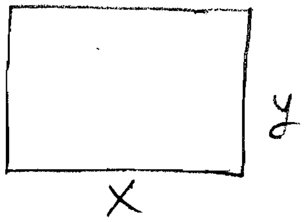
MA 180 Professor Fred Katiraie Test II Form A (Fall 2007)

Name: Solution **Total Possible Points = 150 Points**

1) David has 980 yards of fencing and wishes to enclose a rectangular area.

(2.5 points)

a) Express the area A of the rectangle as a function of the width x of the rectangle.



$$2x+2y=980 \Rightarrow x+y=490$$

$$y=490-x$$

$$A=xy=x(490-x)=\boxed{490x-x^2}$$

b) What is the domain of A?

(2.5 Points)

$$\boxed{0<x<490}$$

2) Find the center and radius of the circle with the given equation

(6 Points)

$$5x^2+5y^2-40x+60y-35=0$$

$$x^2+y^2-8x+12y-7=0$$

$$x^2-8x+16+y^2+12y+36=7+16+36$$

$$(x-4)^2+(y+6)^2=59$$

$$\boxed{\text{Center}=(4,-6) \qquad \text{Radius}=\sqrt{59}}$$

3) If (5a, 2a) is a point on the graph of 3x – 2y =17, what is a?

(5 points)

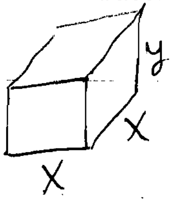
$$3(5a)-2(2a)=17$$

$$15a-4a=17$$

$$11a=17 \qquad \boxed{a=\frac{17}{11}}$$

4) An **open box with a square base** is required to have a volume of 40 cubic feet. Express the amount A of material used to make such a box as a function of the length x of a side of the base.

(5 points)



$$x^2y=40 \Rightarrow y=\frac{40}{x^2}$$

$$A=x^2+4x\frac{40}{x^2}=\boxed{x^2+\frac{160}{x}}$$

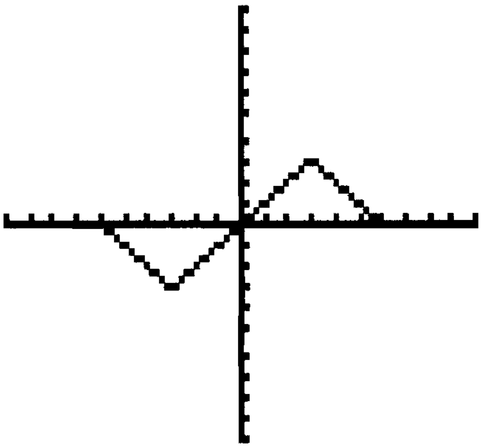
a) shift left three

b) Multiply the y-values by -2 and subtract 1

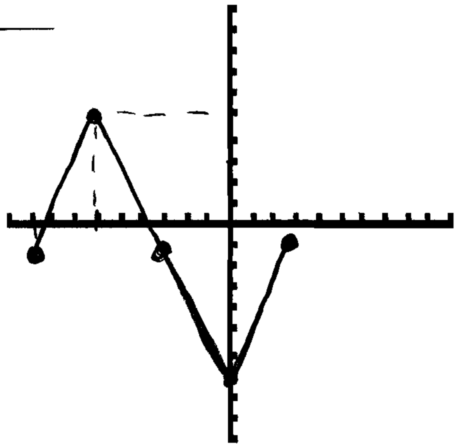
5) The graph of $y = f(x)$ is given below; Sketch a graph of $y = -2f(x+3)-1$

(10 points)

X	y
-6	0
-3	-3
0	0
3	3
6	0



X	y
-9	-1
-6	5
-3	-1
0	-7
3	-1



6) Find the value of $\frac{f(x+h)-f(x)}{h}$ assuming h is not zero for the function $f(x) = 2x^2 - 3x$

(Clearly state each of the steps of the process.) (10 points)

$$f(x+h) = 2(x+h)^2 - 3(x+h) = 2(x^2+2xh+h^2) - 3x - 3h$$

$$= 2x^2 + 4xh + 2h^2 - 3x - 3h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(\cancel{2x^2} + 4xh + 2h^2 \cancel{-3x} - 3h) - (\cancel{2x^2} \cancel{-3x})}{h}$$

$$= \frac{h(4x+2h-3)}{h} = \boxed{4x+2h-3}$$

7) Write an equation of the line passing through the point (6 , 5) and perpendicular to the line y = 3x – 5. (9 points)

$m_{\perp} = \frac{-1}{3}$

$5 = \frac{-1}{3}(6) + b$

$5 = -2 + b$

$+7 = b$

$\boxed{y = \frac{-1}{3}x + 7}$

8) A ball is thrown vertically upward with an initial velocity of 192 feet per second. The distance in feet of the ball from the ground after t seconds is $s = 192t - 16t^2$
For what interval of time is the ball more than 432 feet above the ground? **(6 points)**

$$192t - 16t^2 > 432 \implies 0 > 16t^2 - 192t + 432$$

$$0 > 16(t^2 - 12t + 27)$$

time is between 3 to 9 seconds

$$0 > 16(t-9)(t-3)$$

$$\boxed{3 < t < 9 \text{ seconds}}$$

9) A developer wants to enclose a rectangular grassy lot that borders a city – street for parking. If the developer has 600 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed? **(9 points)**

$$2X + y = 600 \implies y = 600 - 2X$$

$$A = X(600 - 2X) = 600X - 2X^2 = -2X^2 + 600X$$

$$X_{vertex} = \frac{-b}{2a} = \frac{-600}{2(-2)} = 150 \text{ feet}$$

The Largest Area = 45000 ft.^{2}

10) For the following function $f(x) = -2x(x-1)^2(x+2)$ Determine the following **(10 points)**

a) Find the x intercept

(0,0); (1,0); (−2,0)

b) Find the y intercept (0,0)

c) Determine whether the graph crosses or touches the x axis at each x intercept.

Cross at $x=0$; touch at $x=1$; cross at $x=-2$

d) End behavior: Find the power function that the graph of f(x) resembles for large values of |x|

$-2x^4$ ↙ ↘

e) Determine the number of turning points on the graph of f(x)

3

$2(x^2-7x+12)$

11) For the rational function $R(x)=\dfrac{2x^2-14x+24}{x^2+6x-40} = \dfrac{2(x-3)(x-4)}{(x+10)(x-4)}$ (16 points)

a) Find the domain of the rational function

Domain is all Reals except 4 and −10

b) Write R(x) in the lowest terms

$\boxed{R(x)= \dfrac{2(x-3)}{x+10}}$

c) Locate the x-intercept(s)

$(3,0)$

$0 = \dfrac{2(x-3)}{x+10}$

$2(x-3)=0$

$\boxed{x=3}$

d) Locate the y-intercept

$\left(0,\dfrac{-24}{40}\right) = \left(0,\dfrac{-3}{5}\right)$

e) Locate the vertical asymptote

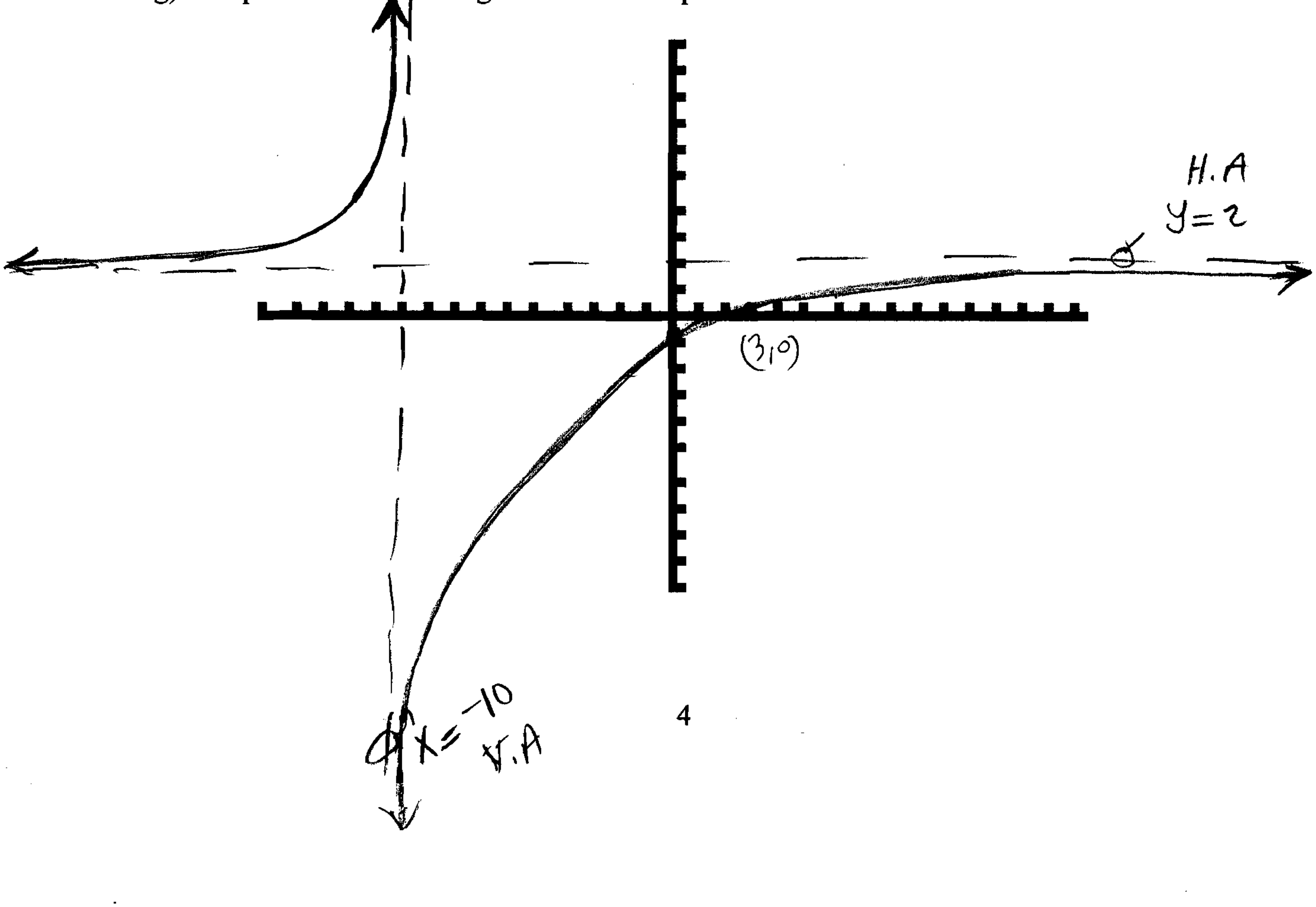
$x+10=0 \Rightarrow \quad x=-10$

f) Locate the horizontal or oblique asymptote if any

Since Degrees are Same

$y =$ Ratio of Coefficient $\Rightarrow y=\dfrac{2}{1}=2$

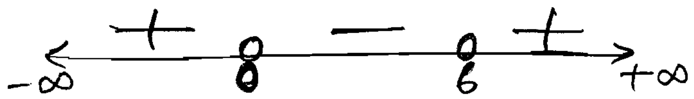
g) Graph the function using results from steps a to f



12) Solve the following inequalities algebraically,
and **write your answer in interval notation.** (10 points)

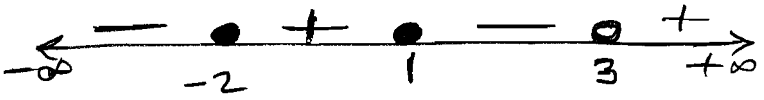
a) $x^2-6x>0$

$X(X-6)>0$



$\boxed{(-\infty, 0) \cup (6, \infty)}$

b) $\frac{(x-1)(x+2)}{(x-3)} \le 0$



$\boxed{(-\infty, -2] \cup [1, 3)}$

13) Find the real zeros of $f(x)=2x^3+5x^2-28x-15$ (use the following steps please)
(10 points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros

Max No of Zeros $= 3$

Step 2) Use the Rational Zeros Theorem to identify rational numbers that potentially can be zeros $\frac{p}{q} = \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

Step 3) Using your calculator, graph the polynomial function to identify the rational zero.

$X=-5$ and $X=3$ $\quad (X+5)(X-3) = X^2+2X-15$

Step 4) Identify the other real zeros of $f(x)$ by using long division, and quadratic formula.

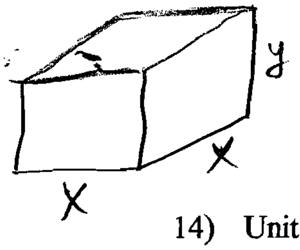
$$\begin{array}{r} 2X+1 \\ X^2+2X-15 \,\overline{)\,2X^3+5X^2-28X-15} \\ \ominus 2X^3 \overset{\ominus}{+}4X^2 \overset{\oplus}{-}30X \\ \hline X^2+2X-15 \\ \ominus X^2 \overset{\ominus}{+}2X \overset{\oplus}{-}15 \\ \hline 0 \end{array}$$

$2X+1=0$

$\boxed{X=-\frac{1}{2}}$

Step 5) Now, list all the real zeros of $f(x)$
(Please No Decimal Answers)

$\boxed{X=-5}$ $\boxed{X=3}$ $\boxed{X=-\frac{1}{2}}$



$$x^2 y = 10000 \Rightarrow y = \frac{10000}{x^2}$$

14) United Parcel service has contracted you to design a closed box with a square base that has a volume of 10000 cubic inches.

a) Find a function for the surface area of the box. (4 points)

$$A = 2x^2 + 4xy$$

$$= 2x^2 + 4x\left(\frac{10000}{x^2}\right) = \boxed{2x^2 + \frac{40000}{x}}$$

b) What are the dimensions of the box that minimize the surface area? (3 points)

21.5 by 21.5 by 21.6 inches

c) What is the minimum amount of cardboard that can be used to construct the box? (3 points)

Area = 2784.97 ≈ 2785 inch^2

15) Find the complex zeros of the following polynomial function
$f(x) = x^3 - 8x^2 + 25x - 26$ (10 points)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros

Max no of zeros = 3

Step 2) Use the Rational Zeros Theorem to identify those rational numbers that potentially can be zeros.

$$\frac{p}{q} = \pm 1, \pm 2, \pm 13, \pm 26$$

Step 3) Now use long division to find the other factors $f(2) = 0 \Rightarrow (x-2)$ is a factor

$$\begin{array}{r|l} & x^2 - 6x + 13 \\ \hline x - 2 & x^3 - 8x^2 + 25x - 26 \\ & \ominus x^3 \oplus 2x^2 \\ \hline & -6x^2 + 25x \\ & \oplus -6x^2 \mp 12x \\ \hline & 13x - 26 \\ & 13x - 26 \\ \hline & 0 \end{array}$$

$$x^2 - 6x + 13 = 0$$

$$x = \frac{+6 \pm \sqrt{36 - 4(1)(13)}}{2} = \frac{6 \pm 4i}{2}$$

Step 4) List all the zeros (real and complex) of $f(x) = x^3 - 8x^2 + 25x - 26$

$$\boxed{\begin{array}{l} x = 2 \\ x = 3 \pm 2i \end{array}}$$

16) For the following problems, give the equation of the specified asymptotes:

a) $g(x)=\dfrac{(2x-1)(x-2)}{(5x-2)(x+5)}$ Is there a Horizontal Asymptote? $\boxed{\text{Yes}}$ (2 points)

If so, what is the Equation of Horizontal Asymptote? $\boxed{y=\frac{2}{5}}$

b) $h(x)=\dfrac{8x-9x-7}{16x^2-8x+9}$ Is there a Horizontal Asymptote? $\boxed{\text{Yes}}$ (2 points)

If so, what is the Equation of the Horizontal Asymptote? $\boxed{y=0}$

c) $w(x)=\dfrac{x^2+x-30}{x+6}$ Is there an Oblique Asymptote? $\boxed{\text{Yes}}$ (4 points)

If so, what is the Equation of the Oblique Asymptote?

$$\begin{array}{r} x-5 \\ x+6\,\overline{)\,x^2+x-30} \\ x^2+6x \\ \hline -5x-30 \\ -5x-30 \\ \hline 0 \end{array}$$

$\boxed{y=x-5}$

d) $u(x)=\dfrac{x-1}{x^4-1}$ Are there any Vertical Asymptote(s)? $\boxed{\text{Yes}}$ (2 points)

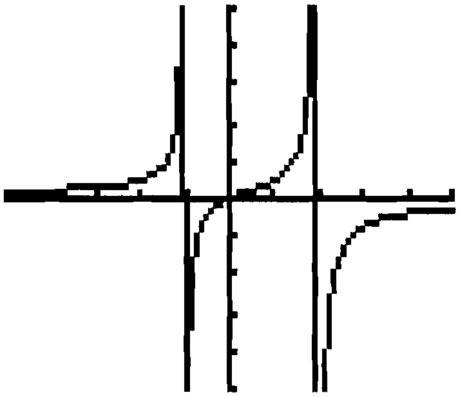
If so, what are the Equation(s) of the Vertical Asymptote(s)? $\boxed{x=-1}$

$$\frac{x-1}{(x^2+1)(x^2-1)}=\frac{\cancel{x-1}}{(x^2+1)(x+1)\cancel{(x-1)}}=\frac{1}{(x^2+1)(x+1)}$$

e) $f(x)=\dfrac{x-7}{x^2+49}$ Are there any Vertical Asymptote(s)? [✓ NO] (2 points)

If so, what are the Equation(s) of the Vertical Asymptote(s)?

17) Given the following graph



List the equation(s) of the following if any:

a) Horizontal Asymptote $\boxed{y = 0}$ (1/2 point)

b) Vertical Asymptote(s) $\boxed{X = -1}$, $\boxed{X = 2}$ (1/2 point)

c) Oblique Asymptote $\boxed{\text{NONE}}$ (1/2 point)

d) X intercept (if any) $\boxed{(0, 0)}$ (1/2 point)

e) Y intercept (if any) $\boxed{(0, 0)}$ (1/2 point)

f) What is a possible equation of a function whose graph is given above.
(Hint: Use your answers for parts a to e) (4.5 points)

$$\boxed{y = \frac{-X}{(X+1)(X-2)}}$$