MA 180 Professor Fred Katiraie Test III Form A (Spring 2007)

Name: **Total Possible Points = 150 Points**

$$A(t) = A_0 e^{kt} \qquad u(t) = T + (u_0 - T)e^{kt} \qquad P(t) = \frac{c}{1 + ae^{-bt}}$$

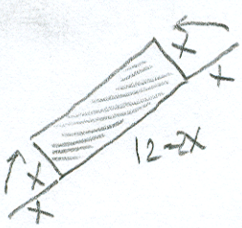
1) Find the value of $\frac{f(x+h)-f(x)}{h}$ assuming h is not zero for the function $f(x) = x^2 - 3x$
(Clearly state each of the steps of the process.) (10 points)

$$f(x+h) = (x+h)^2 - 3(x+h) = x^2 + 2xh + h^2 - 3x - 3h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{x^2} + 2xh + h^2 \cancel{-3x} - 3h \cancel{-x^2} \cancel{+3x}}{h}$$

$$= \frac{\cancel{h}(2x - 3 + h)}{\cancel{h}} = \boxed{2x - 3 + h}$$

2) A piece of rectangular sheet metal is 12 inches wide. It is to be made into a rain gutter by turning up equal edges to form parallel sides. Let x represent the length of each of the parallel sides. For what value of x will the area of the cross section be a maximum (and thus maximize the amount of water that the gutter will hold)? (6 points)

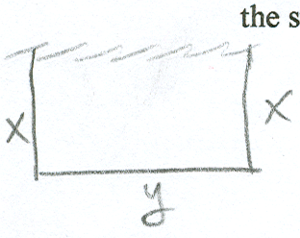


$$A = x(12 - 2x) = 12x - 2x^2$$

$$A = -2x^2 + 12x$$

$$x_{\text{Vertex}} = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = \boxed{3 \text{ inches}}$$

3) A developer wants to enclose a rectangular grassy lot that borders a city – street for parking. If the developer has 700 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed? (9 points)



$$2x + y = 700 \Rightarrow y = 700 - 2x$$

$$A = xy = x(700 - 2x) = -2x^2 + 700x$$

$$x = \frac{-b}{2a} = \frac{-700}{2(-2)} = 175 \text{ feet}$$

$$\boxed{A_{max} = 61250 \text{ ft}^2}$$

4) For the rational function $R(x)=\dfrac{x^3-8}{x^2-5x+6}$ $=\dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x-3)}$ (15 points)

a) Find the domain of the rational function

2pts

$\boxed{\{x \mid x \text{ is all Reals and } x \neq 2 \text{ and } x \neq 3\}}$

b) Write R(x) in the lowest terms

2pts

$\boxed{R(x)=\dfrac{x^2+2x+4}{x-3}}$

c) Locate the x-intercept(s)

2pts

$\boxed{\text{None}}$

Locate the y-intercept

$\boxed{(0, -\tfrac{4}{3})}$

d) Test for symmetry

2pts

$R(-x)=\dfrac{(-x)^3-8}{(-x)^2-5(-x)+6} \neq \begin{cases} -R(x) \\ R(x) \end{cases}$

$\boxed{\text{No symmetry}}$

e) Locate the vertical asymptote

2pts

$\boxed{\text{V.A} \quad x=3}$

f) Locate the horizontal or oblique asymptote if any

3pts

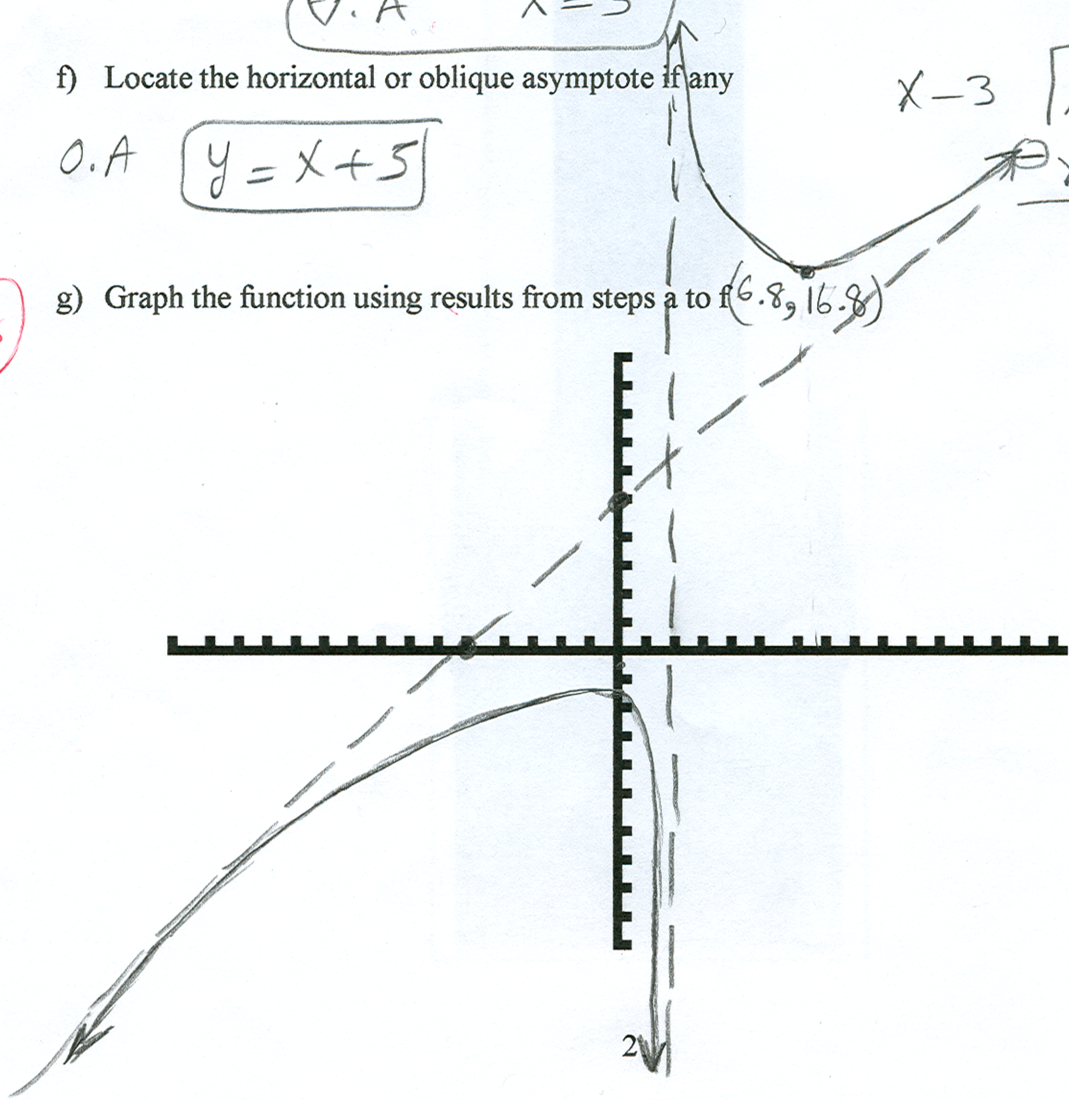
O.A $\boxed{y=x+5}$

$$
\begin{array}{r|l}
 & x+5 \\
\hline
x-3 & x^2+2x+4 \\
 & \ominus x^2 \oplus 3x \\
\hline
 & 5x+4 \\
 & \ominus 5x \oplus 15 \\
\hline
 & 19
\end{array}
$$

g) Graph the function using results from steps a to f

2pts

(6.8, 16.8)



MA 180 **Professor Fred Katiraie** **Test III Form B** **(Spring 2007)**

Name: **Total Possible Points = 150 Points**

$A(t) = A_0 e^{kt}$ $u(t) = T + (u_0 - T)e^{kt}$ $P(t) = \dfrac{c}{1 + ae^{-bt}}$

1) Find the value of $\dfrac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = x^2 - 3x$

(Clearly state each of the steps of the process.) (10 points)

$f(x+h) = (x+h)^2 - 3(x+h) = x^2 + 2xh + h^2 - 3x - 3h$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{x^2} + 2xh + h^2 \cancel{-3x} - 3h \cancel{-x^2} \cancel{+3x}}{h}$

$= \dfrac{\cancel{h}(2x - 3 + h)}{\cancel{h}} = \boxed{2x - 3 + h}$

2) Find the complex zeros of the following polynomial function

$f(x) = x^3 + 13x^2 + 57x + 85$ (10 points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros $= 3$

Step 2) Use the Rational Zeros Theorem to identify those rational numbers that potentially can be zeros.

2 pts $\boxed{\pm 1, \pm 5, \pm 17, \pm 85}$

Step 3) Now use long division to find the other factor

since $f(-5) = 0$ then $(x+5)$ is a factor

$$
\begin{array}{r}
x^2 + 8x + 17 \\
x + 5 \enclose{longdiv}{x^3 + 13x^2 + 57x + 85} \\
\ominus x^3 \ominus 5x^2 \\
\hline
8x^2 + 57x \\
\ominus 8x^2 \ominus 40x \\
\hline
17x + 85 \\
17x + 85
\end{array}
$$

4 pts

$x^2 + 8x + 17 = 0$

$x = \dfrac{-8 \pm \sqrt{64 - 4(1)(17)}}{2}$

$x = \dfrac{-8 \pm \sqrt{-4}}{2} = \dfrac{-8 \pm 2i}{2}$

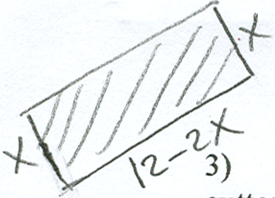
$\boxed{x = -4 \pm i}$

Step 4) List the Complex Zeros

2 pts

$x = -5 \rightarrow$ Real Zero

$x = -4 \pm i \rightarrow$ Complex Zeros



3) A piece of rectangular sheet metal is 12 inches wide. It is to be made into a rain gutter by turning up equal edges to form parallel sides. Let x represent the length of each of the parallel sides. For what value of x will the area of the cross section be a maximum (and thus maximize the amount of water that the gutter will hold)? (6 points)

$A = x(12-2x) = -2x^2+12x$ $\quad x_{vertex} = \frac{-12}{2(-2)} = \boxed{3 \text{ inches}}$

4) Find the real zeros of $f(x)=3x^3-x^2-15x+5$ use the following steps please)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros (2 points)

Max. No of Zeros = 3

Step 2) Use the Rational Zeros Theorem to identify rational numbers that potentially can be zeros (2 points)

$\pm 1, \pm \frac{1}{3}, \pm 5, \pm \frac{5}{3}$

Step 3) Using your calculator, graph the polynomial function to identify the rational zero. (2 points)

$f(\frac{1}{3}) = 0 \Rightarrow x - \frac{1}{3}$ is a factor $\Rightarrow$ $\boxed{(3x-1) \text{ is a factor}}$

Step 4) Identify the other real zeros of $f(x)=3x^3-x^2-15x+5$ by using long division, and quadratic formula. (6 points)

$$\begin{array}{r|l} & x^2 - 5 \\ 3x-1 & 3x^3 - x^2 - 15x + 5 \\ & \ominus 3x^3 \oplus x^2 \\ \hline & -15x + 5 \\ & \oplus 15x \ominus 5 \\ \hline & 0 \end{array}$$

$\Rightarrow x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

Step 5) Now, list all the real zeros of $f(x)=3x^3-x^2-15x+5$ (Please No Decimal Answers) (2 points)

The Zeros are: $\boxed{x = \frac{1}{3}, \quad x = \sqrt{5}, \quad x = -\sqrt{5}}$

Rational Zero

Irrational Zeros

5) Form a polynomial f(x) with real coefficients having a degree of 5 and the following zeros 1, i, 3i

(10 points)

$=(x-1)(x-i)(x+i)(x-3i)(x+3i)$ 2pts

$=(x-1)(x^2+1)(x^2+9)$ 2pts

$=(x-1)(x^4+9x^2+1x^2+9)$

$=(x-1)(x^4+10x^2+9)$ 2pts

$=x^5+10x^3+9x-x^4-10x^2-9$ 2pts

$\boxed{=x^5-x^4+10x^3-10x^2+9x-9}$ 2pts

6) Find the inverse of the following functions. (Must Show All the Appropriate Steps)

(20 points)

a) $y=\sqrt[3]{x+3}+6$

$y-6=\sqrt[3]{x+3}$

Solve for x: $(y-6)^3=x+3$

$(y-6)^3-3=x$

$x=(y-6)^3-3$

$\boxed{f^{-1}(x)=(x-6)^3-3}$ ✓

b) $f(x)=\dfrac{2x+5}{x-4}$

$y=\dfrac{2x+5}{x-4}$

Solve for x

$xy-4y=2x+5$

$xy-2x=4y+5$

$x(y-2)=4y+5$

$x=\dfrac{4y+5}{y-2}$

$\boxed{f^{-1}(x)=\dfrac{4x+5}{x-2}}$

(15 points)

7) Solve the following algebraically:

a) $7^x - 49^x = 0$

$7^x = 49^x$

$7^x = (7^2)^x$

$7^x = 7^{2x} \Longrightarrow x = 2x$

$\boxed{x = 0}$

b) $e^{x^2} = (e^{5x}) \bullet \frac{1}{e^{-6}}$

$e^{x^2} = e^{5x+6}$

$x^2 = 5x + 6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$\boxed{x = 6}$ $\boxed{x = -1}$

c) If $3^x = 49$, what does 3^{-2x} equal?

$3^{-2x} = (3^x)^{-2} = (49)^{-2}$

$= \boxed{\frac{1}{2401}}$

8) Write each of the following expressions as a sum and / or difference of logarithms. (Express the powers as factors.)

(10 points)

a) $\ln\left(\frac{\sqrt[5]{(x^2+5)}}{x^2-8}\right)$

$\ln\left(\sqrt[5]{x^2+5}\right) - \ln(x^2-8) = \ln(x^2+5)^{\frac{1}{5}} - \ln(x^2-8)$

$\boxed{= \frac{1}{5}\ln(x^2+5) - \ln(x^2-8)}$

b) $\log\left(\frac{u^2v^3}{w^5x^3}\right) = \log u^2 + \log v^3 - (\log w^5 + \log x^3)$

$\boxed{= 2\log u + 3\log v - 5\log w - 3\log x}$

9) Write the following expression as a single logarithm, and simplify (if possible)

(10 points)

(Must Show All the Appropriate Steps)

$$\ln\left(\frac{x}{x-1}\right)-\ln\left(\frac{x+1}{x}\right)+\ln\left(x^2-1\right)$$

3pts $\ln\left(\frac{x}{x-1} \div \frac{x+1}{x} \cdot (x^2-1)\right)$

$\ln\left(\frac{x}{\cancel{(x-1)}} \cdot \frac{x}{\cancel{(x+1)}} \cdot \cancel{(x-1)}\cancel{(x+1)}\right) = \boxed{\ln x^2 \quad \text{OR} \quad 2\ln x}$

6 pts

1 pt

10) Solve the following equations algebraically. (15 points)
(Must Show All the Appropriate Steps)

a) $\log x+\log(x+15)=2$

$\log x(x+15)=2 \longrightarrow \log_{10}(x^2+15x)=2$

$x^2+15x=100$

$x^2+15x-100=0 \longrightarrow (x+20)(x-5)=0$

$\cancel{x=-20}$ $\boxed{x=5}$

b) $\ln(3+x)-\ln(x-4)=\ln(2)$

$\cancel{\ln}\left(\frac{3+x}{x-4}\right)=\cancel{\ln}\,2$

$\frac{3+x}{x-4}=\frac{2}{1} \longrightarrow 3+x=2x-8$

$+8 \;\; -x \qquad -x \;\; +8$

$\boxed{11=x}$

c) $\log\left(4x\right)=\log 2+\log\left(x-1\right)$

$\log(4x)=\log(2(x-1))$

$4x=2x-2$

$-2x \qquad -2x$

$2x=-2 \longrightarrow \cancel{\boxed{x=-1}}$ No Solution

$$A = A_0 e^{kt}$$

11) A fossilized leaf contains 20% of its normal amount of carbon-14. How old is the fossil (to the nearest year)? (Use 5600 years as the half – life of carbon 14)

(10 points)

$$\frac{1}{2} = 1e^{5600k}$$

$$\ln\left(\frac{1}{2}\right) = 5600k$$

$$\frac{\ln(1/2)}{5600} = k$$

$$-1.238\times10^{-4} = k$$

$$A = A_0 e^{kt}$$

$$0.20 = 1e^{-1.2377628\times10^{-4}t}$$

$$\frac{\ln 0.20}{-1.238\times10^{-4}} = t$$

$$13003 \text{ years} = t$$

Answers vary Based on Round off errors

12) A thermometer reading 80 degrees F is placed inside a cold storage room with a constant temperature of 32 degrees F. If the thermometer reads 75 degrees F in 15 minutes, how long will it take for the thermometer to reach 60 degrees F? Assume the cooling follows Newton's Law of Cooling (and Round your answer to the nearest whole minute)

$$u(t) = T + (u_0 - T)e^{kt}$$

(10 points)

$$75 = 32 + (80-32)e^{15k}$$

$$43 = 48e^{15k}$$

$$\ln\left(\frac{43}{48}\right) = e^{15k}$$

$$-0.0073 = k$$

$$60 = 32 + (80-32)e^{-0.0073t}$$

$$28 = 48e^{-0.0073t}$$

$$\frac{\ln\left(\frac{28}{48}\right)}{-0.0073} = t$$

$$73.8 = t$$

$$t \approx 74 \text{ min}$$

13) The logistic growth model $P(t) = \dfrac{2100}{1+41e^{-0.3t}}$ represents the population of a bacterium in a culture tube after t hours.

a) What was the initial amount of bacteria in the population? (2 points)

$$P(0) = \frac{2100}{1+41e^{0}} = \frac{2100}{42} = \boxed{50}$$

b) What is the carrying capacity of this population? (2 points)

The carrying capacity of this population is 2100

that means the population can not go beyond 2100.

c) What is the population of bacteria after 7 hours? (2 points)

$$P(7) = \frac{2100}{1+41e^{-0.3*7}} = 348.796 \ \boxed{\approx 349}$$

d) When will the population reach 2000? (Must Show Algebraic Procedure) (4 points)

$$2000 = \frac{2100}{1+41e^{-0.3t}}$$

$$2100 = 2000\left(1+41e^{-0.3t}\right)$$

$$\frac{2100}{2000} = 1+41e^{-0.3t}$$

$$1.05 = 1+41e^{-0.3t}$$

$$0.05 = 41e^{-0.3t}$$

$$\frac{0.05}{41} = e^{-0.3t}$$

$$\ln\frac{0.05}{41} = -0.3t$$

$$22.4 = t$$

Hours