MA180 Professor Katiraie Exam Four [redacted]

All Problems are worth 7 points, unless listed otherwise.

Name Key

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

olve the problem.

1) A pick-up truck is fitted with new tires which have a diameter of 44 inches. How fast will the pick-up truck be moving when the wheels are rotating at 285 revolutions per minute? Express the answer in miles per hour rounded to the nearest whole number. (8 Points)

$$285 \frac{\cancel{\text{rev}}}{\text{min}} * \frac{2\pi \text{ radians}}{1 \cancel{\text{rev}}} =$$

$$V = r\omega = \left(\frac{44}{2}\right)(570\pi) = 12540\pi \frac{\text{inches}}{\text{min}} * \frac{1 \text{ feet}}{12 \text{ inch}} * \frac{1 \text{ mile}}{5280 \text{ feet}} * \frac{60 \text{ m}}{1 \text{ Hr}}$$

$$\boxed{\cong 37 \text{ mph}}$$

2) A circle has a radius of 5 yards. Find the area of the sector of the circle formed by an angle of 30°. Round your answer to two decimal places.

$$A = \frac{1}{2}\theta r^2 = \frac{1}{2}\left(\frac{\pi}{6}\right)(5)^2 = \frac{25\pi}{12} \text{ yards}^2$$

3) A ship in the Pacific Ocean measures its position to be 28°44' north latitude. Another ship is reported to be due north of the first ship at 41°20' north latitude. Approximately how far apart are the two ships? Round to the nearest mile. Assume that the radius of the Earth is 3960 miles.

$$28°44' = 28 + 44' * \frac{1°}{60'} = 28.73°$$

$$41°20' = 41.33°$$

$$S = r\theta = (3960)(41.33° - 28.73°) * \frac{1\pi}{180°} =$$

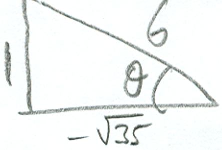
$$\boxed{S = 871 \text{ miles}}$$

4) If friction is ignored, the time t (in seconds) required for a block to slide down an inclined plane is given by the formula

$$t = \sqrt{\frac{2a}{g \sin\theta \cos\theta}}$$

where a is the length (in feet) of the base and g ≈ 32 feet per second per second is the acceleration of gravity. How long does it take a block to slide down an inclined plane with base a = 8 when θ = 45°? Round the final answer to the nearest tenth of a second.

$$t = \sqrt{\frac{2 * 8}{32 \sin 45° \cos 45°}} = 1 \text{ second}$$



$1^2 + x^2 = 6^2$

$x^2 = 35$

$x = -\sqrt{35}$

5) Given $\sin \theta = \frac{1}{6}$ and $\sec \theta < 0$, find

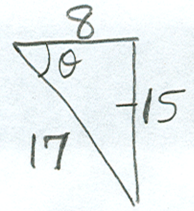
a) $\cos \theta = \boxed{-\frac{\sqrt{35}}{6}}$

b) $\tan \theta. = \frac{-1}{\sqrt{35}} \cdot \frac{\sqrt{35}}{\sqrt{35}} = \boxed{\frac{-\sqrt{35}}{35}}$

Find the exact value of the requested trigonometric function of θ.

6) $\cos\theta = \frac{8}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\cot \theta$.



$8^2 + y^2 = 17^2$

$y^2 = 17^2 - 8^2$

$y = -15$

$\cot\theta = \frac{8}{-15} = -\frac{8}{15}$

Solve the problem.

7) The average daily temperature T of a city in the United States is approximated by

$$T = 55 - 23 \cos \frac{2\pi}{365}(t - 30)$$

where t is in days, $1 \le t \le 365$, and $t = 1$ corresponds to January 1. Find the period of T.

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/365} = \frac{2\pi}{1} \cdot \frac{365}{2\pi} = \boxed{365 \text{ days}}$

Write the equation of a sine function with the given characteristics.

8) Amplitude: 2
Period: π
Phase Shift: $\frac{3}{2}$

$y = A \sin(\omega x - \phi) = A \sin \omega\left(x - \frac{\phi}{\omega}\right)$

$\frac{2\pi}{\omega} = \pi$

$\pi\omega = 2\pi \Rightarrow \boxed{\omega = 2}$

$y = 2 \sin 2\left(x - \frac{3}{2}\right) = \boxed{2 \sin(2x - 3)}$

Find the amplitude, period, and phase shift of the sinusoidal funtion.

9) $y = -\frac{3}{4} \sin\left(\frac{1}{4}x + \frac{\pi}{2}\right) = -\frac{3}{4} \sin\left[\frac{1}{4}(x + 2\pi)\right]$

$A = \frac{3}{4}$

$\text{Period} = \frac{2\pi}{\frac{1}{4}} = 8\pi$

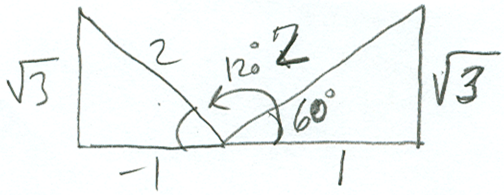
$\text{Phase shift} = -2\pi$

Find the exact value of the expression.

10) $\sec^{-1}(-2)$

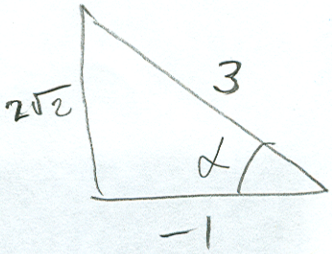
$\frac{-2}{1} = \boxed{\frac{2\pi}{3}}$

$\cos^{-1}\left(\frac{-1}{2}\right)$



$60° = \frac{\pi}{3}$

11) $\tan[\cos^{-1}(-\frac{1}{3})] = \boxed{-2\sqrt{2}}$



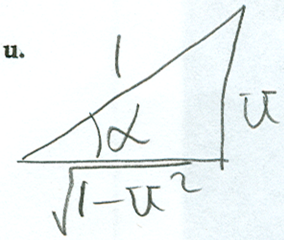
$(-1)^2 + y^2 = 9$

$y^2 = 8$

$\boxed{y = 2\sqrt{2}}$

Write the problem as an expression in u.

12) $\cos(\sin^{-1} u)$



$\boxed{\cos(\sin^{-1} u) = \sqrt{1-u^2}}$

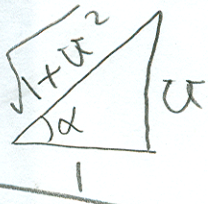
$x^2 + u^2 = 1$

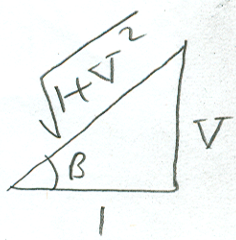
$x^2 = 1 - u^2$

$x = \sqrt{1-u^2}$

Write the trigonometric expression as an algebraic expression containing u and v.

13) $\sin(\tan^{-1} u + \tan^{-1} v)$





$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{u}{\sqrt{1+u^2}} \cdot \frac{1}{\sqrt{1+v^2}} + \frac{1}{\sqrt{1+u^2}} \cdot \frac{v}{\sqrt{1+v^2}} = \boxed{\frac{u+v}{\sqrt{1+u^2}\sqrt{1+v^2}}}$

Complete the identity.

14) $\frac{\cos(\alpha - \beta)}{\sin\alpha\cos\beta} = ?$

$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\cos\beta} = \frac{\cos\alpha\cancel{\cos\beta}}{\sin\alpha\cancel{\cos\beta}} + \frac{\cancel{\sin\alpha}\sin\beta}{\cancel{\sin\alpha}\cos\beta} = \boxed{\cot\alpha + \tan\beta}$

15) $\cos(\alpha + \beta)\cos(\alpha - \beta) = ?$

$$(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$= (\cos\alpha\cos\beta)^2 - (\sin\alpha\sin\beta)^2 = \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$$

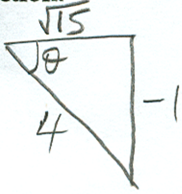
$$= (1-\sin^2\alpha)(\cos^2\beta) - \sin^2\alpha\sin^2\beta = \cos^2\beta - \sin^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$$

$$= \cos^2\beta - \sin^2\alpha\,[\cos^2\beta + \sin^2\beta) = \boxed{\cos^2\beta - \sin^2\alpha}$$

Using the information given, find the exact value of the trigonometric function.

16) Find $\cos\frac{\theta}{2}$, given that $\sin\theta = -\frac{1}{4}$, and $\frac{3\pi}{2} < \theta < 2\pi$.

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$ Quad 2



$16 = 1 + x^2$

$x = \sqrt{15}$

$$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\cos\theta}{2}}$$

$$= -\sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = -\sqrt{\frac{4+\sqrt{15}}{8}} = -\frac{\sqrt{4+\sqrt{15}}}{\sqrt{8}} = -\frac{\sqrt{4+\sqrt{15}}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{-\frac{\sqrt{8+2\sqrt{15}}}{4}}$$

Solve the equation for solutions in the interval $0 \le \theta < 2\pi$.

17) $\cos 2x = \sqrt{2} - \cos 2x$

$2\cos 2x = \sqrt{2}$

$\cos 2x = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + 2k\pi$

$x = \frac{\pi}{8} + k\pi \Longrightarrow \boxed{x = \frac{\pi}{8}}, \boxed{x = \frac{9\pi}{8}}$

OR

$2x = \frac{7\pi}{4} + 2k\pi \Longrightarrow \boxed{x = \frac{7\pi}{8}}$ $\boxed{x = \frac{15\pi}{8}}$

$x = \frac{7\pi}{8} + k\pi$

Solve the equation on the interval $0 \le \theta < 2\pi$. Round answer to two decimal places.

18) $6\cos^2\theta - 7\cos\theta - 3 = 0$

$$(3\cos\theta + 1)(2\cos\theta - 3) = 0$$

$3\cos\theta + 1 = 0$

$\cos\theta = \frac{-1}{3}$

$2\cos\theta = 3$

$\cos\theta = \frac{3}{2}$ Impossible

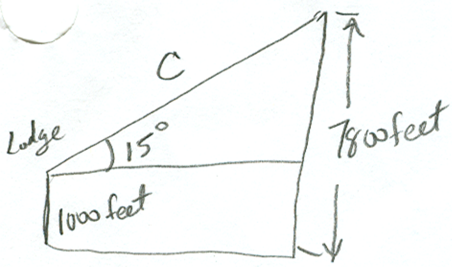
$\boxed{\theta = 109.47^\circ = 1.91 \text{ radians}} \Longrightarrow$

$\theta = 250.53^\circ$

$\theta \overset{4}{=} 4.37$ radians

Solve the problem.

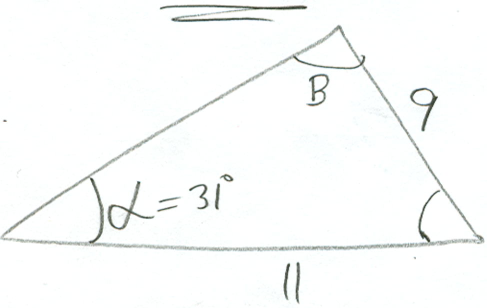
19) A straight trail with a uniform inclination of 15° leads from a lodge at an elevation of 1000 feet to a mountain lake at an elevation of 7800 feet. What is the length of the trail (to the nearest foot)?
(8 Points)



$$\sin 15^\circ = \frac{(7800 - 1000)}{C}$$

$$\Rightarrow C = \frac{6800}{\sin 15^\circ} = \boxed{26273 \text{ feet}}$$

20) Given a triangle with $a = 9$, $b = 11$, $\alpha = 31°$, what is (are) the possible length(s) of c ? Round your answer to two decimal places.



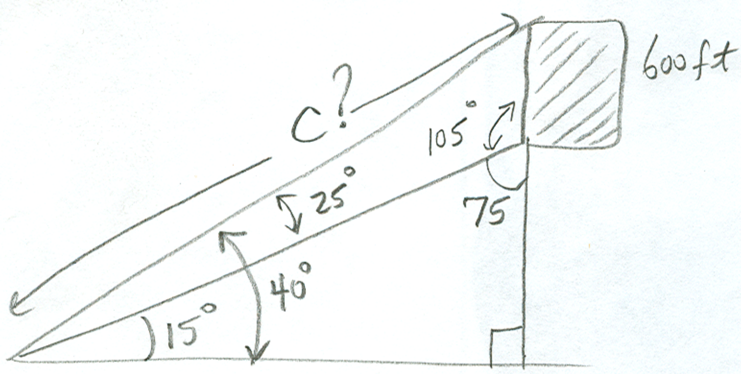
$$\frac{\sin 31}{9} = \frac{\sin B}{11} \Longrightarrow B = \sin^{-1}\left(\frac{11 \sin 31}{9}\right)$$

$$\boxed{C = 16.42} \quad \boxed{\gamma = 109.99^\circ} \Longleftarrow \boxed{B = 39.01^\circ}$$

OR

$$\boxed{C = 2.43} \quad \boxed{\gamma = 8.01^\circ} \Longleftarrow \boxed{B = 140.99^\circ}$$

21) A hill slopes at an angle of 15° with the horizontal. From the base of the hill, the angle of elevation of a 600 ft tower at the top of the hill is 40°. How much rope would be required to reach from the top of the tower to the bottom of the hill? Round answer to the nearest foot. (8 points)



AAS Law of Sines

$$\frac{\sin 25^\circ}{600} = \frac{\sin 105^\circ}{C}$$

$$C = \frac{600 \sin 105^\circ}{\sin 25^\circ} = 1371.35$$

$$\boxed{= 1371 \text{ foot}}$$

$V_0 = 180$ ft/sec $\theta = 40°$ $h = 3$

EXTRA CREDITS

22) Sammy Sosa hit a baseball with an initial speed of 180 feet per second at an angle of 40° to the horizontal. The ball was hit at a height of 3 feet off the ground. Find parametric equations that describe the motion of the ball as a function of time.

(5 Points)

$x = 180 \cos 40° t$ $y = -\frac{1}{2}(32)t^2 + 180 \sin 40° t + 3 = -16t^2 + (180 \sin 40)t + 3$

a) How long is the ball in the air?

$-16t^2 + 180 \sin 40° t + 3 = 0$ $\boxed{t = 7.26 \text{ seconds}}$

b) When is the ball at its maximum height?

$t = \frac{-b}{2a} = \frac{-180 \sin 40°}{2(-16)} = \boxed{3.62 \text{ seconds later}}$

c) What is the distance the ball traveled?

$x = (180 \cos 40) \times 7.26 = \boxed{1001.1 \text{ feet}}$

Solve the problem.

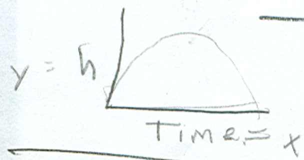
23) Ron throws a ball straight up with an initial speed of 40 feet per second from a height of 3 feet. Find parametric equations that describe the motion of the ball as a function of time. $\theta = 90°$

(5 Points)

$x = \boxed{(40 \cos 90°)t = 0}$ $y = -\frac{1}{2}(32)t^2 + (40 \sin 90°)t + 3 = \boxed{-16t^2 + 40t + 3}$

a) How long is the ball in the air?

$\boxed{t = 2.57 \text{ seconds}}$



b) When is the ball at its maximum height?

$t = \frac{-40}{2(-16)} = \frac{40}{32} = \boxed{1.25 \text{ sec}}$

c) What is the maximum height of the ball? $y_{max} = \boxed{28 \text{ feet}}$

Find a rectangular equation for the plane curve defined by the parametric equations.

24) x = 2t, y = t + 4, −2 ≤ t ≤ 3 (Three Points)

$t = \frac{x}{2}$

$y = t + 4$ $-2 \le t \le 3$

$\boxed{y = \frac{x}{2} + 4 \quad -4 \le x \le 6}$