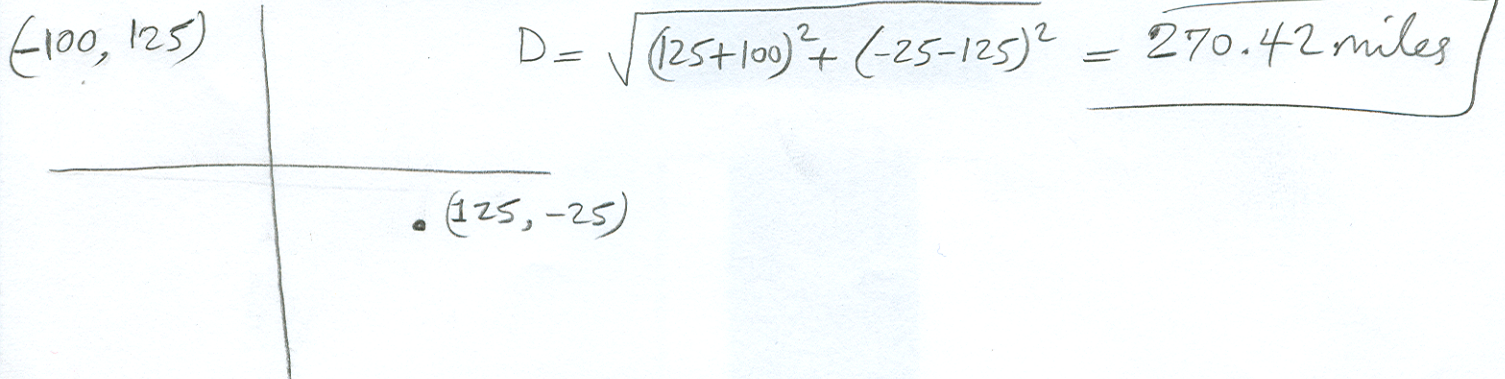


Name: Solution

Total Possible Points = 150 Points

1) City B is located at 125 miles east and 25 miles south of city A. City C is located at 100 miles west and 125 miles north of city A. Find the distance between city B and city C. You can choose city A as the origin of the rectangular coordinate system. Write your answer rounded to two decimal places, if necessary. (5 Points)



2) Find the standard form of equation of a circle with endpoints of a diameter at (5, 9) and (-3, -3) (5 Points)

(5, 9) and (-3, -3)

Center = $(\frac{5-3}{2}, \frac{9-3}{2}) = (1, 3)$

$(x-1)^2 + (y-3)^2 = 52$

Radius = $\frac{\sqrt{(5+3)^2 + (9+3)^2}}{2} = 52$

Algebraically Solve:

3) $\sqrt{5x+6} - x + 500 = 500$ (6 points)

$\sqrt{5x+6} = x$

$5x+6 = x^2 \implies x^2 - 5x - 6 = 0$
 $(x-6)(x+1) = 0$

$x=6$
 ~~$x=-1$~~ Extraneous sol.

4) Find the average rate of change for the function $f(x) = 4x^3 - 5x + 2$ between -4 to X

X	f(x)
-4	-234
X	$4x^3 - 5x + 2$

\implies Arg Rate of Change = $\frac{4x^3 - 5x + 2 - (-234)}{x - (-4)}$ (5 points)
 $= \frac{4x^3 - 5x + 236}{x + 4}$

5) Each month a gas station sells x gallons of gas at \$2.19 per gallon. The cost to the owner of the gas station for each gallon of gas is \$1.99, and the monthly fixed cost for running the gas station is \$37000. (10 points)

a) Find the cost function. (Hint: Cost = Variable Cost + Fixed Cost)

$$C(x) = 1.99x + 37000$$

b) Find the revenue function. (Hint: Revenue = Price * Quantity)

$$R(x) = 2.19x$$

c) Write an equation that relates the monthly profit, in dollars, to the number of gallons of gasoline sold. (Hint: Profit = Revenue - Cost)

$$P(x) = 2.19x - (1.99x + 37000) = 0.2x - 37000$$

d) If the monthly profit is \$113000, find the number of gallons of gas that are sold in that month.

$$113000 = 0.2x - 37000$$

$$0.2x = 150000 \implies x = 750000 \text{ Gallons of Gas}$$

6) Find the value of $\frac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = x^2 + 3x$

(Clearly state each of the steps of the process.)

$$f(x+h) = (x+h)^2 + 3(x+h) = x^2 + 2xh + h^2 + 3x + 3h \quad (10 \text{ points})$$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + \cancel{3x} + 3h - x^2 - \cancel{3x}}{h} = \frac{h(2x + h + 3)}{h} = 2x + h + 3$$

7) Write an equation of the line passing through the point (6, 5) and perpendicular to the line $y = 3x - 5$. (9 points)

$$m \perp = -\frac{1}{3}$$

$$y = mx + b$$

$$5 = -\frac{1}{3}(6) + b$$

$$5 = -2 + b$$

$$7 = b$$

$$y = -\frac{1}{3}x + 7$$

- 8) A ball is thrown vertically upward with an initial velocity of 192 feet per second. The distance in feet of the ball from the ground after t seconds is $s = 192t - 16t^2$. For what interval of time is the ball more than 432 feet above the ground? (6 points)

$$192t - 16t^2 > 432$$

$$0 > 16t^2 - 192t + 432$$

$$0 > 16(t^2 - 12t + 27)$$

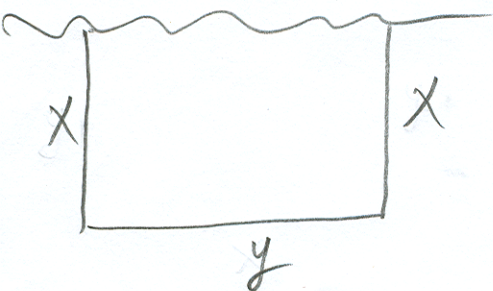
$$0 > 16(t-3)(t-9)$$



$$3 < t < 9$$

$$(3, 9) \text{ seconds}$$

- 9) A developer wants to enclose a rectangular grassy lot that borders a city - street for parking. If the developer has 500 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed? (9 points)



$$2x + y = 500 \Rightarrow y = 500 - 2x$$

$$A = xy = x(500 - 2x) = -2x^2 + 500x$$

$$x_{\text{vertex}} = \frac{-b}{2a} = \frac{-500}{2(-2)} = 125 \text{ feet}$$

$$A_{\text{max}} = 31250 \text{ ft}^2$$

- 10) For the following function $f(x) = -x^2(x^2 - 4)(x + 2)$ Determine the following (10 points)

- a) Find the x intercept $x=0$ $x=+2$ $x=-2$

$$(0,0) (2,0) (-2,0)$$

- b) Find the y intercept $(0,0)$

- c) Determine whether the graph crosses or touches the x axis at each x intercept.

Touch the x axis at $x=0$ and -2 ; Cross the x axis at $x=+2$

- d) End behavior: Find the power function that the graph of $f(x)$ resembles for large values of $|x|$

$$-x^5$$

- e) Determine the number of turning points on the graph of $f(x)$

$$4$$

11) For the rational function $R(x) = \frac{x^3}{x^4 - 1} = \frac{x^3}{(x^2 + 1)(x + 1)(x - 1)}$

(16 points)

a) Find the domain of the rational function

Domain is all Reals except $x = \pm 1$

b) Write $R(x)$ in the lowest terms

Already in lowest term

c) Locate the x-intercept(s)

$(x = 0, y = 0)$

2pts Locate the y-intercept

$(0, 0)$

d) Test for symmetry

$R(-x) = \frac{(-x)^3}{(-x)^4 - 1} = \frac{-x^3}{x^4 - 1}$

symmetric w.r.t origin

e) Locate the vertical asymptote

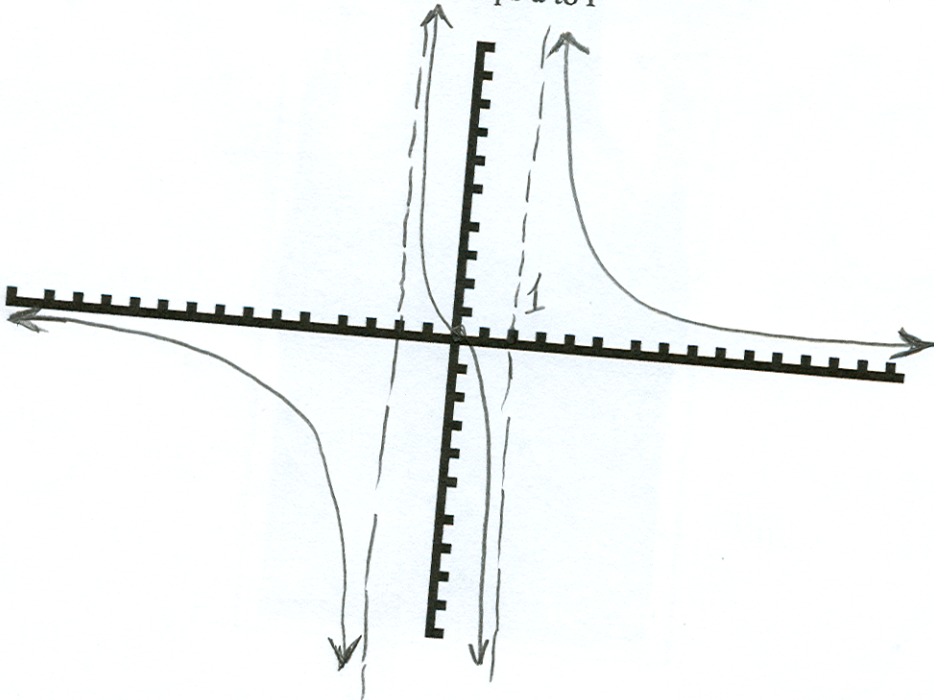
$x = 1$ $x = -1$

f) Locate the horizontal or oblique asymptote if any

H.A $y = 0$

O.A
None

g) Graph the function using results from steps a to f



12) Solve the following inequalities algebraically, and give your answer in interval notation.

(10 points)

a) $x^2 - 4x > 0$

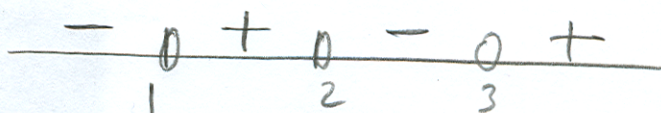
$x(x-4) > 0$



$(-\infty, 0) \cup (4, \infty)$

b) $(x-1)(x-2)(x-3) > 0$

5pts



$(1, 2) \cup (3, \infty)$

13) Find the real zeros of $f(x) = 3x^3 - x^2 - 15x + 5$ use the following steps please)

(10 points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros

Max No of Zeros = 3

Step 2) Use the Rational Zeros Theorem to identify rational numbers that potentially can be zeros

$\frac{p}{q} = \pm 5, \pm 1, \pm \frac{5}{3}, \pm \frac{1}{3}$

Step 3) Using your calculator, graph the polynomial function to identify the rational zero.

$x = \frac{1}{3} \implies$ the factor must be $(3x-1)$

Step 4) Identify the other real zeros of $f(x) = 3x^3 - x^2 - 15x + 5$ by using long division, and quadratic formula.

3x-1 $\overline{) 3x^3 - x^2 - 15x + 5}$
 $\underline{-(3x^3 \oplus x^2)}$
 $-15x + 5$
 $\underline{+(15x \oplus 5)}$
 0

3pts

$x^2 - 5 = 0$
 $x^2 = 5$
 $x = \pm \sqrt{5}$

Step 5) Now, list all the real zeros of $f(x) = 3x^3 - x^2 - 15x + 5$ (Please No Decimal Answers)

$x = \frac{1}{3}, x = +\sqrt{5}, x = -\sqrt{5}$

14) Form a polynomial $f(x)$ with real coefficients having a degree of 5 and the following zeros 1, i , $3i$
 (Hint: multiply the factors and find the polynomial)

(10 points)

$$\begin{aligned} & (x-1)(x-i)(x+i)(x-3i)(x+3i) \quad (2pts) \\ & = (x-1)(x^2+1)(x^2+9) \quad (2pts) \\ & = (x-1)(x^4+9x^2+1x^2+9) \quad (2pts) \\ & = (x-1)(x^4+10x^2+9) \quad (2pts) \\ & = x^5+10x^3+9x-x^4-10x^2-9 = x^5-x^4+10x^3-10x^2+9x-9 \quad (2pts) \end{aligned}$$

15) Find the complex zeros of the following polynomial function
 $f(x) = x^3 + 13x^2 + 57x + 85$

(10 points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros

(1pt) Maximum No of zeros = 3

Step 2) Use the Rational Zeros Theorem to identify those rational numbers that potentially can be zeros.

(1pt) $\frac{p}{q} = \frac{\pm 1, \pm 5, \pm 17, \pm 85}{\pm 1} = \pm 1, \pm 5, \pm 17, \pm 85$

Step 3) Now use long division to find the other factors

$(x+5)$ is a factor

(3pts)

$$\begin{array}{r} x^2+8x+17 \\ x+5 \overline{) x^3+13x^2+57x+85} \\ \underline{\ominus x^3+5x^2} \\ 8x^2+57x \\ \underline{\ominus 8x^2+40x} \\ 17x+85 \\ \underline{\ominus 17x+85} \\ 0 \end{array}$$

(3pts)

$$\begin{aligned} & \rightarrow x^2 + 8x + 17 = 0 \\ & x = \frac{-8 \pm \sqrt{64 - 4(17)}}{2} = \frac{-8 \pm \sqrt{-4}}{2} \\ & = \frac{-8 \pm 2i}{2} = -4 \pm i \end{aligned}$$

Step 4) List all the zeros (real and complex) of $f(x) = x^3 + 13x^2 + 57x + 85$

(2pts) $-5, -4+i, -4-i$

16) For the following problems, give the equation of the specified asymptotes:

a) $g(x) = \frac{x^2 + 5x - 3}{x + 5}$ Is there a Horizontal Asymptote? No (2 points)

If so, what is the Equation of Horizontal Asymptote? N/A

b) $h(x) = \frac{8x^2 - 9x - 7}{16x^2 - 8x + 9}$ Is there a Horizontal Asymptote? Yes (2 points)

If so, what is the Equation of the Horizontal Asymptote? $y = \frac{8}{16} = \frac{1}{2}$

c) $w(x) = \frac{x^2 + 2x - 2}{x - 2}$ Is there an Oblique Asymptote? Yes (4 points)

If so, what is the Equation of the Oblique Asymptote?

$$\begin{array}{r} x+4 \\ x-2 \overline{) x^2 + 2x - 2} \\ \underline{\ominus x^2 - 2x} \\ 4x - 2 \end{array}$$

$y = x + 4$

d) $u(x) = \frac{x^2 - 8x + 6}{x^2 - 1}$ Are there any Vertical Asymptote(s)? Yes (2 points)

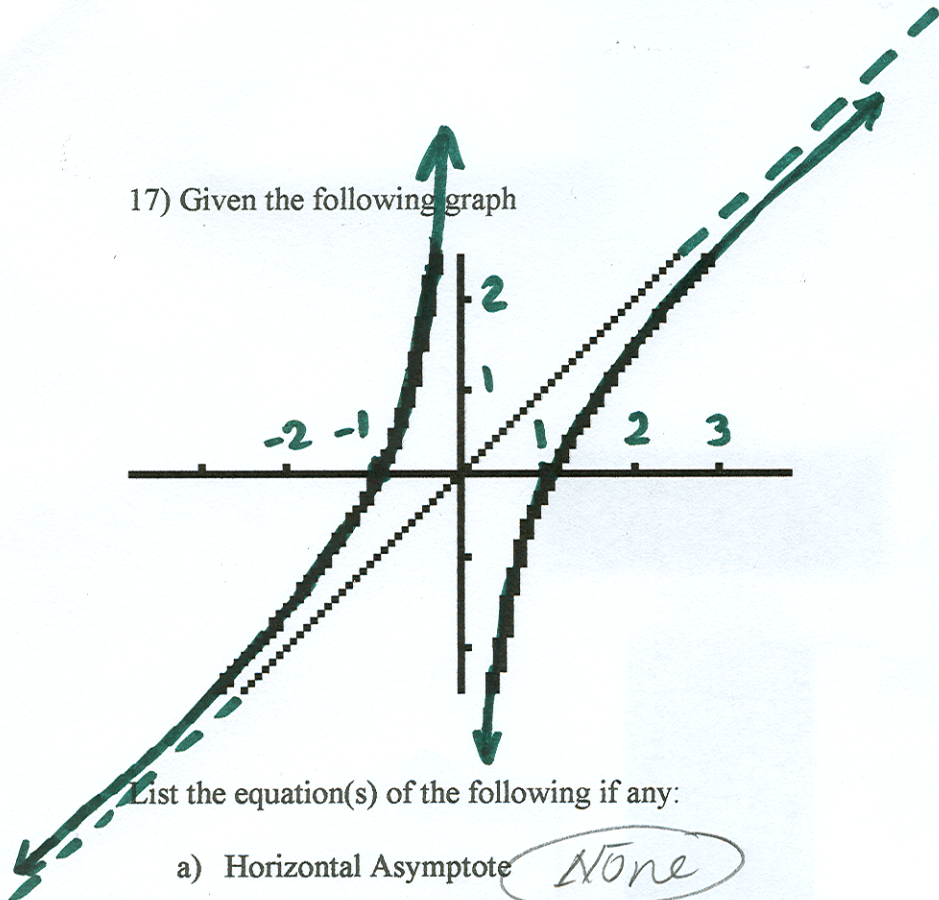
If so, what are the Equation(s) of the Vertical Asymptotes?

$$u(x) = \frac{\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{1}{x+1} \Rightarrow \underline{\text{only V.A. } x = -1}$$

e) $f(x) = \frac{x - 9}{x^2 + 81}$ Are there any Vertical Asymptote(s)? No (2 points)

If so, what are the Equation(s) of the Vertical Asymptotes? N/A

17) Given the following graph



List the equation(s) of the following if any:

a) Horizontal Asymptote None (1/2 point)

b) Vertical Asymptote $x = 0$ (1/2 point)

c) Oblique Asymptote $y = x$ (1/2 point)

d) X intercept (if any) $(1, 0)$ and $(-1, 0)$ (1/2 point)

e) Y intercept (if any) None (1/2 point)

f) What is a possible equation of a function whose graph is given above.
(Hint: Use your answers for parts a to e) (4.5 points)

$$y = x + \frac{(x-1)(x+1)}{x}$$