Name: **Total Possible Points = 150 Points**

$$A(t) = A_0 e^{kt} \qquad\qquad u(t) = T + (u_0 - T)e^{kt} \qquad\qquad P(t) = \frac{c}{1 + ae^{-bt}}$$

1) Find the value of $\dfrac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = x^2 - 3x$

(Clearly state each of the steps of the process.)   (10 points)

2)   A piece of rectangular sheet metal is 12 inches wide. It is to be made into a rain gutter by turning up equal edges to form parallel sides. Let x represent the length of each of the parallel sides. For what value of x will the area of the cross section be a maximum (and thus maximize the amount of water that the gutter will hold)?   (6 points)

3) A developer wants to enclose a rectangular grassy lot that borders a city – street for parking. If the developer has 700 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed?   (9 points)

4)   For the rational function $R(x) = \dfrac{x^3 - 8}{x^2 - 5x + 6}$ (15 points)

a) Find the domain of the rational function

b) Write R(x) in the lowest terms

c) Locate the x-intercept(s)   Locate the y-intercept

d) Test for symmetry

e) Locate the vertical asymptote

f) Locate the horizontal or oblique asymptote if any

g) Graph the function using results from steps a to f

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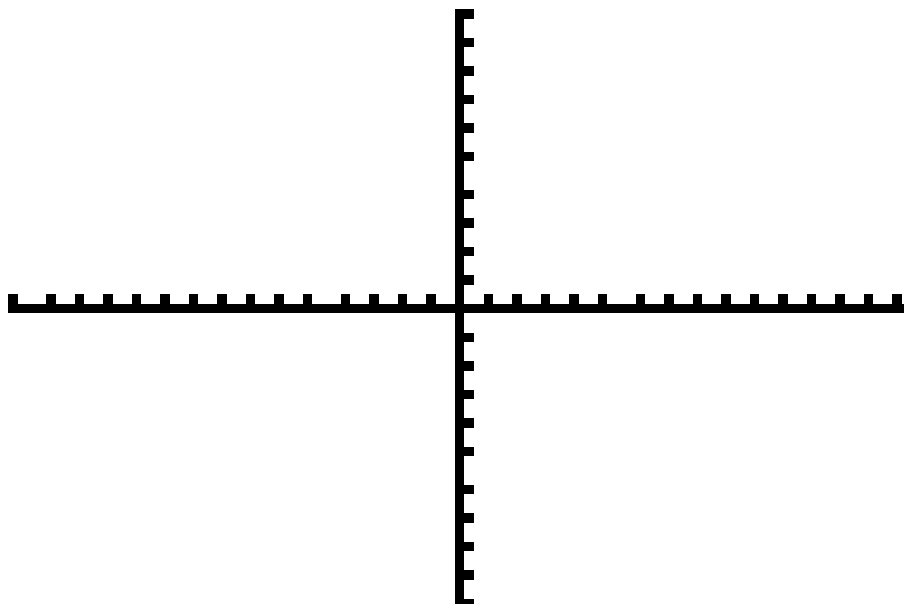
5) Form a polynomial f(x) with real coefficients having a
   degree of 5 and the following zeros $1, i, 3i$ (10 points)

6)   Find the inverse of the following functions.   (20 points)
      (Must Show All the Appropriate Steps)

a) $y = \sqrt[3]{x+3} + 6$

b) $f(x) = \dfrac{2x+5}{x-4}$

7) Solve the following algebraically: (15 points)

a) $7^x - 49^x = 0$

b) $e^{x^2} = \left(e^{5x}\right) \bullet \dfrac{1}{e^{-6}}$

c) If $3^x = 49$, what does 3^{-2x} equal?

8) Write each of the following expressions as a sum and / or difference of logarithms. (Express the powers as factors.) (10 points)

a) $\ln\left(\dfrac{\sqrt[5]{(x^2+5)}}{x^2-8}\right)$

b) $\log\left(\dfrac{u^2 v^3}{w^5 x^3}\right)$

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9)   Write the following expression as a single logarithm, and simplify (if possible) (10 points)

(Must Show All the Appropriate Steps)

$$\ln\left(\frac{x}{x-1}\right) - \ln\left(\frac{x+1}{x}\right) + \ln\left(x^2 - 1\right)$$

10) Solve the following equations algebraically.   (15 points)
(Must Show All the Appropriate Steps)

a) $\log x + \log(x+15) = 2\square\square$

b) $\ln(3+x) - \ln(x-4) = \ln(2)$

c) $\log(4x) = \log 2 + \log(x-1)$

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11) A fossilized leaf contains 20% of its normal amount of carbon-14. How old is the fossil (to the nearest year)? (Use 5600 years as the half – life of carbon 14)

(10 points)

12) A thermometer reading 80 degrees F is placed inside a cold storage room with a constant temperature of 32 degrees F. If the thermometer reads 75 degrees F in 15 minutes, how long will it take for the thermometer to reach 60 degrees F? Assume the cooling follows Newton's Law of Cooling (and Round your answer to the nearest whole minute)

(10 points)

13) The logistic growth model $P(t) = \dfrac{2100}{1 + 41e^{-0.3t}}$ represents the population of a

bacterium in a culture tube after t hours.

a)   What was the initial amount of bacteria in the population?   (2 points)

b)   What is the carrying capacity of this population?   (2 points)

c)   What is the population of bacteria after 7 hours?   (2 points)

d)   When will the population reach 2000?   (4 points)