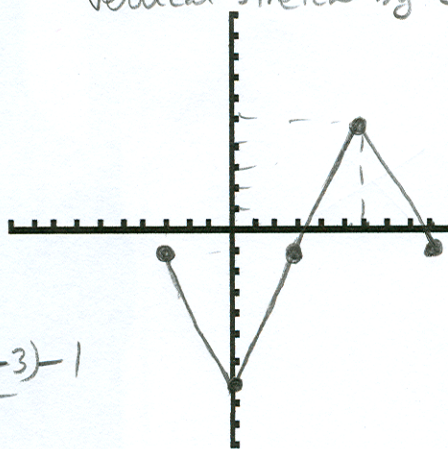
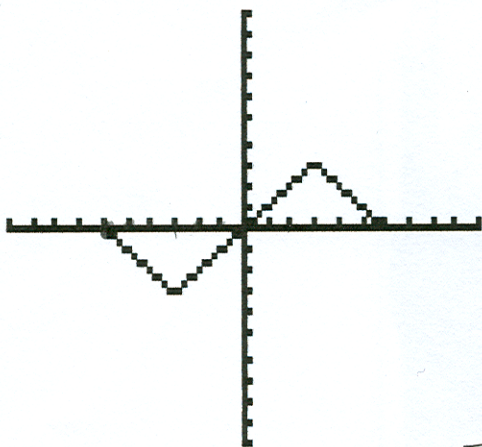
7) The graph of $y = f(x)$ is given below; (9 points)

Sketch a graph of $y = 2f(x-3) - 1$

Shift Right 3 down 1

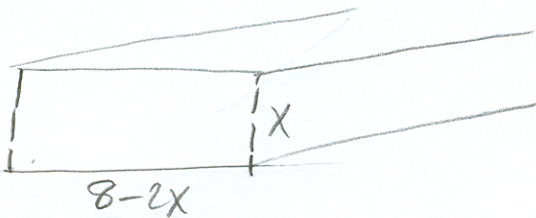
Vertical stretch by 2

x	y
-6	0
-3	-3
0	0
3	3
6	0



x	$2f(x-3) - 1$
-3	-1
0	-7
3	-1
6	5
9	-1

- 8) A piece of rectangular sheet metal is 8 inches wide. It is to be made into a rain gutter by turning up equal edges to form parallel sides. Let x represent the length of each of the parallel sides. For what value of x will the area of the cross section be a maximum (and thus maximize the amount of water that the gutter will hold)? (6 points)

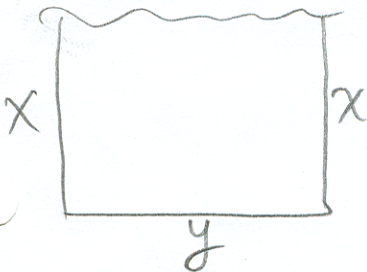


$$A = x(8 - 2x) = 8x - 2x^2$$

$$A = -2x^2 + 8x$$

$$x = \frac{-b}{2a} = \frac{-8}{2(-2)} = \frac{-8}{-4} = 2 \text{ inches}$$

- 9) A developer wants to enclose a rectangular grassy lot that borders a city - street for parking. If the developer has 500 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed? (9 points)



$$2x + y = 500$$

$$y = 500 - 2x$$

$$A = xy = x(500 - 2x)$$

$$A = 500x - 2x^2$$

$$= -2x^2 + 500x$$

$$x_{\text{vertex}} = \frac{-500}{2(-2)} = 125 \text{ feet}$$

$$\text{Max Area} = 31250 \text{ ft}^2$$

- 10) For the following function $f(x) = -x^2(x^2 - 1)(x + 1)$ Determine the following (10 points)

- a) Find the x intercept

$$(0,0) (1,0) (-1,0)$$

- b) Find the y intercept

$$(0,0)$$

- c) Determine whether the graph crosses or touches the x axis at each x intercept.

Touch at $x = 0$ and $x = -1$; Cross at $x = 1$

- d) End behavior: Find the power function that the graph of $f(x)$ resembles for large values of $|x|$

$$-x^5$$

- e) Determine the number of turning points on the graph of $f(x)$

4

11) For the rational function $R(x) = \frac{x^3 - 8}{x^2 - 5x + 6} = \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}(x-3)}$ (16 points)

2pts

a) Find the domain of the rational function

All Reals except $x=2$ and $x=3$

2pts

b) Write $R(x)$ in the lowest terms

$$R(x) = \frac{x^2 + 2x + 4}{x - 3}$$

2pts

c) Locate the x-intercept(s)

None

2pts

Locate the y-intercept

$$(0, -\frac{4}{3})$$

2pts

d) Test for symmetry

$$R(-x) = \frac{(-x)^3 - 8}{(-x)^2 - 5(-x) + 6} = \frac{-x^3 - 8}{x^2 + 5x + 6}$$

No symmetry

2pts

e) Locate the vertical asymptote

$$x = 3$$

2pts

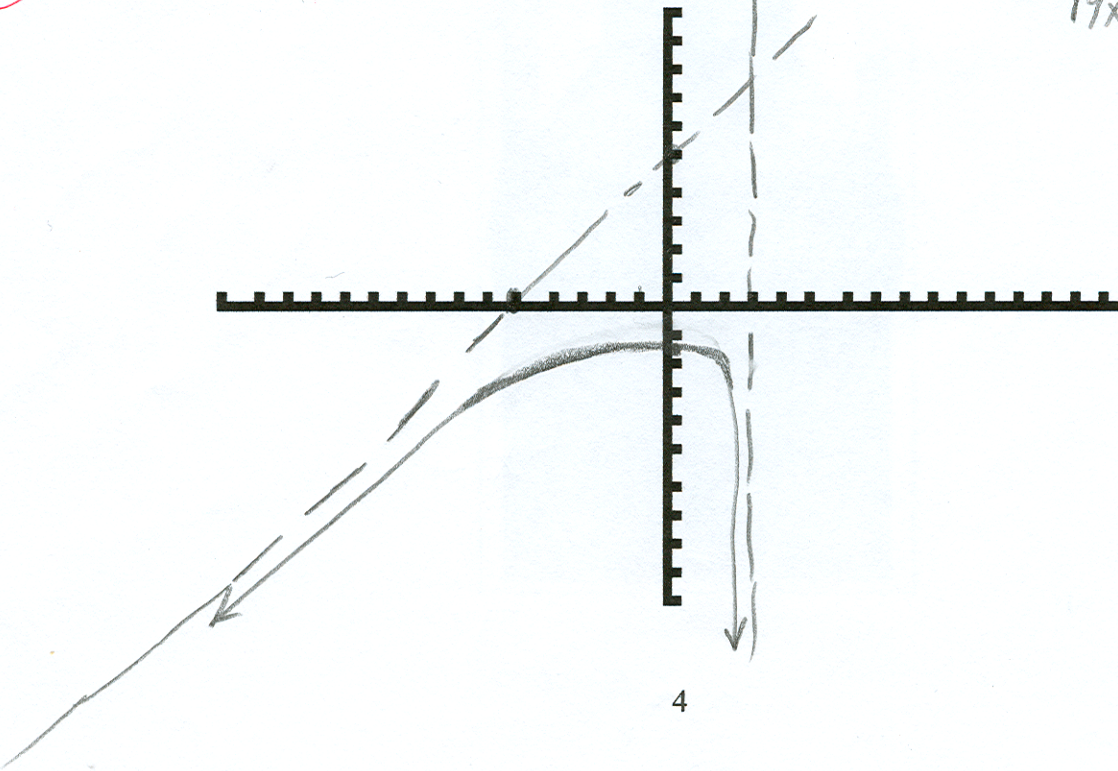
f) Locate the horizontal or oblique asymptote if any

O.A. $y = x + 5$

$$\begin{array}{r} x + 5 \\ x^2 - 5x + 6 \overline{) x^3 + 0x^2 + 0x - 8} \\ \underline{\ominus x^3 + 5x^2 + 6x} \\ 5x^2 - 6x - 8 \\ \underline{\ominus 5x^2 + 25x + 30} \\ 19x - 38 \end{array}$$

2pts

g) Graph the function using results from steps a to f



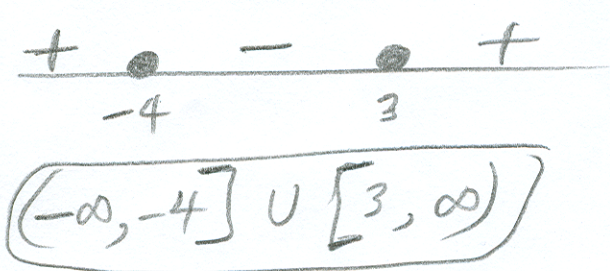
12) Solve the following inequalities algebraically, and give your answer in interval notation.

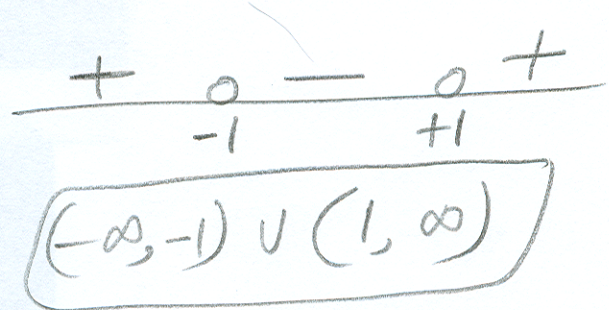
(10 points)

a) $x^2 + x \geq 12$

b) $\frac{x+1}{x-1} > 0$

$$x^2 + x - 12 \geq 0$$

$$(x+4)(x-3) \geq 0$$




13) Find the real zeros of $f(x) = 3x^3 + 4x^2 - 7x + 2$ use the following steps please)

(10 points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros

(1 pt)

Max no of zeros = 3

Step 2) Use the Rational Zeros Theorem to identify rational numbers that potentially can be zeros

(1 pt)

$$\frac{p}{q} = \pm 2, \pm 1, \pm \frac{1}{3}, \pm \frac{2}{3}$$

Step 3) Using your calculator, graph the polynomial function to identify the rational zero.

(2 pts)

$x = \frac{2}{3} \Rightarrow$ factor is $(3x - 2)$

Step 4) Identify the other real zeros of $f(x) = 3x^3 + 4x^2 - 7x + 2$ by using long division

(4 pts)

$$\begin{array}{r}
 x^2 + 2x - 1 \\
 3x - 2 \overline{) 3x^3 + 4x^2 - 7x + 2} \\
 \underline{\ominus 3x^3 + 2x^2} \\
 6x^2 - 7x + 2 \\
 \underline{\ominus 6x^2 + 4x} \\
 -3x + 2 \\
 \underline{ 3x - 2} \\
 4
 \end{array}$$

$$x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2}$$

$$= \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$= -1 \pm \sqrt{2}$$

Step 5) Now, list all the real zeros of $f(x) = 3x^3 + 4x^2 - 7x + 2$

(2 pts)

$$x = \frac{2}{3}, x = -1 + \sqrt{2}, x = -1 - \sqrt{2}$$

14) Form a polynomial $f(x)$ with real coefficients having a degree of 5 and the following zeros 1, i , $2i$

(10 points)

(2pts) $(x-1)(x-i)(x+i)(x-2i)(x+2i)$

(2pts) $(x-1)(x^2+1)(x^2+4)$

(2pts) $(x-1)(x^4+5x^2+4)$

(2pts) $x^5+5x^3+4x-x^4-5x^2-4$

(2pts) $x^5-x^4+5x^3-5x^2+4x-4$

15) Find the complex zeros of the following polynomial function

$$f(x) = x^3 - 8x^2 + 25x - 26$$

(10 points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros

(1pt) Max No of Zeros = 3

Step 2) Use the Rational Zeros Theorem to identify those rational numbers that potentially can be zeros.

(1pt) $\frac{p}{q} = \pm 1, \pm 2, \pm 13, \pm 26$

Step 3) Now use long division to find the other factors. and $x=2$ is a zero

(8pts)

$$\begin{array}{r}
 x^2 - 6x + 13 \quad \text{find the zeros} \\
 x-2 \overline{) x^3 - 8x^2 + 25x - 26} \\
 \underline{\ominus x^3 \oplus 2x^2} \\
 -6x^2 + 25x \\
 \underline{\oplus 6x^2 \ominus 12x} \\
 13x - 26 \\
 \underline{\ominus 13x^2 \oplus 26} \\
 0 \quad 0
 \end{array}$$

Thus $(x-2)$ is a factor

$$x^2 - 6x + 13 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(13)}}{2}$$

$$\frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$$

$$= 3 \pm 2i$$

Zeros are $x=2$ $x=3+2i$

(4pts)

(4pts)

16) For the following problems, give the equation of the specified asymptotes:

a) $g(x) = \frac{x^2 + 5x - 3}{x - 3}$ Is there a Horizontal Asymptote? NO (2 points)

If so, what is the Equation of Horizontal Asymptote? N/A

b) $h(x) = \frac{8x^2 - 9x - 7}{5x^2 - 8x + 9}$ Is there a Horizontal Asymptote? Yes (2 points)

If so, what is the Equation of the Horizontal Asymptote? $y = \frac{8}{5}$

c) $w(x) = \frac{x^2 + 2x - 2}{x - 8}$ Is there an Oblique Asymptote? Yes (4 points)

If so, what is the Equation of the Oblique Asymptote? $y = x + 10$

$$\begin{array}{r}
 x + 10 \\
 x - 8 \overline{) x^2 + 2x - 2} \\
 \underline{-(x^2 + 8x)} \\
 10x - 2 \\
 \underline{-(10x + 80)} \\
 72
 \end{array}$$

d) $u(x) = \frac{x^2 - 10x + 80}{x^2 + 16}$ Are there any Vertical Asymptote(s)? NO (2 points)

If so, what are the Equation(s) of the Vertical Asymptotes?

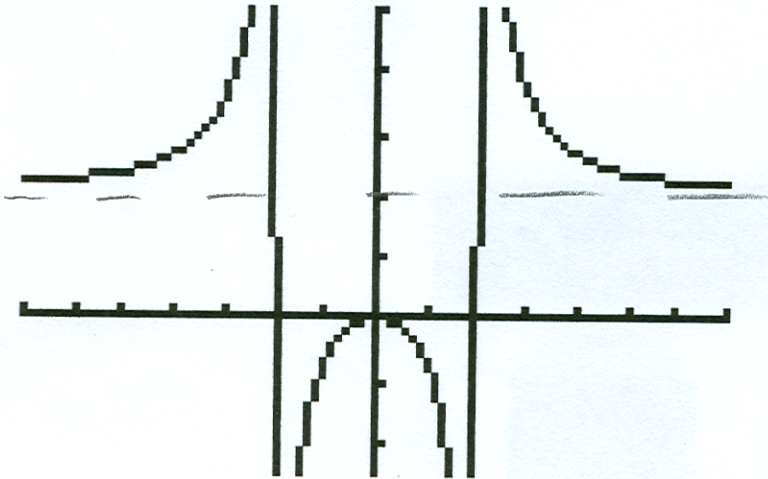
N/A

e) $f(x) = \frac{x - 9}{x^2 - 81}$ Are there any Vertical Asymptote(s)? Yes (2 points)

If so, what are the Equation(s) of the Vertical Asymptotes? $x = -9$

$$f(x) = \frac{\cancel{x-9}}{(\cancel{x-9})(x+9)} = \frac{1}{x+9}$$

17) Given the following graph



List the equation(s) of the following if any:

a) Horizontal Asymptote $y = 2$ (1/2 point)

b) Vertical Asymptote(s) $x = -2$ (1/2 point)

$x = 2$

c) c) Oblique Asymptote (1/2 point)

None

d) d) X intercept (if any) $(0, 0)$ (1/2 point)

e) Y intercept (if any) $(0, 0)$ (1/2 point)

f) What is a possible equation of a function whose graph is given above.
 (Hint: Use your answers for parts a to e) (4.5 points)

$$y = \frac{2x^2}{(x+2)(x-2)}$$