Professor Katiraie MA181 Fall 2006 Quiz Three

Name Key

Evaluate the limit.

1) $\lim_{t\to 4} \frac{t^2 - 7t + 12}{t-4} = \frac{16-28+12}{4-4} = \frac{0}{0}$

$\lim_{t\to 4} \frac{\cancel{(t-4)}(t-3)}{\cancel{(t-4)}} = \lim_{t\to 4} (t-3) = \boxed{1}$

1) $\boxed{1}$

2) $\lim_{t\to 4} \frac{2-\sqrt{t}}{4-t} \cdot \frac{2+\sqrt{t}}{2+\sqrt{t}} = \frac{\cancel{(4-t)}}{\cancel{(4-t)}(2+\sqrt{t})} = \lim_{t\to 4} \frac{1}{2+\sqrt{t}} = \boxed{\frac{1}{4}}$

2) $\boxed{\frac{1}{4}}$

Evaluate $\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ for the given "a" and function f(x)

3) $f(x) = \frac{x}{5} + 10$ for $a = 7$

3) $\boxed{\frac{1}{5}}$

step I) find $f(7+h) = \frac{7+h}{5} + 10$

step II) find $f(7) = \frac{7}{5} + 10$

step III) find $f(7+h) - f(7) = \left(\frac{7+h}{5} + 10\right) - \left(\frac{7}{5} + 10\right) = \frac{h}{5}$

$\lim_{h\to 0} \frac{f(7+h) - f(7)}{h} = \lim_{h\to 0} \frac{\frac{h}{5}}{h} = \lim_{h\to 0} \frac{\cancel{h}}{5} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{1}{5}}$

4) $f(x) = 2\sqrt{x} + 3$ for $a = 9$

4) ______

step I) find $f(9+h) = 2\sqrt{9+h} + 3$

step II) find $f(9) = 2\sqrt{9} + 3 = 9$

step III) find $f(9+h) - f(9) = 2\sqrt{9+h} + 3 - 9 = 2\sqrt{9+h} - 6$

$\lim_{h\to 0} \frac{f(9+h) - f(9)}{h} = \lim_{h\to 0} \frac{2\sqrt{9+h} - 6}{h} \cdot \frac{2\sqrt{9+h} + 6}{2\sqrt{9+h} + 6} = \frac{4(9+h) - 36}{h(2\sqrt{9+h} + 6)}$

$= \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}(2\sqrt{9+h} + 6)} = \frac{4}{12} = \boxed{\frac{1}{3}}$

e the problem.

5) Find equation of the tangent line to the curve $f(x) = \sqrt{x}$ at the point $x = 4$. 5) ______

$X = 4\ ;\ y = \sqrt{4} = 2$

$$m = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h\to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} = \lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

Multiply Top & Bottom by $\sqrt{4+h} + 2$; we get

$$= \lim_{h\to 0} \frac{\cancel{4}+h-\cancel{4}}{h(\sqrt{4+h}+2)} = \frac{1}{4} \Rightarrow m = \frac{1}{4}$$

$$\therefore\ y - y_1 = m(x - x_1) \Rightarrow y - 2 = \frac{1}{4}(x-4) \Rightarrow y = \frac{1}{4}x + 1$$

6) **Assume that a watermelon dropped from a tall building falls $y = 16t^2$ ft in t sec.** 6) ______

a) **Find the watermelon's average speed during the first 4 sec of fall**

t	y
0	0
4	256

$$\text{Avg Speed} = \frac{256 - 0}{4 - 0} = \frac{256}{4} = 64\ \frac{ft}{sec}$$

b) **Using the definition of derivative find the speed of the watermelon at the instant t = 4 sec.**

$$\text{Speed} = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h\to 0} \frac{16(4+h)^2 - 256}{h} = \lim_{h\to 0} \frac{16(16+8h+h^2) - 256}{h}$$

$$= \lim_{h\to 0} \frac{128h + 16h^2}{h} = \frac{\cancel{h}(128 + 16h)}{\cancel{h}} = 128\ ft/sec$$

Find the limit, if it exists.

7) $\displaystyle\lim_{x\to\infty} \frac{4x^3 - 5x^2 + 3x}{5 + 2x - x^3}$ $\div$ by x^3 / $\div$ by x^3

$$= \lim_{x\to\infty} \frac{4 - \frac{5}{x} + \frac{3}{x^2}}{\frac{5}{x^3} + \frac{2x}{x^3} - \frac{x^3}{x^3}}$$ 7) ______

$$= \frac{4}{-1} = -4$$