MA 181 Professor Katiraie Quiz Five (Take Home) Fall 2005

Name____________________________

(Students May Get Help From Tutors in Math Science Center.)

Find the related rate equation.

1) The range R of a projectile is related to the initial velocity v and projection angle θ by the equation $R = \frac{v^2 \sin 2\theta}{g}$, where g is a constant. How are dR/dt, dv/dt, and $d\theta/dt$ related if none of R, v, θ are constant?

Solve the problem.

2) Water is falling on a surface, wetting a circular area that is expanding at a rate of 9 mm^2/s. How fast is the radius of the wetted area expanding when the radius is 186 mm? (Round approximations to four decimal places.)

3) A piece of land is shaped like a right triangle. Two people start at the right angle at the same time, and walk at the same speed along different legs of the triangle while spraying the land. If the area covered is changing at 3 m^2/s, how fast are the people moving when they are 2 m from the right angle? (Round approximations to two decimal places.)

4) A wheel with radius 2 m rolls at 19 rad/s. How fast is a point on the rim of the wheel rising when the point is $\pi/3$ radians above the horizontal? (Round approximations to one decimal place.)

5) A container is the shape of an inverted right circular cone has a radius of 6.00 inches at the top and a height of 7.00 inches. At the instant when the water in the container is 5.00 inches deep, the surface level is falling at the rate of −0.700 in./s. Find the rate at which water is being drained.

6) A product sells by word of mouth. The company that produces the product has noticed that revenue from sales is given by $R(t) = 5\sqrt{x}$, where x is the number of units produced and sold. If the revenue keeps changing at a rate of \$400 per month, how fast is the rate of sales changing when 1400 units have been made and sold? (Round to the nearest dollar per month.)

7) A man 6 ft tall walks at a rate of 2 ft/s away from a lamppost that is 23 ft high. At what rate is the length of his shadow changing when he is 70 ft away from the lamppost?

8) One airplane is approaching an airport from the north at 193 km/hr. A second airplane approaches from the east at 209 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 26 km away from the airport and the westbound plane is 15 km from the airport.

9) The radius of a right circular cylinder is increasing at the rate of 2 in./s, while the height is decreasing at the rate of 9 in./s. At what rate is the volume of the cylinder changing when the radius is 7 in. and the height is 16 in.?

Answer the question.

10) A curve is parameterized by $x = \cos t$ and $y = 3t^3 - 4t$.

(a) Find $\frac{dy}{dx}$ as a function of t.

(b) Find all values of t for which the curve has a horizontal tangent.

(c) Find all values of t for which the curve has a vertical tangent.

MA 181 Professor Katiraie Quiz Five (Take Home) Fall 2005

Name Key

(Students May Get Help From Tutors in Math Science Center.)

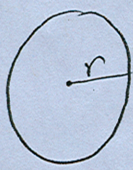
Find the related rate equation.

1) The range R of a projectile is related to the initial velocity v and projection angle θ by the equation $R = \frac{v^2 \sin 2\theta}{g}$, where g is a constant. How are dR/dt, dv/dt, and dθ/dt related if none of R, v, θ are constant?

$$\frac{dR}{dt} = \frac{2v\,v'}{g}\sin(2\theta) + \frac{v^2}{g}\left(2\cos(2\theta)\right)\cdot\frac{d\theta}{dt}$$

Solve the problem.

2) Water is falling on a surface, wetting a circular area that is expanding at a rate of 9 mm^2/s. How fast is the radius of the wetted area expanding when the radius is 186 mm? (Round approximations to four decimal places.)



$A = \pi r^2$

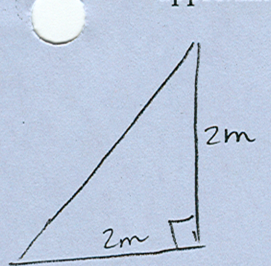
$\frac{dA}{dt} = 9 \frac{mm^2}{sec}$

$\frac{dr}{dt} = ?$ $r = 186 mm$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{dA/dt}{2\pi r} = \frac{9 \frac{mm^2}{sec}}{2\pi(186)} = 0.0077 \frac{mm}{sec}$

3) A piece of land is shaped like a right triangle. Two people start at the right angle at the same time, and walk at the same speed along different legs of the triangle while spraying the land. If the area covered is changing at 3 m^2/s, how fast are the people moving when they are 2 m from the right angle? (Round approximations to two decimal places.)



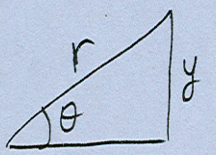
$\frac{dA}{dt} = 3 \frac{m^2}{sec}$

$A = \frac{1}{2} b h$

$A = \frac{1}{2} b b = \frac{1}{2} b^2$

$\frac{dA}{dt} = 2 \cdot \frac{1}{2} b \frac{db}{dt} \Rightarrow \frac{db}{dt} = \frac{3 m^2/sec}{2} = 1.5 m/sec$

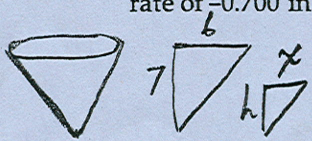
4) A wheel with radius 2 m rolls at 19 rad/s. How fast is a point on the rim of the wheel rising when the point is π/3 radians above the horizontal? (Round approximations to one decimal place.)



$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

$\frac{dy}{dt} = \frac{dr}{dt}\sin\theta + r\cos\theta\frac{d\theta}{dt} = 0 + 2\cos\left(\frac{\pi}{3}\right)\left(19 \frac{rad}{sec}\right) = 19 m/sec$

5) A container is the shape of an inverted right circular cone has a radius of 6.00 inches at the top and a height of 7.00 inches. At the instant when the water in the container is 5.00 inches deep, the surface level is falling at the rate of -0.700 in./s. Find the rate at which water is being drained.



$\frac{6}{7} = \frac{x}{h} \Rightarrow x = \frac{6}{7} h$

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{6}{7}h\right)^2 h = \frac{36}{147}\pi h^3$

$\frac{dV}{dt} = \frac{36\pi}{147} 3h^2 \frac{dh}{dt} = \frac{36\pi}{147}(3)(5)^2(-0.7) = -40.39 \frac{inch^3}{sec}$

6) A product sells by word of mouth. The company that produces the product has noticed that revenue from sales is given by $R(t) = 5\sqrt{x}$, where x is the number of units produced and sold. If the revenue keeps changing at a rate of \$400 per month, how fast is the rate of sales changing when 1400 units have been made and sold? (Round to the nearest dollar per month.)

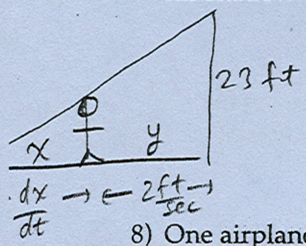
$R(t) = 5\sqrt{x} = 5x^{1/2}$

$\frac{dR}{dt} = 5 \cdot \frac{1}{2}(x)^{-1/2}\frac{dx}{dt}$

$\frac{\$400}{\text{month}} = \frac{5}{2}(1400)^{-1/2}\frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = \frac{\$5986.65}{\text{month}}}$

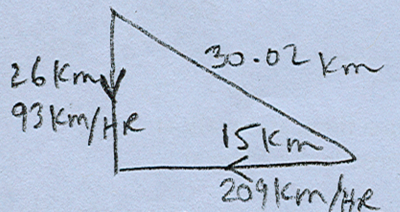
7) A man 6 ft tall walks at a rate of 2 ft/s away from a lamppost that is 23 ft high. At what rate is the length of his shadow changing when he is 70 ft away from the lamppost?



$\frac{6}{x} = \frac{23}{x+y} \Rightarrow y = \frac{17}{6}x \Rightarrow \frac{dy}{dt} = \frac{17}{6}\frac{dx}{dt}$

$\frac{2\,ft}{sec} = \frac{17}{6}\left(\frac{dx}{dt}\right) \Rightarrow \boxed{\frac{dx}{dt} = \frac{12}{17}\frac{ft}{sec}}$

8) One airplane is approaching an airport from the north at 193 km/hr. A second airplane approaches from the east at 209 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 26 km away from the airport and the westbound plane is 15 km from the airport.



$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{26^2 + 15^2} = 30.02$

$\cancel{2}c\frac{dc}{dt} = \cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt}$

$(30.02)\left(\frac{dc}{dt}\right) = -26(193) + (-15)(209) \Rightarrow \boxed{\frac{dc}{dt} = -271.59\,\frac{km}{HR}}$

9) The radius of a right circular cylinder is increasing at the rate of 2 in./s, while the height is decreasing at the rate of 9 in./s. At what rate is the volume of the cylinder changing when the radius is 7 in. and the height is 16 in.?

$V = \pi r^2 h \Rightarrow \frac{dV}{dt} = 2\pi r\frac{dr}{dt}h + \pi r^2\frac{dh}{dt}$

$= (2\pi)(7\,inch)\left(2\frac{in}{sec}\right)(16) + \pi(7^2)(-9)$

$\boxed{= 7\pi\,\frac{inch^3}{sec}}$

Answer the question.

10) A curve is parameterized by $x = \cos t$ and $y = 3t^3 - 4t$.

(a) Find $\frac{dy}{dx}$ as a function of t. $\Rightarrow$ $\boxed{\frac{dy}{dx} = \frac{9t^2 - 4}{-\sin t}}$

(b) Find all values of t for which the curve has a horizontal tangent.

$9t^2 - 4 = 0$

$t = \pm\frac{2}{3}$

(c) Find all values of t for which the curve has a vertical tangent.

$-\sin t = 0$

then $t = n\pi$ where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$