

Name: Solution

1) Differentiate the following functions:

(2 Points Each)

$$A) f(x) = \frac{x^3 + 4x^2 + 3}{\sqrt{x}} = x^{3-\frac{1}{2}} + 4x^{2-\frac{1}{2}} + 3x^{-\frac{1}{2}} \quad B) y(t) = ae^t + \frac{b}{t} + \frac{c}{t^2} = ae^t + bt^{-1} + ct^{-2}$$

$$f(x) = x^{\frac{5}{2}} + 4x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$$

$$f'(x) = \frac{5}{2}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$$

$$f'(x) = \frac{5}{2}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - \frac{3}{2\sqrt{x^3}}$$

$$y'(t) = ae^t - bt^{-2} - 2ct^{-3}$$

$$y' = ae^t - \frac{b}{t^2} - \frac{2c}{t^3}$$

$$C) V(t) = \sqrt[3]{t^2} + 2\sqrt{t^3}$$

$$V(t) = t^{\frac{2}{3}} + 2t^{\frac{3}{2}}$$

$$V'(t) = \frac{2}{3}t^{-\frac{1}{3}} + 3t^{\frac{1}{2}}$$

$$V'(t) = \frac{2}{3t^{\frac{1}{3}}} + 3t^{\frac{1}{2}}$$

$$2) \text{ If } f(x) = -2e^x g(x)$$

$$g(0) = -5 \text{ and } g'(0) = 3, \text{ find } f'(0)$$

$$f'(x) = -2e^x g(x) - 2e^x g'(x)$$

$$f'(0) = -2e^0 g(0) - 2e^0 g'(0)$$

$$= -2(-5) - 2(3) = 4$$

3) Differentiate the following function:

$$f(x) = \frac{ax^2 - b}{cx^3 - d}$$

(2 Points)

$$f'(x) = \frac{2ax(cx^3 - d) - 3cx^2(ax^2 - b)}{(cx^3 - d)^2} = \frac{2acx^4 - 2adx - 3acx^4 + 3bcx^2}{(cx^3 - d)^2}$$

$$f'(x) = \frac{-acx^4 - 2adx + 3bcx^2}{(cx^3 - d)^2}$$

4) Algebraically

On what interval is the function  $f(x) = x^3 - 4x^2 + 5x$  concave upward?

(3 Points) (Hint: for Concavity, we check out the second derivative, and .....)

$$f'(x) = 3x^2 - 8x + 5$$

$$f''(x) = 6x - 8$$

$$CU \quad f''(x) > 0 \Rightarrow 6x - 8 > 0 \Rightarrow 6x > 8$$

$$x > \frac{8}{6} = \frac{4}{3}$$

$$\left(\frac{4}{3}, \infty\right)$$

5) Algebraically

$f(x)$   
Increasing means

On what interval is the function  $f(x) = x^5 e^x$  increasing?

(2 Points)

$$f'(x) > 0$$

$$f'(x) = 5x^4 e^x + x^5 e^x = x^4 e^x (5+x)$$

$$f'(x) > 0 \Rightarrow x^4 e^x (5+x) > 0$$

$$(-5, 0) \cup (0, \infty)$$

6) The position of a particle is given by the equation  $S = f(t) = t^3 - 6t^2 + 9t$

Where  $t$  is measured in seconds and  $S$  is measured in meters.

(5 Points)

a) Algebraically, when is the particle at rest?

$$f'(t) = 3t^2 - 12t + 9 = 0$$

$$3(t^2 - 4t + 3) = 0$$

$$3(t-1)(t-3) = 0$$

$$t=1 \text{ sec} \quad t=3 \text{ sec}$$

b) Find the distance traveled by the particle during the first five seconds.

$$S(1) = 4 \text{ m}$$

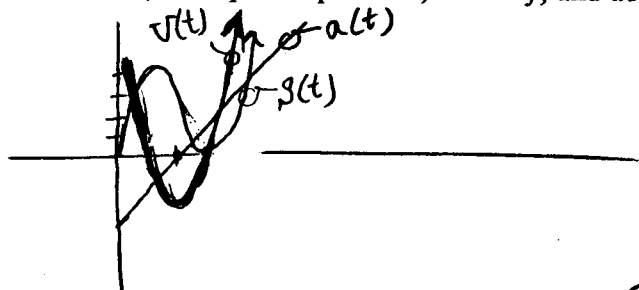
$$S(3) = 0 \text{ m}$$

$$S(5) = 20 \text{ m}$$

$$\Rightarrow \text{Distance} = 4 + 4 + 20 = 28$$

meters

c) Graph the position, velocity, and acceleration functions for  $0 \leq t \leq 5$



d) When is the particle speeding up?

$$1 < t < 2 \text{ seconds}$$

$$3 < t < \infty \text{ seconds}$$

e) When is the particle moving forward?

$$0 \leq t < 1$$

$$3 < t < \infty$$