MA181 Professor Katiraie Calculus I (Sections 3.4 – 3.6) Quiz Five

Name: Solution

1) Differentiate the following functions: (2 Points Each)

A) $f(x)=e^{2x}\csc x$

$f'(x)=2e^{2x}\csc x+e^{2x}(-\csc x\cot x)$

$=2e^{2x}\csc x-e^{2x}\csc x\cot x$

$=e^{2x}\csc x(2-\cot x)$

B) $y=\dfrac{\cot x}{1+\sec x}$

$y'=\dfrac{-\csc^2 x(1+\sec x)-\sec x\tan x\cot x}{(1+\sec x)^2}$

$=\dfrac{-\csc^2 x(1+\sec x)-\sec x\tan x\cot x}{(1+\sec x)^2}$

C) $f(\theta)=\dfrac{1+\sin 2\theta}{\theta+\sin 3\theta}$

$f'(\theta)=\dfrac{2\cos 2\theta(\theta+\sin 3\theta)-(1+3\cos 3\theta)(1+\sin 2\theta)}{(\theta+\sin 3\theta)^2}$

D) $f(\theta)=\sqrt[5]{1+\tan\theta}$

D) $f'(\theta)=\frac{1}{5}(1+\tan\theta)^{-\frac{4}{5}}(\sec^2\theta)$

2) Algebraically find the following $\dfrac{d}{dx}(\csc x)$ using the methods discussed in class.

$(\csc x)'=\left(\dfrac{1}{\sin x}\right)'=\dfrac{0\cdot\sin x-\cos x}{\sin^2 x}=\dfrac{-\cos x}{\sin x}\cdot\dfrac{1}{\sin x}$

$=-\cot x\csc x$

$y' = 2e^{2x}\sin x + e^{2x}\cos x$

$m = 2e^{\pi}\sin\pi/2 + 0 \Rightarrow \boxed{m = 2e^{\pi}}$

3) Find the equation of the tangent line to the curve $y = e^{2x}\sin x$ at the point $\left(\frac{\pi}{2}, e^{\pi}\right)$ (2 Points)

$y - e^{\pi} = 2e^{\pi}(x - \frac{\pi}{2}) \Rightarrow \boxed{y = 2e^{\pi}x - \pi e^{\pi} + e^{\pi}}$

4) A table of values for f, g, f′, and g′ is given: (2 Points each)

x	f(x)	g(x)	f′(x)	g′(x)
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

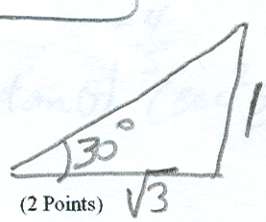
a) If $F(x) = f(g(x))$, find $F'(1)$

$F'(1) = f'(g(1))\,g'(1) = f'(2)g'(1) = (5)(6) = 30$

b) If $G(x) = g(f(x))$, find $G'(1)$

$G'(1) = g'(f(1))\,f'(1) = g'(3)f'(1) = (9)(4) = 36$

5) Find the equation of the tangent line to the curve $x = 2\sin 2t$, $y = 2\sin t$ at the point $(\sqrt{3}, 1)$ (2 Points)



$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos t}{4\cos 2t} = \frac{\cos t}{2\cos 2t} = \frac{\cos 30^\circ}{2\cos 60^\circ} = \frac{\sqrt{3}}{2}$

$y - 1 = \frac{\sqrt{3}}{2}(x - \sqrt{3}) \Rightarrow y = \frac{\sqrt{3}}{2}x - \frac{3}{2} + 1 = \frac{\sqrt{3}}{2}x - \frac{1}{2}$

6) Find the equation of the tangent to the circle $x^2 + y^2 = 4$ at the point (0 , -2). (2 Points)

$2x + 2yy' = 0 \Rightarrow y' = \frac{-2x}{2y} = \frac{-x}{y} = \frac{-0}{-2} = 0$

$y - -2 = 0(x - 0)$ $\boxed{y = -2}$