Solution to Practice test II

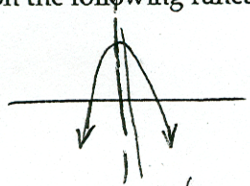
MA 180 **Professor Fred Katiraie** **Pre Calculus Test II** (Practice)

Name: KEY **Date:**

$x = -\frac{3}{4}$

1) Graph the following function $f(x) = -4x^2 - 6x + 2$, and determine the following: (16 Points)

(2)



(4) a) Vertex $x = \frac{+6}{2(-4)} = -\frac{3}{4}$; $y_{vertex} = 4.25$

(4) b) Axis of symmetry $x = -\frac{3}{4}$

(2) c) y-intercept $(0,2)$

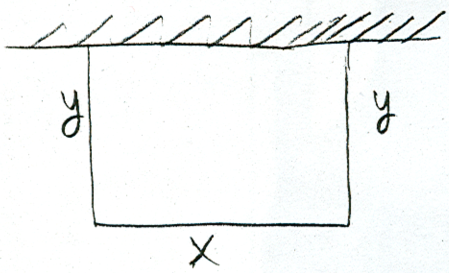
d) x-intercepts (if any)

(4)

$(-1.78, 0)$ & $(0.28, 0)$

2) A developer wants to enclose a rectangular grassy lot that borders a city-street for parking. If the developer has 380 feet of fencing and **does not fence the side along the street**, what is the largest area that can be enclosed?

(Hint: Draw a picture, and label each side, i.e length is x and width is y, and remember that we do not want to fence the side along the street) (12 Points)



(2) $x + 2y = 380 \Rightarrow 2y = -x + 380 \Rightarrow y = \frac{-x}{2} + 190$ (2)

(2)

$\text{Area} = x \cdot y = x \cdot \left(-\frac{1}{2}x + 190\right)$

$\text{Area} = -\frac{1}{2}x^2 + 190x$ (2)

(2)

$x_{vertex} = \frac{-190}{2(-\frac{1}{2})} = 190$ feet (2) $\Rightarrow$ $\boxed{\text{Area} = 18050 \text{ Feet}^2}$

3) For the following function $f(x) = (x-2)^2(x+2)(x+4)$ determine the following: **(15 Points)**

(4) a) Find the x- intercepts $(2,0)$ $(-2,0)$ and $(-4,0)$

(2) b) Find the y-intercepts $(0, 32)$

(4) c) Determine whether the graph crosses or touches the x-axis at each x-intercept

Graph touches x axis at $x=2$; graph crosses at $x=-2$ and $x=-4$

(2) d) End behavior: Find the power function that the graph of f resembles for large values of |x|

$y = x^4$

(3) e) Determine the number of turning points on the graph of f.

Three Turning Points

4) For the rational function $R(x) = \dfrac{x^3-1}{x^2+2x} = \dfrac{(x-1)(x^2+x+1)}{x(x+2)}$ **(25 Points)**

(3) a) Find the domain of the rational function

All Reals except 0 and -2

(3) b) Write R in the lowest terms

Already in lowest form

(3) c) Locate the x-intercept $(1,0)$

(3) and the y- intercept None

(3) d) Test for symmetery

$R(-x) = \dfrac{-x^3-1}{x^2-2x}$ No symmetry

(3) e) Locate the vertical asymptotes.

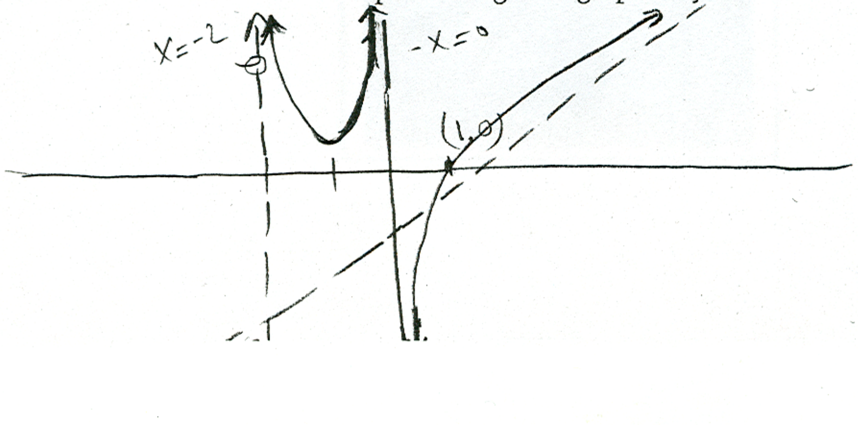
$x=0$ and $x=2$

(3) f) Locate the horizontal or oblique asymptotes, if any.

oblique Asymptote $\boxed{y = x-2}$

$$\begin{array}{r|l} & x-2 \\ \hline x^2+2x+0 & x^3+0x^2+0x-1 \\ & -(x^3+2x^2+0x) \\ \hline & -2x^2+0x-1 \\ & -(-2x^2-4x) \\ \hline & 4x-1 \end{array}$$

(4) g) Use the results obtained in steps a through f to graph R by hand.



5) Solve the following inequality algebraically $6x - 5 < \frac{6}{x}$ **(14 Point)**

2 $6x - 5 - \frac{6}{x} < 0$

1 $\frac{6x^2 - 5x - 6}{x} < 0$

2 $\frac{(3x+2)(2x-3)}{x} < 0$

$- - - - - 0 + + + + + +$ $3x+2$ 2
$-\frac{2}{3}$ $\quad 0$

$- - - - - - 0 + + +$ $2x-3$ 2
$\frac{3}{2}$

$- - - - 0 + + + + +$ x
0

$- - - 0 + 0 - 0 + + +$ $\frac{(3x+2)(2x-3)}{x}$ 2
$-\frac{2}{3}$ $\quad 0$ $\quad \frac{3}{2}$

$x < -\frac{2}{3}$ or $0 < x < \frac{3}{2}$ 3

$(-\infty, -\frac{2}{3}) \cup (0, \frac{3}{2})$

6) Find the real zeros of $f(x) = x^3 + 2x^2 - 5x - 6$ (Must use the following steps): **(10 Points)**

Step 1) Use the degree of the polynomial to determine the maximum number of zeros.

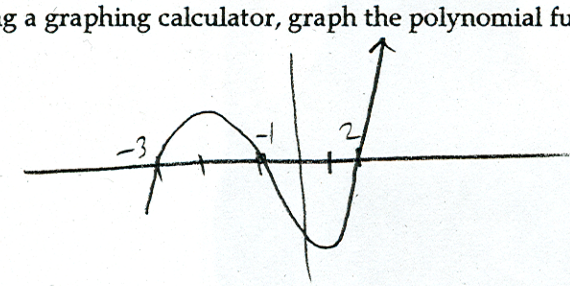
2 Maximum Zero are 3

Step 2) Use the Rational Zeros Theorem to identify those rational numbers that potentially can be zeros.

2 $\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 6$

Step 3) Using a graphing calculator, graph the polynomial function.

2



Step 4) Identify the real zeros of $f(x) = x^3 + 2x^2 - 5x - 6$.
(Hint: You could use the "2nd trace value" of your calculator, or use other methods!)

2 $x = -3$, $x = -1$, $x = 2$

Step 5) Now, use the real zeros to factor the function $f(x) = x^3 + 2x^2 - 5x - 6$.

$x^3 + 2x^2 - 5x - 6 = (x+3)(x+1)(x-2)$

2

7) Form a polynomial f(x) with real coefficients having the following degree and zeros:

Degree 4; Zeros: 3, multiplicity 2; −i

(10 Points)

(Hint: After you set up your factors, you must multiply (FOIL) the factors to get the desired polynomial)

$$= (x-3)^2(x+i)(x-i)$$

$$= (x-3)^2(x^2+1)$$

$$= (x^2-6x+9)(x^2+1)$$

$$= \cancel{x^4} + \cancel{x^2} - \cancel{6x^3} - 6x + \cancel{9x^2} + 9 = \boxed{x^4 + 10x^2 - 6x^3 - 6x + 9}$$

8) Find the complex zeros of the following polynomial function $f(x) = x^3 + 13x^2 + 57x + 85$

(20 Points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros.

2 Maximum No of Zeros = 3

Step 2) Use the Rational Zeros Theorem to identify those rational numbers that potentially can be zeros.

4 $\frac{p}{q} = \frac{\pm 1, \pm 5, \pm 17, \pm 85}{\pm 1} = \pm 1, \pm 5, \pm 17, \pm 85$

Step 3) Now use either synthetic division or long division to find the other factors $(x+5)$ is a factor

2

$$\begin{array}{r|rrrr} -5 & 1 & 13 & 57 & 85 \\ & & -5 & -40 & -85 \\ \hline & +1 & 8 & 17 & 0 \end{array}$$

$(x+5)(x^2+8x+17)$

$$x^2 + 8x + 17 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 4(17)}}{2} = \frac{-8 \pm \sqrt{-4}}{2}$$

$$= \frac{-8 \pm 2i}{2} = \boxed{-4 \pm 1i}$$

3

Step 4) List the Zeros of the above polynomial

$-5, -4+i, -4-i$

1 1 1

9) Factor Completely (10 points)

a) x^3+2x^2-x-2

$= x^2(x+2)-1(x+2)$

$=(x^2-1)(x+2)$

$\boxed{=(x-1)(x+1)(x+2)}$

b) x^4-1 $=(x^2-1)(x^2+1)$

$\boxed{=(x-1)(x+1)(x^2+1)}$

10) Perform the following: (10 points)

$4x^3-3x^2+x+1$ divided by $4x^2+1$

$$
\begin{array}{r|l}
 & x-3/4 \\
4x^2+0x+1 & 4x^3-3x^2+x+1 \\
 & -4x^3 - 0x^2 - 1x \\
\hline
 & -3x^2 + 1 \\
 & +3x^2 + 0x + \frac{3}{4} \\
\hline
 & 1+\frac{3}{4} = \frac{7}{4}
\end{array}
$$

$$\boxed{4x^3-3x^2+x+1 = \left(x-\frac{3}{4}\right)(4x^2+1)+\frac{7/4}{4x^2+1}}$$

11) Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is p dollars, the revenue R (in dollars) is $R(p) = -4p^2 + 4000p$ (8 points)

What unit price should be established for the dryer to maximize revenue?

(4)

$$P = \frac{-4000}{2(-4)} = \frac{-4000}{-8} = \boxed{\$500}$$

What is the maximum revenue? Maximum Revenue $= \boxed{\$1,000,000}$

(4)