Calculus I   Quiz One   Form A   Summer 2007   **(20 Points)**

Name: _Solution_

1) Given $f(x) = \dfrac{3}{x+1}$ find $\dfrac{f(x+h) - f(x)}{h}$ (5 Points)

$$f(x+h) = \frac{3}{x+h+1}$$

$$f(x+h) - f(x) = \frac{3}{x+h+1} - \frac{3}{x+1} = \frac{3(x+1) - 3(x+h+1)}{(x+1)(x+h+1)} = \frac{3x+3-3x-3h-3}{(x+1)(x+h+1)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-3h}{(x+1)(x+h+1)} \div h = \frac{-3h}{(x+1)(x+h+1)} \cdot \frac{1}{h}$$

$$= \frac{-3}{(x+1)(x+h+1)}$$

2) A box with an <u>open top</u> is to be constructed from a rectangular piece of cardboard with dimensions 15 inches by 25 inches by cutting out equal squares of side x at each corner and then folding up the sides. Express the volume V of the box as a function of x and state the domain of this function. (5 Points)

$$V = x(15-2x)(25-2x)$$

$$= (15x - 2x^2)(25-2x) = 375x - 30x^2 - 50x^2 + 4x^3$$

$$= 4x^3 - 80x^2 + 375x$$

Domain: $0 < x < 7.5$

(100, 2200)   (300, 4800)

3) The manager of a furniture factory finds that it costs $2200 to manufacture 100 chairs in one day, and $4800 to produce 300 chairs in one day.   (2 Points Each)

a) Express the cost as a function of the number of chairs produced, assuming that it is linear.

$$M = \frac{4800 - 2200}{300 - 100} = \frac{2600}{200} = 13$$

$$y - 2200 = 13(x - 100)$$

$$y = 13x - 1300 + 2200$$

$$\underline{\quad\quad\quad\quad\quad\quad\quad\quad} \text{OR}$$

$$\boxed{y = 13x + 900}, \boxed{C(x) = 13x + 900}$$

b) What is the slope of the graph and what does it represent?

Slope is 13 and it means that each additional chair costs $13 to make.

c) What is the y-intercept of the graph and what does it represent?   y intercept is (0, 900)

the fixed cost is $900

4) Use the following table to evaluate the expressions.   (4 Points)

x	1	2	3	4	5	6
f(x)	3	1	4	2	2	5
g(x)	6	3	2	1	2	3

a) $(f \circ g \circ g)(1) = f(g(6))$

$$= f(3) = \boxed{4}$$

b) $(g \circ g \circ f)(1) = g(g(3))$

$$= g(2) = \boxed{3}$$

c) $(g \circ f)(3) =$

$$g(4) = \boxed{1}$$

d) $(f \circ g)(6) =$

$$= f(3) = \boxed{4}$$

Name: _Solution_

1) Given $f(x) = \dfrac{x+2}{5}$ Find $\dfrac{f(x+h) - f(x)}{h}$ (5 Points)
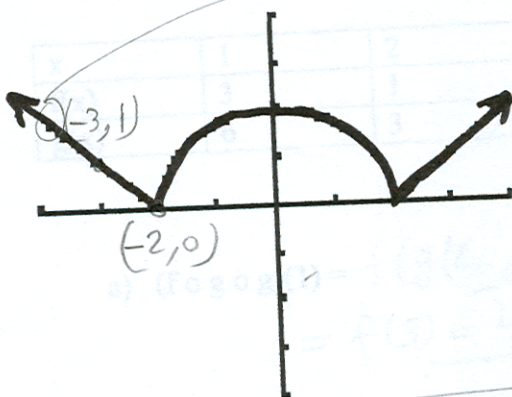
I) $f(x+h) = \dfrac{x+h+2}{5}$

II) $f(x+h) - f(x) = \dfrac{x+h+2}{5} - \dfrac{x+2}{5}$

$= \dfrac{x+h+2-x-2}{5} = \dfrac{h}{5}$

III) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{h}{5}}{h} = \dfrac{h}{5} \cdot \dfrac{1}{h} = \boxed{\dfrac{1}{5}}$

2)   Find a formula that describes the following function: (5 Points)

$M = \dfrac{0-1}{-2--3} = \dfrac{-1}{1} = -1$

$(x-0)^2 + (y-0)^2 = 2^2$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$

$f(x) = \begin{cases} -x-2 & x \leq -2 \\ \sqrt{4-x^2} & -2 < x \leq 2 \\ x-2 & 2 < x \end{cases}$

$$(1000, 9000)$$
$$(1500, 12000)$$
$$m = \frac{12000 - 9000}{1500 - 1000} = \frac{3000}{500}$$
$$= 6$$

3) A small-appliance manufacturer finds that it costs \$9000 to produce 1000 toaster ovens a week and \$12000 to produce 1500 toaster ovens a week. (6 Points)

   a) Express the cost as a function of the number of the toaster ovens produced, assuming that it is linear.

$$y - 9000 = 6(x - 1000)$$
$$y = 6x - 6000 + 9000$$
$$y = 6x + 3000 \qquad \boxed{C(x) = 6x + 3000}$$

   b) What is the slope of the graph and what does it represent?

The slope is 6 and it means that each additional toaster costs \$6 to make.

   c) What is the y-intercept of the graph and what does it represent?

The y-intercept is (0, 3000). This means that the fixed cost is \$3000.

4) Use the following table to evaluate the expressions. (4 Points)

x	1	2	3	4	5	6
f(x)	3	1	4	2	2	5
g(x)	6	3	2	1	2	3

   a) $(f \circ g \circ g)(1) = f(g(6))$
   $$= f(3) = \underline{4}$$

   b) $(g \circ g \circ f)(1) =$
   $$g(g(3)) = g(2) = 3$$

   c) $(g \circ f)(3) =$
   $$g(4) = \underline{1}$$

   d) $(f \circ g)(6) =$
   $$f(3) = 4$$