Name: **Total Possible Points = 150 Points**

$$A(t)=A_0e^{kt} \qquad u(t)=T+(u_0-T)e^{kt} \qquad P(t)=\frac{c}{1+ae^{-bt}}$$

1) Find the value of $\frac{f(x+h)-f(x)}{h}$ for the function $f(x)=2x^2-2x+3$

(Assume h is not zero, Clearly state each of the steps of the process.) (10 points)

$f(x+h)=2(x+h)^2-2(x+h)+3=2x^2+4xh+2h^2-2x-2h+3$

$\frac{f(x+h)-f(x)}{h}=\frac{2x^2+4xh+2h^2-2x-2h+3-2x^2+2x-3}{h}$

$=\frac{h(4x+2h-2)}{h}=4x+2h-2$

2) Find the inverse of the following functions. (10 points)
(Must Show All the Appropriate Steps)

a) $y=(x+2)^3-5$

$y+5=(x+2)^3$

$\sqrt[3]{y+5}=x+2$

$x=\sqrt[3]{y+5}-2$

$f^{-1}(x)=\sqrt[3]{x+5}-2$

b) $f(x)=\frac{2x-5}{x+6}$

$y=\frac{2x-5}{x+6}$

$xy+6y=2x-5$

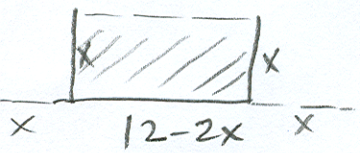
$xy-2x=-5-6y$

$x(y-2)=-5-6y$

$x=\frac{-5-6y}{y-2}$

$f^{-1}(x)=\frac{-5-6x}{x-2}$

3) A piece of rectangular sheet metal is 12 inches wide. It is to be made into a rain gutter by turning up equal edges to form parallel sides. Let x represent the length of each of the parallel sides. For what value of x will the area of the cross section be a maximum (and thus maximize the amount of water that the gutter will hold)? (5 points)



$A = x(12-2x) = 12x - 2x^2$

$A(x) = -2x^2 + 12x$

$x_{vertex} = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{12}{4} = 3$ inches

4) Solve the following algebraically: (6 points)

a) $7^x - 49^x = 0$

$7^x = 49^x$

$7^x = 7^{2x} \Rightarrow x = 2x$

$2x - x = 0$

$x = 0$

b) $e^{x^2} = \left(e^{5x}\right) \bullet \frac{1}{e^{-6}}$

$e^{x^2} = e^{5x+6}$

$x^2 = 5x + 6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6$ $x = -1$

c) If $3^x = 49$, what does 3^{-2x} equal?

$3^x = 49$ $3^{-2x} = (3^x)^{-2} = 49^{-2} = \frac{1}{49^2} = \frac{1}{2401}$

5) Solve the following equations algebraically. (10 points)
(Must Show All the Appropriate Steps)

a) $\log x + \log(x+15) = 2$

$\log(x(x+15)) = 2$ $\log_{10}(x^2+15x) = 2 \Rightarrow x^2+15x = 10^2$

$x^2 + 15x - 100 = 0$ $(x+20)(x-5) = 0$

$x = -20$ $x = 5$

b) $\ln(3+x) - \ln(x-4) = \ln(2)$

$\frac{3+x}{x-4} = \frac{2}{1} \Rightarrow 2x - 8 = 3 + x$

$x = 11$

c) $\log_3(x) - \log_3(x-6) = 4$

$\log_3\left(\frac{x}{x-6}\right) = 4 \Rightarrow \frac{x}{x-6} = 81 \Rightarrow 81x - 486 = x$

$80x = 486 \Rightarrow x = 6.075$

6) A fossilized leaf contains 75% of its normal amount of carbon-14.
How old is the fossil (to the nearest year)?
(Use 5600 years as the half – life of carbon 14)
(Must Show All the Appropriate Steps) (9 points)

$A = A_0 e^{kt}$

$\frac{1}{2} = e^{5600k}$

$k = \frac{\ln(\frac{1}{2})}{5600} = -1.238 \times 10^{-4}$

$0.75 = 1.00\, e^{-1.238 \times 10^{-4} t}$

$\ln(0.75) = -1.238 \times 10^{-4}\, t$

$t = \frac{\ln(0.75)}{-1.238 \times 10^{-4}} = 2324.21$

≈ 2324 years

7) An object is traveling around a circle with a radius of 20 meters. If in 10 seconds a central angle of $\frac{1}{5}$ radian is swept out, what is the linear speed of the object? (5 points)

$v = r\omega = r\frac{\theta}{t} = (20)\left(\frac{\frac{1}{5}}{10}\right) = 20\left(\frac{1}{50}\right) = \frac{2}{5}$ m/sec

8) An irrigation sprinkler in a field of lettuce sprays water over a distance of 40 feet as it rotates through an angle of 155°. What area of the field receives water? Round the answer to two decimal places. (5 points)

$155^\circ * \frac{\pi}{180^\circ} = \frac{155\pi}{180}$ radians

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(40)^2\left(\frac{155\pi}{180}\right) = 2164.21\ ft^2$

9) Salt Lake City, Utah, is due north of Flagstaff, Arizona. Find the distance between Salt Lake City ($40^\circ 45'$ north latitude) and Flagstaff ($35^\circ 16'$ north latitude). Assume that the radius of the Earth is 3960 miles. Round to nearest whole mile. (5 points)

$\theta = 40^\circ 45' - 35^\circ 16' = 5.48^\circ * \frac{\pi}{180^\circ} = 0.03\pi$ radians

$S = r\theta = (3960 \text{ miles})(0.030\pi) = 120.6\pi = 379$ miles

10) TRUE/FALSE. Circle one. (2.5 pts each)

a) True (False) $\sin(\sin^{-1}\pi)=\pi$

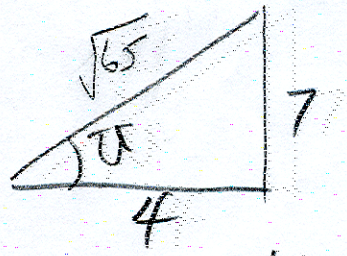
b) (True) False The period of $y=\cos\left(\frac{\pi}{3}x\right)$ is greater than the period of $y=\tan\left(\frac{\pi}{3}x\right)$

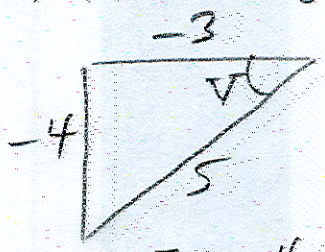
c) True (False) If $0<\theta<\frac{\pi}{2}$, *then* $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\pi$

d) True (False) If $\sin\theta=-\frac{3}{5}$, *then the* $\csc(-\theta)=-\frac{5}{3}$

11) Given: $\cot u=\frac{4}{7}$ for $0<u<\frac{\pi}{2}$ and $\cos v=-\frac{3}{5}$ for $\pi<v<\frac{3\pi}{2}$ (9 pts)

Find the **exact** value of $\tan(u+v)$ (Hint: Draw triangles in appropriate quadrants)

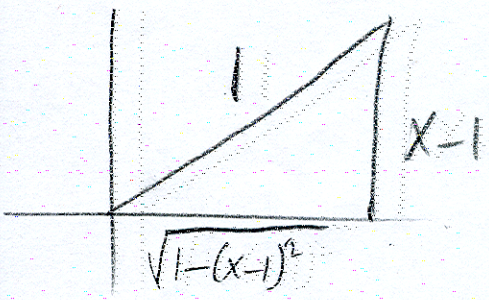




$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}=\frac{\frac{7}{4}+\frac{4}{3}}{1-\frac{7}{4}\cdot\frac{-4}{-3}}=\frac{-37}{16}$$

12) Find an algebraic expression for: $\sec(\sin^{-1}(x-1))$

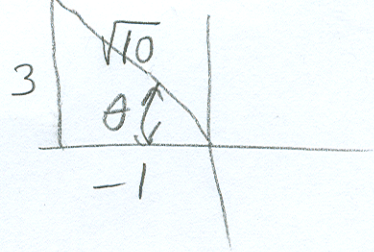
(Hint: Draw a triangle in the first quadrant) (8 pts)



$$\sec(\sin^{-1}(x-1))=\frac{1}{\sqrt{-x^2+2x}}=\frac{\sqrt{-x^2+2x}}{-x^2+2x}$$

$=\sqrt{1-(x^2-2x+1)}$

$=\sqrt{-x^2+2x}$



13) Given: $\tan\theta = -3$ *and* $\cos\theta < 0$

Find the **exact value** for $\sin 2\theta$. (8 pts)

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{-1}{\sqrt{10}}\right) = \frac{-6}{10} = \frac{-3}{5}$

14) Algebraically solve $2\cos(2x)+1=0$ over the interval $[0,2\pi)$: (8 pts)

$\cos(2x) = \frac{-1}{2} \Rightarrow 2x = \frac{2\pi}{3} + 2k\pi \qquad 2x = \frac{4\pi}{3} + 2k\pi$

$x = \frac{\pi}{3} + k\pi \qquad x = \frac{2\pi}{3} + k\pi$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

15) Solve algebraically over the interval $[0,2\pi)$: $4\sin^2(x)-3=0$ (10 pts)

$\sin^2(x) = \frac{3}{4} \Rightarrow \sin(x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2}$ OR $\sin x = -\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ $\qquad$ $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

16) Check whether the following identity is correct
(Is it possible to change the left hand side to look like the right side?) (8 pts)

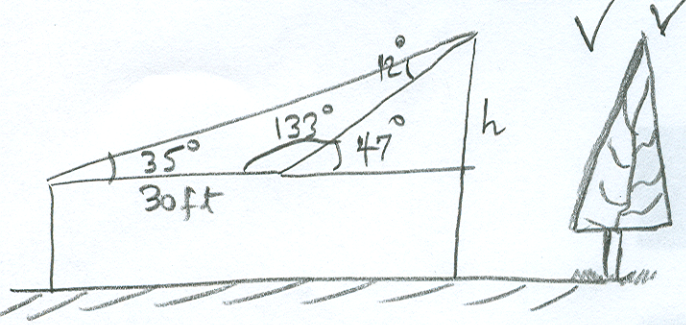
$\cos\left(\frac{\pi}{2}-x\right)\csc x = 1$

$\left(\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x\right)\frac{1}{\sin x} = \sin x\frac{1}{\sin x} = 1$

(0 = $\cos\frac{\pi}{2}$, 1 = $\sin\frac{\pi}{2}$)

Yes, Identity is valid

17) John (whose line of sight is 6 ft above horizontal) is trying to estimate the height of a tall oak tree. He first measures the angle of elevation from where he is standing as 35^0. He walks 30 feet closer to the tree and finds that the angle of elevation has increased by 12^0. Estimate the height of the tree rounded to the nearest whole number. (8 pts)



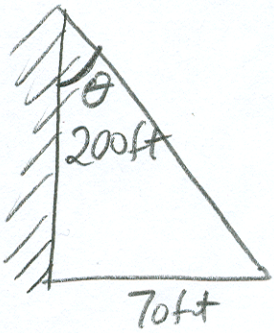
$\frac{\sin 12^\circ}{30} = \frac{\sin 35^\circ}{c} \Rightarrow c = 82.76$

$\sin 47^\circ = \frac{h}{c}$

$h = c \sin 47^\circ = 82.76 \sin 47^\circ = 60.5$ ft

Height of tree $= 60.5 + 6 = 66.5 \cong 67$ feet

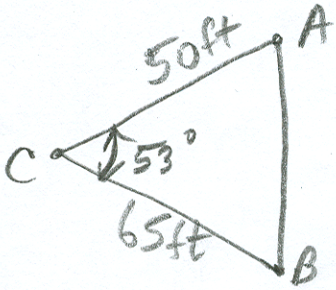
18) A building 200 feet tall casts a 70 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.) (8 pts)



$\tan\theta = \frac{70}{200}$

$\theta = \tan^{-1}\left(\frac{70}{200}\right) = 19.29 \approx 19^\circ$

19) Two points A and B are on opposite sides of a building.
A surveyor selects a third point C to place a transit. Point C is 50 feet from point A and 65 feet from point B. The angle ACB is 53^0. How far apart are points A and B? (8 pts)



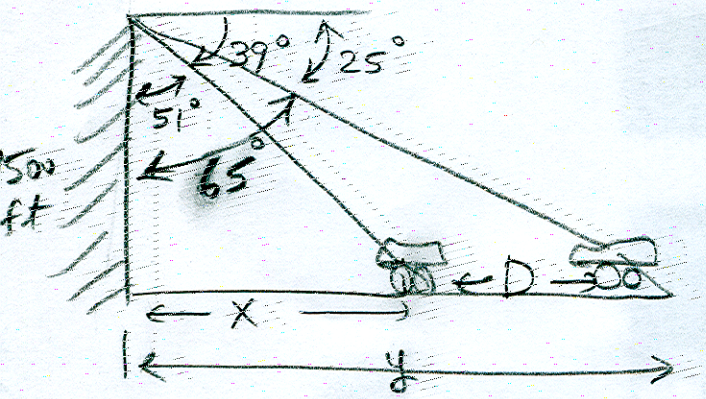
$AB^2 = 65^2 + 50^2 - 2(65)(50)\cos 53^\circ$

$AB^2 = 2813.2$

$A = 53.04 \approx 53$ ft

(8 POINTS EXTRA CREDIT)

From the edge of a 1500-foot cliff, the angles of depression to two cars in the valley below are 25^0 and 39^0. How far apart are the cars? (8 pts)
Round your answers to the nearest 0.1 ft.



$\tan 51^\circ = \frac{X}{1500} \Rightarrow X = 1500 \tan 51^\circ = 1852.35$ ft

$\tan 65^\circ = \frac{y}{1500} \Rightarrow y = 1500 \tan 65^\circ = 3216.76$ ft

$D = y - X = 3216.76 - 1852.35 = 1364.41$ feet