MA 180 Professor Katiraie Test IV Fall 2007

Name: **Total Possible Points 150 + 10 pts Extra Credits**

$A(t)=A_0e^{kt}$ $u(t)=T+(u_0-T)e^{kt}$ $P(t)=\frac{c}{1+ae^{-bt}}$

1) Find the value of $\frac{f(x+h)-f(x)}{h}$ assuming h is not zero for the function (10 points)

$$f(x)=-x^2+2x-3$$

(Clearly state each of the steps of the process.)

$f(x+h)=-(x+h)^2+2(x+h)-3=-x^2-2xh-h^2+2x+2h-3$

$f(x+h)-f(x)=-x^2-2xh-h^2+2x+2h-3-(-x^2+2x-3)$

$f(x+h)-f(x)=-2xh-h^2+2h$

$\frac{f(x+h)-f(x)}{h}=\frac{-2xh-h^2+2h}{h}=-2x-h+2$

2) Find the inverse of the following functions. (20 points)
(Must Show All the Appropriate Steps)

a) $y=\sqrt[5]{x+2}+4$

swap x and y

$x=\sqrt[5]{y+2}+4$

$\sqrt[5]{y+2}=x-4$

$y+2=(x-4)^5$

$y=(x-4)^5-2$

b) $f(x)=\frac{7x+5}{x-6}$

$y=\frac{7x+5}{x-6}$ $\Rightarrow$ swap x and y

$x=\frac{7y+5}{y-6}$ $\Rightarrow xy-6x=7y+5$

$y(x-7)=6x+5$

$y=\frac{6x+5}{x-7}$

$f^{-1}(x)=\frac{6x+5}{x-7}$

3) A fossilized leaf contains 65% of its normal amount of carbon-14. How old is the fossil (to the nearest year)? (Use 5600 years as the half – life of carbon 14)

(Must Show All the Appropriate Steps) (10 points)

$A = A_0 e^{5600K}$

$\frac{1}{2} = e^{5600K}$

$\frac{\ln \frac{1}{2}}{5600} = K$

$K = -1.238 \times 10^{-4}$

$0.65 = e^{-1.238 \times 10^{-4} t}$

$\ln 0.65 = -1.238 \times 10^{-4} t$

$t = 3480.33$ years

4) A thermometer reading 100 degrees F is placed inside a cold storage room with a constant temperature of 32 degrees F. If the thermometer reads 85 degrees F in 20 minutes, how long will it take for the thermometer to reach 55 degrees F? Assume the cooling follows Newton's Law of Cooling (and Round your answer to the nearest whole minute) (Must Show All the Appropriate Steps) (10 points)

$U(t) = T + (U_0 - T)e^{Kt}$

$85 = 32 + (100 - 32)e^{20K}$

$0.7794 = e^{20K}$

$-0.0125 = K$

$55 = 32 + (100 - 32)e^{-0.0125t}$

$23 = 68 e^{-0.0125t}$

$\frac{\ln(\frac{23}{68})}{-0.0125} = t$

$t = 86.7$ minutes

$t \approx 87$ minutes

$\frac{\pi}{6}$ Radians

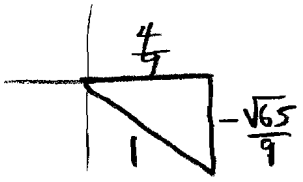
5) A pendulum swings though an angle of 30° each second. If the pendulum is 60 inches long, how far does its tip move each second? (5 points)

$S = r\theta = (60)\left(\frac{\pi}{6}\right) = 10\pi$ inches

$= 31.42$ inches

6) In the problem, t is a real number and P = (x, y) is the point on the unit circle that corresponds to t. Find the exact value of the indicated trigonometric functions of t.

$\left(\frac{4}{9}, \frac{-\sqrt{65}}{9}\right)$ (Hint: Draw a triangle)



(4 points)

a) Find $\sec(t) = \frac{\frac{4}{9}}{1} = \frac{9}{4}$

b) Find $\cot(t) = \frac{\frac{4}{9}}{\frac{-\sqrt{65}}{9}} = \frac{4}{9} \cdot \frac{9}{-\sqrt{65}} = \frac{-4\sqrt{65}}{65}$

7a) For the equation $y = -\frac{1}{2}\cos(2x - 2\pi)$, identify (6 points)

(i) the amplitude $= \frac{1}{2}$

(ii) the phase shift $= \frac{2\pi}{2} = \pi$

(iii) the period. $= \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

$y = A\sin(wx - \phi)$

7b) Write the equation of a sine function that has the given characteristics. (5 points)

Amplitude: 3

Period: 4π

Phase Shift: $\frac{\pi}{4}$

$\frac{2\pi}{w} = \text{Period}$

$\frac{2\pi}{w} = 4\pi \Rightarrow \frac{2\pi}{4\pi} = w$

$\frac{1}{2} = w$

$\text{Phase shift} = \frac{\phi}{w}$

$\frac{\pi}{4} = \frac{\phi}{1/2} \Rightarrow \phi = \frac{\pi}{8}$

$y = 3\sin\left(\frac{1}{2}x - \frac{\pi}{8}\right)$

8) Name the Quadrant in which the angle θ lies (5 points)

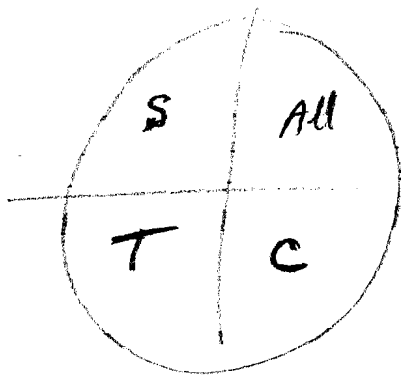
a) $\sin\theta > 0,\ \cos\theta < 0$ Quad II

b) $\cos\theta > 0,\ \tan\theta < 0$ Quad IV

c) $\sec\theta < 0,\ \sin\theta > 0$ Quad II

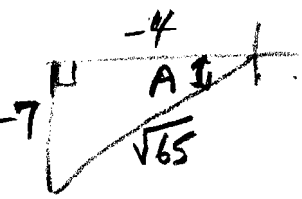
d) $\csc\theta > 0,\ \cos\theta < 0$ Quad II

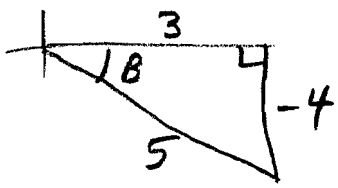
e) $\sin\theta < 0,\ \tan\theta < 0$ Quad IV



9) Given: $\cot A = \frac{4}{7}$ for $\pi < A < \frac{3\pi}{2}$ and $\cos B = \frac{3}{5}$ for $\frac{3\pi}{2} < B < 2\pi$ (10 pts)

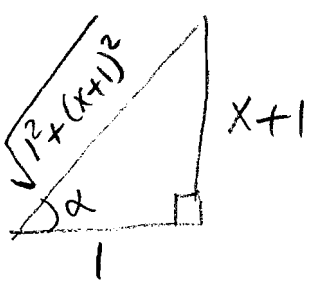
Find the **exact** value of $\tan(A+B)$ (Hint: Draw triangles in appropriate quadrants)





$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{7}{4} + \frac{-4}{3}}{1 - \frac{7}{4}\frac{-4}{3}} = \boxed{\frac{1}{8}}$$

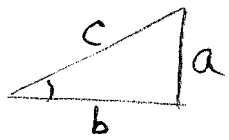
10) Find an algebraic expression for: $\sec(\tan^{-1}(x+1))$
(Hint: Draw a triangle in the first quadrant) (10 pts)



$$\sec(\tan^{-1}(x+1)) = \sec\alpha = \sqrt{1^2+(x+1)^2}$$

$$= \sqrt{1+x^2+2x+1}$$

$$\boxed{\sec(\tan^{-1}(x+1)) = \sqrt{x^2+2x+2}}$$



11) TRUE/FALSE. Circle one. (2.5 pts each)

a) (True) False $\csc(\sin^{-1}\frac{a}{c}) = \frac{c}{a}$

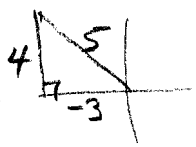
b) True (False) The period of $y = \tan\left(\frac{\pi}{3}x\right)$ is greater than the period of $y = \cos\left(\frac{\pi}{3}x\right)$

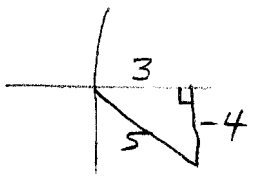
$\frac{\pi}{\frac{\pi}{3}} = 3$ $\quad \frac{2\pi}{\frac{\pi}{3}} = \frac{6\pi}{\pi} = 6$

c) True (False) If $0 < \theta < \frac{\pi}{2}$, *then* $\cos\left(\frac{\pi}{2} - \theta\right) = -\sin\theta$

$\cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta = \sin\theta$

d) True (False) If $\cot\theta = -\frac{3}{4}$, *then the* $\csc(\theta) = -\frac{5}{3}$





12) Solve the following equation on the interval $0 \le \theta < 2\pi$, (10 pts)
Round your answer(s) to two decimal places. (Hint: Factor)

$6\cos^2\theta - 7\cos\theta - 3 = 0$

$(2\cos\theta - 3)(3\cos\theta + 1) = 0$

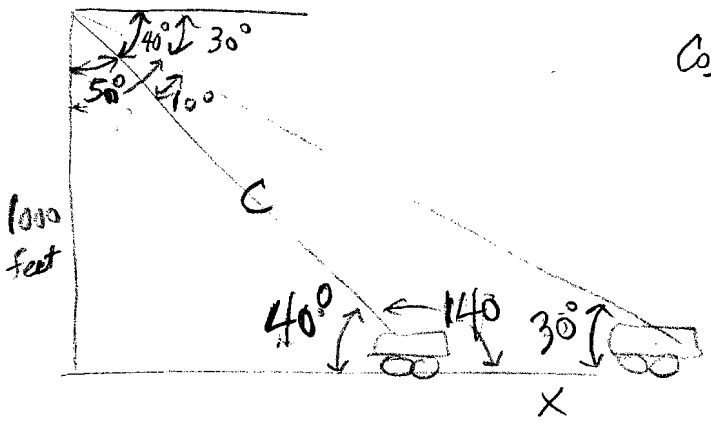
$\cos\theta = \frac{3}{2}$
Impossible

$3\cos\theta = -1$
$\cos\theta = -\frac{1}{3}$

$\theta = \cos^{-1}\left(-\frac{1}{3}\right) = 109.47°$ and
$\theta = 250.53°$

OR
1.91 Radians
4.37 Radians

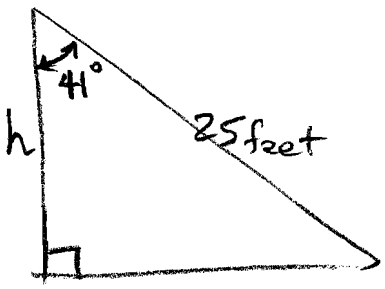
13) From the edge of a 1000 foot cliff, the angles of depression to two cars in the valley below are 30^0 and 40^0. How far apart are the cars? (10 pts)
Round your answers to the nearest 0.1 ft.



$\cos 50^\circ = \frac{1000}{C} \Rightarrow C = 1555.7$ feet

$\frac{\sin 30^\circ}{1555.7} = \frac{\sin 10^\circ}{x} \rightarrow x = 540.3$ feet

14) A twenty-five foot ladder just reaches the top of a house and forms an angle of 41^0 with the wall of the house. How tall is the house? Round your answer to the nearest 0.1 foot. (5 pts)

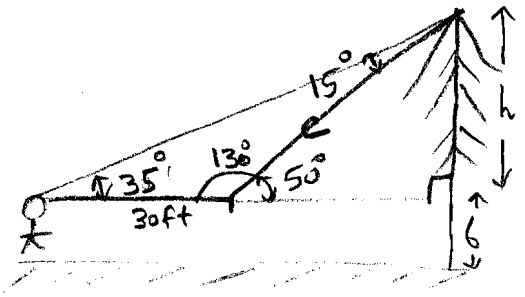


$\cos 41^\circ = \frac{h}{25}$

$h = 25 \cos 41^\circ = 18.87$ feet

$= 18.9$ feet

15) John (whose line of sight is 6 ft above horizontal) is trying to estimate the height of a tall oak tree. He first measures the angle of elevation from where he is standing as 35^0. He walks 30 feet closer to the tree and finds that the angle of elevation has increased by 15^0. Estimate the height of the tree rounded to the nearest whole number. (10 pts)



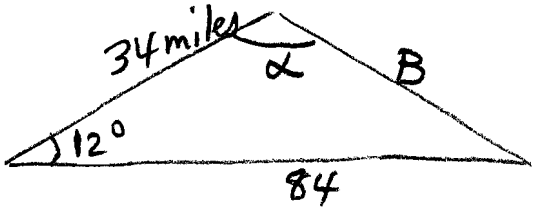
$$\frac{\sin 15^\circ}{30} = \frac{\sin 35^\circ}{c} \Rightarrow c = 66.5 \text{ feet}$$

$$\sin 50^\circ = \frac{h}{66.5}$$

$$h = 50.93 \text{ feet}$$

$$H = h + 6 = 50.93 + 6 = 56.93 \text{ feet}$$

16) In flying the 84 miles from Champaign to Peoria, a student pilot sets a heading that is 12^0 off course and maintains an average speed of 136 miles per hour. After 15 minutes, the instructor notices the course error and tells the student to correct his heading. (10 pts)



a) Through what angle will the plane move to correct the heading?

$$\frac{\sin 12^\circ}{51.23} = \frac{\sin \alpha}{84} \Rightarrow \alpha = 19.93^\circ$$

b) How many miles away is Peoria when the plane turns?

$$B^2 = 34^2 + 84^2 - 2(34)(84)\cos 12^\circ \Rightarrow B = 51.23 \text{ Miles}$$

Extra Credits (10 Points)

17) A baseball pitcher throws a baseball with an initial speed of 138 feet per second at an angle of 20^0 to the horizontal. The ball leaves the pitcher's hand at a height of 4 feet.

a) Find parametric equations that describe the motion of the ball as a function of time.

$X = 138 \cos 20^\circ t$

$y = -\frac{1}{2}(32)t^2 + 138 \sin 20^\circ t + 4$

b) How long is the ball in the air?

let $y = 0$

$-16t^2 + 138 \sin 20^\circ t + 4 = 0$

$t = 3.03$ seconds

c) When is the ball at its maximum height?

$t = \frac{-b}{2a} = \frac{-138 \sin 20^\circ}{2(-16)} = 1.47$ seconds

d) What is the maximum height of the ball?

$y = 38.8$ feet