Calculus I; 2007 Professor Katiraie Quiz Two Summer

Name: Solution

Date: 6,6,07

1) A ball is thrown into the air with a velocity of 30 feet per second, its height in feet after t seconds is given by $y = 30t - 16t^2$

t	y
2	-4
2.5	-25

a) Find the average velocity for the time period beginning when t = 2 and lasting (3 Pts)

i) 0.5 s

Avg Velocity $= \dfrac{-25+4}{2.5-2} = \dfrac{-21}{0.5} = -42 \dfrac{ft}{sec}$

t	y
2	-4
2.05	-5.74

j) 0.05 s

Avg Velocity $= \dfrac{-5.74+4}{0.05} = -34.8 \dfrac{ft}{sec}$

t	y
2	-4
2.01	-4.3416

k) 0.01 s

Avg Velocity $= \dfrac{-4.3416+4}{2.01-2.00} = -34.16 \dfrac{ft}{sec}$

b) Find the instantaneous velocity when t = 2 (1 Pt)

$V = -34.00$ ft/sec

2) Find the inverse of the following functions. (Must Show All the Appropriate Steps) (6 points)

a) $y = \sqrt[3]{x+3} + 6$

$y - 6 = \sqrt[3]{x+3}$

$(y-6)^3 = x+3$

$x = (y-6)^3 - 3$

$f^{-1}(x) = (x-6)^3 - 3$

b) $f(x) = \dfrac{2x+5}{x-4}$

$\dfrac{y}{1} = \dfrac{2x+5}{x-4}$

$xy - 4y = 2x+5$

$xy - 2x = 4y+5$

$x(y-2) = 4y+5$

$x = \dfrac{4y+5}{y-2} \Rightarrow f^{-1}(x) = \dfrac{4x+5}{x-2}$

(3 Points)

3) If $f(x) = 5x + \log(x+10)$, find $f^{-1}(1)$

$1 = 5X + \log(X+10)$

let $Y_1 = 1$ $Y_2 = 5X + \log(X+10)$

$X = 0$

(3 points)

4) Express the function $F(x) = \dfrac{1}{\sqrt{x+\sqrt{x}}}$

as a composition of three functions (namely (fogoh)(x)).

(Hint: Find f(x), g(x), and h(x) so that (fogoh)(x) = $\dfrac{1}{\sqrt{x+\sqrt{x}}}$)

Answers Vary for Example let

$$\begin{cases} f(x) = \dfrac{1}{X} \\ g(x) = \sqrt{X+\sqrt{X}} \\ h(x) = X \end{cases}$$

Solve the following algebraically:

(4 points)

a) $2^x - 8^x = 0$

$2^X = 8^X$

$2^X = 2^{3X}$

$X = 3X$

$2X = 0$

$X = 0$

b) $e^{x^2} = \left(e^{5x}\right) \bullet \dfrac{1}{e^{-6}}$

$e^{X^2} = e^{5X+6}$

$X^2 = 5X + 6$

$X^2 - 5X - 6 = 0$

$(X-6)(X+1) = 0$

$X = 6$ $X = -1$

Calculus I; Professor Katiraie Quiz Three

Name: Solution Date: 6/7/07

1) $f(x)=\begin{cases} x^3+2; x\le -1 \\ x^2+x+1; -1<x<1 \\ x^4+2; x\ge 1\end{cases}$ Find the following limits **(4 Points)**

a) $\lim_{x\to -1^-} f(x) = (-1)^3+2 = 1$

b) $\lim_{x\to -1} f(x) = 1$

c) $\lim_{x\to 1^+} f(x) = 3$

d) $\lim_{x\to 1} f(x) = 3$

2) Algebraically find the following limits. (please show all your work, you may verify your answer with your calculator, but for credit you must do this problem algebraically):

$$\lim_{t\to 7} \frac{\sqrt{t+2}-3}{t-7} \cdot \frac{\sqrt{t+2}+3}{\sqrt{t+2}+3} = \frac{\cancel{(t+2)-9}}{\cancel{(t-7)}(\sqrt{t+2}+3)}$$ **(3 Pts)**

$$= \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

$$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4} \frac{\cancel{x+4}}{4x} \cdot \frac{1}{\cancel{x+4}}$$ **(3 Pts)**

$$\boxed{= \frac{-1}{16}}$$

3) Given $f(x)=\begin{cases} 2x^3+16; x\le -2 \\ x^2+bx+c; -2<x<2 \\ 3x^4-48; x\ge 2 \end{cases}$ **(3 Pts)**

Determine the values for b and c so that $f(x)$ is continuous everywhere.

$2(-2)^3+16=0$ $\quad 3(2)^{2}-48=0$

$\begin{cases} 4-2b+c=0 \\ 4+2b+c=0 \end{cases}$ $\quad 4-2b+(-4)=0$

$8+2c=0 \implies \boxed{c=-4}$ $\quad \boxed{b=0}$

4) Use the intermediate Value Theorem to show that there is a root for the equation $x^3+2x^2-42=0$ on the interval (0,3). let $f(x)=x^3+2x^2-42$ **(3 Points)**

Observe that $f(x)$ is continuous over the Interval $[0,3]$

$f(0)=-42$ and $f(3)=3^3+2(3)^2-42=3$

Since $f(0)<0<f(3)$

Then by IVT there exists a value c in $(0,3)$

Such that $f(c)=0$

5) Algebraically find the following limit, if it exits. **(4 Points)**

$$\lim_{x\to\infty}\frac{-x-2+9x^2}{3x^2+4x+1}$$

$$\lim_{x\to\infty}\frac{-\frac{x}{x^2}-\frac{2}{x^2}+\frac{9x^2}{x^2}}{\frac{3x^2}{x^2}+\frac{4x}{x^2}+\frac{1}{x^2}}=\lim_{x\to\infty}\frac{-\frac{1}{x}-\frac{2}{x^2}+9}{3+\frac{4}{x}+\frac{1}{x^2}}$$

$$\boxed{=3}$$