MA181 Professor Katiraie Calculus I (Sections 3.4 – 3.6) Quiz Five

Name: Key

1) Differentiate the following functions: (2 Points Each)

A) $f(x) = xe^x \csc x$

$f'(x) = (1e^x + xe^x)(\csc x) + xe^x(-\csc x \cot x)$

B) $y = \dfrac{\sec x}{1+\cos x}$

$y' = \dfrac{\sec x \tan x (1+\cos x) + \sin x \sec x}{(1+\cos x)^2}$

C) $f(\theta) = \dfrac{1+\sin\theta}{\theta+\cos\theta}$

$f'(\theta) = \dfrac{\cos\theta(\theta+\cos\theta) - \underbrace{(1-\sin\theta)(1+\sin\theta)}_{1-\sin^2\theta}}{(\theta+\cos\theta)^2}$

$= \dfrac{\theta\cos\theta + \cancel{\cos^2\theta} - \cancel{1} + \cancel{\sin^2\theta}}{(\theta+\cos\theta)^2} = \boxed{\dfrac{\theta\cos\theta}{(\theta+\cos\theta)^2}}$

D) $f(\theta) = \sqrt[3]{1+\tan\theta}$

$= (1+\tan\theta)^{1/3}$

$f'(\theta) = \frac{1}{3}(1+\tan\theta)^{-\frac{2}{3}}(\sec^2\theta)$

$= \boxed{\dfrac{\sec^2\theta}{3(1+\tan\theta)^{\frac{2}{3}}}}$

2) Prove that $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

$\dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x}\right)$

$= \dfrac{-\sin x \sin x - \cos x \cos x}{\sin^2 x} = \dfrac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= \dfrac{-1}{\sin^2 x} = \boxed{-\csc^2 x}$

3) Find the equation of the tangent line to the curve $y = e^x \cos x$ at the point $(0\ ,\ 1)$ (2 Points)

$y' = e^x \cos x - e^x \sin x$

$y'(0) = e^0 \cos 0 - \cancel{e^0 \sin 0} = 1$

$y - 1 = 1(x - 0)$

$\boxed{y = x + 1}$

4) A table of values for f, g, f′, and g′ is given: (2 Points each)

| x | f(x) | g(x) | f′(x) | g′(x) |
|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 6 |
| 2 | 1 | 8 | 5 | 7 |
| 3 | 7 | 2 | 7 | 9 |

a) If $F(x) = f(g(x))$, find $F'(1)$

$F'(x) = f'(g(x))\,g'(x) \Rightarrow F'(1) = f'(g(1))\,g'(1) = f'(2)\,g'(1)$

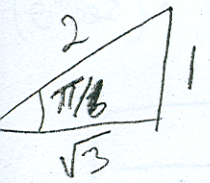
$= (5)(6) = \boxed{30}$

b) If $G(x) = g(f(x))$, find $G'(1)$

$G'(1) = g'(f(1))\,f'(1)$

$= g'(3)(4) = 9(4) = \boxed{36}$

5) Find the equation of the tangent line to the curve $x = 2\sin 2t$, $y = 2\sin t$ at the point $(\sqrt{3}\ ,\ 1)$ (2 Points)




$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2\cos t}{4\cos 2t}$

$= \dfrac{1}{2}\,\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \dfrac{\sqrt{3}}{2}$

$y - 1 = \frac{\sqrt{3}}{2}(x - \sqrt{3})$

$y = \frac{\sqrt{3}}{2}x - \frac{3}{2} + 1 = \frac{\sqrt{3}}{2}x - \frac{1}{2}$

6) Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (3 , 4). (2 Points)

$2x + 2yy' = 0 \quad y' = \frac{-2x}{2y} = \frac{-x}{y} \Rightarrow \boxed{m = \frac{-3}{4}}$

$y - 4 = \frac{-3}{4}(x - 3)$

$y = \frac{-3}{4}x + \frac{9}{4} + 4 \Rightarrow \boxed{y = \frac{-3}{4}x + \frac{25}{4}}$