MA 180 **Professor Fred Katiraie** **Test III Forms** A & B (Fall 2007)

Name: **Total Possible Points = 150 Points**

$A(t) = A_0 e^{kt}$ $u(t) = T + (u_0 - T)e^{kt}$ $P(t) = \dfrac{c}{1 + ae^{-bt}}$

1) Find the value of $\dfrac{f(x+h) - f(x)}{h}$ assuming h is not zero for the function $f(x) = \dfrac{1}{x} - 3$
(Clearly state each of the steps of the process.) (10 points)

step I) $f(x+h) = \dfrac{1}{x+h} - 3$

$f(x) = \dfrac{1}{x} - 3$

step II) $f(x+h) - f(x) = \dfrac{1}{x+h} - 3 - \left(\dfrac{1}{x} - 3\right)$

$= \dfrac{1}{x+h} - 3 - \dfrac{1}{x} + 3 =$

$= \dfrac{1}{x+h} - \dfrac{1}{x} = \dfrac{\cancel{x} - \cancel{x} - h}{x(x+h)}$

step III) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{-h}{x(x+h)}}{h} = \dfrac{-h}{x(x+h)} \cdot \dfrac{1}{h} = \boxed{\dfrac{-1}{x(x+h)}}$

2) **Find the complex zeros of the following polynomial function** (10 points)
$f(x) = x^3 - 8x^2 + 25x - 26$
Step 1) Use the degree of the polynomial to determine the maximum number of zeros

3

Step 2) Use the Rational Zeros Theorem to identify those rational numbers that potentially can be zeros.

$\pm 1, \pm 2, \pm 13, \pm 26$

Step 3) Now use long division to find the other factor, and $x = 2$ is a Zero of $f(x)$ then $(x-2)$ is a factor

$$\begin{array}{r|l} & x^2 - 6x + 13 \\ x-2 & x^3 - 8x^2 + 25x - 26 \\ & \ominus x^3 \oplus 2x^2 \\ \hline & -6x^2 + 25x - 26 \\ & \oplus 6x^2 \ominus 12x \\ \hline & 13x - 26 \end{array}$$

$x^2 - 6x + 13 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(1)(13)}}{2} = \dfrac{6 \pm \sqrt{-16}}{2}$

$= \dfrac{6 \pm 4i}{2} = \boxed{3 \pm 2i}$

Step 4) List the Complex Zeros

$x = 2$ Real Zero

$x = 3 \pm 2i$ Complex Zeros

3) For the following problems, give the equation of the specified asymptotes:

a) $g(x) = \dfrac{x^2 + 5x - 3}{x + 5}$ Is there a Horizontal Asymptote? $\boxed{\text{NO}}$ (1 point)

If so, what is the Equation of Horizontal Asymptote? $\boxed{\text{N/A}}$

b) $h(x) = \dfrac{8x^2 - 9x - 7}{16x^2 - 8x + 9}$ Is there a Horizontal Asymptote? $\boxed{\text{Yes}}$ (2 points)

If so, what is the Equation of the Horizontal Asymptote? $\boxed{y = \frac{8}{16} = \frac{1}{2}}$

c) $w(x) = \dfrac{x^2 + 2x - 2}{x - 2}$ Is there an Oblique Asymptote? $\boxed{\text{Yes}}$ (4 points)

If so, what is the Equation of the Oblique Asymptote?

$$\begin{array}{r|l} & x+4 \\ x-2 & x^2+2x-2 \\ & \ominus x^2 \oplus 2x \\ \hline & 4x-2 \end{array}$$

$\boxed{y = x+4 \text{ is the oblique Asymptote}}$

d) $u(x) = \dfrac{x-1}{x^2 - 1}$ Are there any Vertical Asymptote(s)? $\boxed{\text{yes}}$ (2 points)

If so, what are the Equation(s) of the Vertical Asymptote(s)? $u(x) = \dfrac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \dfrac{1}{x+1}$

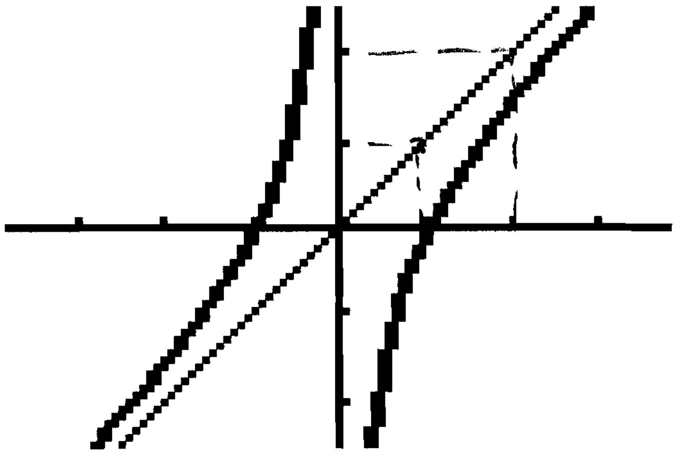
$x + 1 = 0$

$\boxed{x = -1}$

e) $f(x) = \dfrac{x - 9}{x^2 + 81}$ Are there any Vertical Asymptote(s)? $\boxed{\text{No}}$ (1 point)

If so, what are the Equation(s) of the Vertical Asymptote(s)? $\boxed{\text{N/A}}$

4) Given the following graph



List the equation(s) of the following if any:

a) Horizontal Asymptote None (1/2 point)

b) Vertical Asymptote $X=0$ (1/2 point)

c) Oblique Asymptote yes (1/2 point)

$y=x$

d) X intercept (if any) (1/2 point)

$(1,0)$ and $(-1,0)$

e) Y intercept (if any) (1/2 point)

None

a) What is a possible equation of a function whose graph is given above.
(Hint: Use your answers for parts a to e) (5.5 points)

$$y=\frac{(x+1)(x-1)}{x}$$

5) Find the real zeros of $f(x) = x^3 - 8x^2 + 17x - 6$ use the following steps please)

Step 1) Use the degree of the polynomial to determine the maximum number of zeros (2 points)

Max No of Zeros $= 3$

Step 2) Use the Rational Zeros Theorem to identify rational numbers that potentially can be zeros (2 points)

$\pm 1, \pm 2, \pm 3, \pm 6$

Step 3) Using your calculator, graph the polynomial function to identify the rational zero. (2 points)

$x = 3$ is a Rational Zero

therefore $x - 3$ is a factor

Step 4) Identify the other real zeros of $f(x) = x^3 - 8x^2 + 17x - 6$ by using long division, and quadratic formula. (6 points)

$$
\begin{array}{r|l}
 & x^2 - 5x + 2 \\
\hline
x - 3 & x^3 - 8x^2 + 17x - 6 \\
 & \ominus\, x^3 \oplus 3x^2 \\
\hline
 & -5x^2 + 17x \\
 & \oplus\, -5x^2 \ominus 15x \\
\hline
 & \ominus\, 2x \oplus 6 \\
 & \quad 0
\end{array}
$$

Step 5) Now, list all the real zeros of $f(x) = x^3 - 8x^2 + 17x - 6$
(Please No Decimal Answers) (2 points)

$x^2 - 5x + 2 = 0$

$$x = \frac{5 \pm \sqrt{25 - 4(1)(2)}}{2} = \boxed{\frac{5 \pm \sqrt{17}}{2}}$$

there fore the Zeros are

$$\boxed{\begin{array}{l} x = 3 \\ x = \dfrac{5 + \sqrt{17}}{2} \\ x = \dfrac{5 - \sqrt{17}}{2} \end{array}}$$

6) Find the inverse of the following functions. (20 points)
(Must Show All the Appropriate Steps)

a) $y=(x+2)^3-5$

$y+5 = (x+2)^3$

$\sqrt[3]{y+5} = x+2$

$\sqrt[3]{y+5} - 2 = x$

$x = \sqrt[3]{y+5} - 2$

$f^{-1}(x) = \sqrt[3]{x+5} - 2$

b) $f(x)=\dfrac{2x-5}{x+6}$

$y = \dfrac{2x-5}{x+6}$

$xy + 6y = 2x - 5$

$xy - 2x = -6y - 5$

$x(y-2) = -6y-5$

$x = \dfrac{-6y-5}{y-2}$

$f^{-1}(x) = \dfrac{-6x-5}{x-2}$ OR $\boxed{\dfrac{6x+5}{2-x}}$

7) Solve the following algebraically: (15 points)

a) $\left(\dfrac{1}{5}\right)^{2-x} = 25$

$(5^{-1})^{2-x} = 5^2$

$5^{-2+x} = 5^2$

$-2+x = 2 \Rightarrow \boxed{x = +4}$

b) $e^{x^2} \bullet \dfrac{1}{e^6} = (e^{5x})$

$e^{x^2} \cdot e^{-6} = e^{5x}$

$x^2 - 6 - 5x = 0$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$\boxed{x=6}$ $\boxed{x=-1}$

c) If $3^x = \dfrac{1}{49}$, what does 3^{-2x} equal?

$3^{-2x} = (3^x)^{-2} = \left(\dfrac{1}{49}\right)^{-2} = \boxed{2401}$

8) Write each of the following expressions as a sum and / or difference of logarithms. (Express the powers as factors.) (10 points)

a) $$\ln\left(\frac{\sqrt[7]{(x^2-9)}}{\sqrt[5]{(5x^2)}}\right) = \ln\sqrt[7]{x^2-9} - \ln\sqrt[5]{(5x^2)}$$

$$= \frac{1}{7}\ln(x^2-9) - \frac{1}{5}\ln(5x^2)$$

$$\boxed{= \frac{1}{7}\ln(x+3) + \frac{1}{7}\ln(x-3) - \frac{1}{5}\ln(5x^2)}$$

$$\cancel{= \frac{1}{7}\ln(x+3) + \frac{1}{7}\ln(x-3) - \frac{1}{5}\ln 5} - \frac{2}{5}\ln x$$

b) $$\log\left(\frac{u^2-v^3}{w^5+x^3}\right)^5$$

$$= 5\log\left(\frac{u^2-v^3}{w^5+x^3}\right) = \boxed{5\left[\log(u^2-v^3) - \log(w^5+x^3)\right]}$$

9) Write the following expression as a single logarithm, and simplify (if possible) (10 points)

(Must Show All the Appropriate Steps)

$$\ln\left(x^2-9\right) + \ln\left(\frac{x+5}{x-3}\right) - \ln\left(\frac{x+3}{x+5}\right)$$

$$= \ln\left((x+3)(x-3)\cdot\frac{x+5}{x-3} \div \frac{x+3}{x+5}\right)$$

$$= \ln\left(\cancel{(x+3)}\cancel{(x-3)}\frac{(x+5)}{\cancel{(x-3)}}\cdot\frac{x+5}{\cancel{(x+3)}}\right) = \ln(x+5)^2$$

$$\boxed{= 2\ln(x+5)}$$

10) Solve the following equations algebraically. (15 points)
(Must Show All the Appropriate Steps)

a) $\log_{\frac{1}{3}}(x^2+x)-\log_{\frac{1}{3}}(x^2-x)=-1$

$\log_{\frac{1}{3}}\left(\frac{x^2+x}{x^2-x}\right)=-1 \Longrightarrow \frac{x^2+x}{x^2-x}=\left(\frac{1}{3}\right)^{-1}$

$\frac{x^2+x}{x^2-x}=3 \Longrightarrow x^2+x=3x^2-3x$

$\Longrightarrow 2x^2-4x=0$

$2x(x-2)=0$

$\cancel{x=0}$ $\quad (x=2)$

b) $\ln(3+x)-\ln(3-x)=\ln(2)$

$\ln\left(\frac{3+x}{3-x}\right)=\ln 2$

$\frac{3+x}{3-x}=\frac{2}{1} \Longrightarrow 6-2x=3+x$

$3=3x \Longrightarrow \boxed{x=1}$

c) $\log(x)+\log(x+15)=2$

$\log(x^2+15x)=2 \Longrightarrow x^2+15x-100=0$

$x^2+15x=100$

$(x+20)(x-5)=0$

Extraneous solution $\Rightarrow$ $\cancel{x=-20}$ $\quad (x=5)$

11) A fossilized leaf contains 75% of its normal amount of carbon-14. How old is the fossil (to the nearest year)? (Use 5600 years as the half – life of carbon 14)

(Must Show All the Appropriate Steps) (10 points)

$\frac{1}{2}=e^{5600k} \Longrightarrow \ln\frac{1}{2}=5600k \Longrightarrow \left(k=\frac{\ln 1/2}{5600}\right)$

$0.75=1e^{\frac{\ln\frac{1}{2}}{5600}t}$

$\ln 0.75=\frac{\ln\frac{1}{2}}{5600}t \Longrightarrow t=\frac{\ln(0.75)}{\ln\left(\frac{1/2}{5600}\right)}=2324.21$ years

12) A thermometer reading 95 degrees F is placed inside a cold storage room with a constant temperature of 32 degrees F. If the thermometer reads 80 degrees F in 15 minutes, how long will it take for the thermometer to reach 60 degrees F? Assume the cooling follows Newton's Law of Cooling (and Round your answer to the nearest whole minute) (Must Show All the Appropriate Steps) (10 points)

$80 = 32 + (95-32)e^{15K}$

$48 = 63e^{15K}$

$\frac{48}{63} = e^{15K}$

$K = \frac{\ln \frac{48}{63}}{15}$

$60 = 32 + (95-32)e^{\frac{\ln(\frac{48}{63})}{15}t}$

$28 = 63e^{\frac{\ln(\frac{48}{63})}{15}t}$

$\frac{\ln(\frac{28}{63})}{\left(\frac{\ln(\frac{48}{63})}{15}\right)} = t = 44.73$

≈ 45 minutes

13) The logistic growth model $P(t) = \frac{2700}{1+30e^{-0.3t}}$ represents the population of a bacterium in a culture tube after t hours.

a) What was the initial the population? (2 points)

$\frac{2700}{1+30} = 87.1$

b) What is the carrying capacity of this population? (2 points)

2700

c) What is the population of bacteria after 7 hours? (2 points)

$\frac{2700}{1+30e^{-0.3*7}} = 577.7$

d) When will the population reach 2000? (4 points)

(Please show all the algebraic steps for full credit)

$\frac{2000}{1} = \frac{2700}{1+30e^{-0.3t}}$

$1+30e^{-0.3t} = \frac{2700}{2000}$

$30e^{-0.3t} = 0.35$

$e^{-0.3t} = \frac{0.35}{30} \Rightarrow t = 14.84$ HRS

$t \approx 15$ HRS