MA 180 Professor Fred Katiraie Pre Calculus Test [redacted] Three

Name: KEY Date:

$$A(t) = A_0 e^{kt} \qquad u(t) = T + (u_0 - T)e^{kt} \qquad P(t) = \frac{c}{1 + ae^{-bt}}$$

1) The function $f(x) = \frac{3}{x^{\frac{1}{3}}}$ is one-to-one. (15 Points)

a) Find the domain and range of $f(x) = \frac{3}{x^{\frac{1}{3}}}$

3pts Domain: all Reals except $x \neq 0$

3pts Range All Reals except $y \neq 0$

b) Find the inverse of the above function.

5pts $y = \frac{3}{x^{1/3}} \Rightarrow x = \frac{3}{y^{1/3}} \Rightarrow xy^{\frac{1}{3}} = 3 \Rightarrow y^{\frac{1}{3}} = \frac{3}{x}$

$y = \frac{27}{x^3}$

1pts 1pts

$\boxed{f^{-1}(x) = \frac{27}{x^3}}$

c) Find the domain and range of the inverse function.

2pts Domain: All Reals except Zero $(x \neq 0)$

2pts Range: All Reals except $(y \neq 0)$

2) Solve the following: (15 Points)

a) $4^x - 2^x = 0$ 5pts

5pts

$2^{2x} - 2^x = 0$

$2^{2x} = 2^x$

$2x = x \Rightarrow \boxed{x = 0}$

b) $e^{x^2} = \left(e^{3x}\right) \bullet \frac{1}{e^2}$

$e^{x^2} = e^{3x-2}$

$x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$\boxed{x = 2}$ or $\boxed{x = 1}$

c) If $4^x = 7$, what does 4^{-2x} equal?

5pts

$4^{-2x} = (4^x)^{-2} = 7^{-2} = \boxed{\frac{1}{49}}$

3) Write each of the following expressions as a sum and/or difference of logarithms. (Note: Express powers as factors) (15Points)

a) $\ln\left(\dfrac{\sqrt[3]{(x^2+5)}}{x^2-49}\right)^{\frac{1}{5}} = \dfrac{1}{5}\ln\left(\dfrac{\sqrt[3]{x^2+5}}{x^2-49}\right)$ (2pts)

$= \dfrac{1}{5}\left[\ln\sqrt[3]{x^2+5} - \ln(x^2-49)\right]$ (2pts)

$= \dfrac{1}{5}\left[\dfrac{1}{3}\ln(x^2+5) - \ln(x+7) - \ln(x-7)\right]$ (4pts)

(2pts)

$= \dfrac{1}{15}\ln(x^2+5) - \dfrac{1}{5}\ln(x+7) - \dfrac{1}{5}\ln(x-7)$

b) $\log_a\left(\dfrac{u^2v^3}{w^5}\right) = \log_a u^2 + \log_a v^3 - \log_a w^5$ (3pts)

(4pts)

$= 2\log_a u + 3\log_a v - 5\log_a w$

4) **Write the following expression as a single logarithm.** (10 Points)

$$\ln\left(\frac{x-1}{x}\right)+\ln\left(\frac{x}{x+1}\right)-\ln\left(x^2-1\right)$$

2 pts

$= \ln\left(\frac{X-1}{\cancel{X}} \cdot \frac{\cancel{X}}{X+1} \div X^2-1 \right)$

2 pts 2 pts 2 pts

$= \ln\left(\frac{\frac{X-1}{X+1}}{X^2-1} \right) = \ln\left(\frac{\frac{X-1}{X+1}}{(X+1)(X-1)} \right) = \ln\left(\frac{\cancel{X-1}}{X+1} \cdot \frac{1}{(X+1)(\cancel{X-1})} \right)$

$= \ln\left(\frac{1}{(X+1)^2} \right) = \boxed{-2\ln(X+1)}$

2 pts

5) **Find the domain of the following logarithmic function:** (10 Points)

$$\log\left(\frac{x+1}{x-1}\right)$$

$\frac{X+1}{X-1} > 0$ 2 pts

$- - - - 0 + + + +$ (-1, 0) $X+1$ 2 pts

$- - - - - 0 + +$ (0, 1) $X-1$ 2 pts

$+ + + 0 - - 0 + + +$ (-1, 1) $\frac{X+1}{X-1}$ 2 pts

Hence, Domain:

2 pts

$X < -1$ OR $X > 1$

$(-\infty, -1) \cup (1, \infty)$

6) **Solve the following equation algebraically. (Hint: Recall factoring method)** (10 Points)

$4^x - 2^x - 12 = 0$

$2^{2X} - 2^X - 12 = 0$

2 pts 2 pts

$(2^X - 4)(2^X + 3) = 0$

2 pts 2 pts

$2^X = 4$ $\quad$ $2^X = -3$

$\boxed{X = 2}$ $\quad$ No soln

2 pts

$\boxed{\text{Answer: } X = 2}$

7) A fossilized leaf contains 14% of its normal amount of carbon–14. How old is the fossil (to the nearest year)? Use 5600 years as the half–life of carbon 14. (15 Points)

$A(t) = A_0 e^{Kt}$

$\frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{K(5600)}$

6 points

$\ln \frac{1}{2} = K(5600) \cancel{\ln e}^{1}$

$K = \frac{\ln(1/2)}{5600}$

$A(t) = A_0 e^{Kt}$

$14\% = 100\% \, e^{Kt}$

3 pts

$0.14 = 1.00 \, e^{\frac{\ln(\frac{1}{2})}{5600} t}$

$\ln 0.14 = \ln e^{\frac{\ln(\frac{1}{2})}{5600} t}$

$\ln 0.14 = \frac{\ln(\frac{1}{2})}{5600} t$ 3 pts

$t = \frac{\ln(0.14)}{\frac{\ln(1/2)}{5600}}$ 3 pts

$t = 15884$ 15,884 years ago

8) A thermometer reading 79°F is placed inside a cold storage room with a constant temperature of 35°F. If the thermometer reads 74°F in 13 minutes, how long before it reaches 57°F? Assume the cooling follows Newton's Law of Cooling (Round your answer to the nearest whole minute.) (15 Points)

$u(t) = T + (u_0 - T) e^{Kt} \longrightarrow u(t) = 35 + (79 - 35) e^{Kt}$

8 pts

$74 = 35 + (79 - 35) e^{K(13)}$

$39 = 44 e^{K(13)}$

$K = -0.009279076$

4 pts

$57° = 35 + 44 e^{-0.009279076 t}$

3 pts

$t = 74.70$ minutes

Hence, After 74.70 mintes the termometer will read 57°F

9) **The logistic growth model** $P(t)=\dfrac{1240}{1+40.33e^{-0.325t}}$ **represents the population of a bacterium in a culture tube after t hours. What was the initial amount of bacteria in the population?** (10 Points)

2pts

$$P(0)=\frac{1240}{1+40.33e^{-0.325(0)}}$$

8pts

$$\boxed{P(0)=30}$$

So, Initially the population was 30

10) **A life insurance company uses the following rate table for annual premiums for women for term life insurance. Use a graphing utility to fit** **an exponential function to the data**. **Predict the annual premium for a woman aged 70 years.** (15 Points)

Hint: After using your calculator, write your final equation in the form of $A(t)=A_0e^{kt}$

Age	35	40	45	50	55	60	65
Permium	$103	$133	$190	$255	$360	$503	$818

2pts

$y=8.944*(1.070334437)^x$

$y=A_0e^{kx}$ where $A_0=8.944$

$e^{kx}=1.070334437^x$

then $e^k=1.070334437$

$k=\ln(1.070334437)$

8 pts

Then $\boxed{A(t)=8.944e^{0.06797t}}$ $\Rightarrow$ premium is $\$1041.99$ OR

5pts

Also! Acceptable

$y=8.94e^{0.068x}$

$\$1044$

11) After introducing an inhibitor into a culture of luminescent bacteria, a scientist monitors the luminosity produced by the culture. Use a graphing utility to fit a logarithmic function to the data. Predict the luminosity after 20 hours. (10 Points)

Time, hrs	2	3	4	5	8	10	15
Luminosity	77.4	60.8	54.5	45.8	30.0	24.3	10.5

5pts $y = a + b\ln x = 98.75 - 32.66\ln x$

5pts the Luminosity after 20 Hours is

$\boxed{0.9023}$

12) A mechanic is testing the cooling system of a boat engine. He measures the engine's temperature over time. Use a graphing utility to fit a logistic function to the data. What is the carrying capacity of the cooling system? (10 Points)

time, min	5	10	15	20	25
temperature, °F	100	180	270	300	305

$$y = \frac{c}{1 + ae^{-bx}}$$

5pts $$= \frac{314.79}{1 + 7.863e^{-0.2459x}}$$

5pts the carrying capacity is $\boxed{314.79°F \approx 315°F}$