

DIRECTIONS:

Show all your work, use exact values and units where appropriate. Full credit will be given only when the work you show on paper supports your final answer.

1. [4 pts] Simplify the following expression as much as possible.

$$\frac{(2x^2 - 3)5x + (5 - x^2)3x}{(2x^2 - 3)^2} = \frac{10x^3 - 15x + 15x - 3x^4}{(2x^2 - 3)^2} = \frac{x^3(10 - 3x)}{(2x^2 - 3)^2}$$

2. [4 pts] If $f(x) = \frac{5}{x-1}$

a) Find $f(3) = \frac{5}{2}$

b) Evaluate and simplify: $\frac{f(x) - f(3)}{x - 3} = \frac{\frac{5}{x-1} - \frac{5}{2}}{x-3} = \frac{\frac{10 - 5(x-1)}{2(x-1)}}{x-3}$

$$= \frac{10 - 5x + 5}{2(x-1)} \cdot \frac{1}{x-3} = \frac{15 - 5x}{2(x-1)(x-3)} = \frac{5(3-x)}{2(x-1)(x-3)}$$

$$= \frac{-5(\cancel{x-3})}{2(x-1)(\cancel{x-3})} = \frac{-5}{2(x-1)}$$

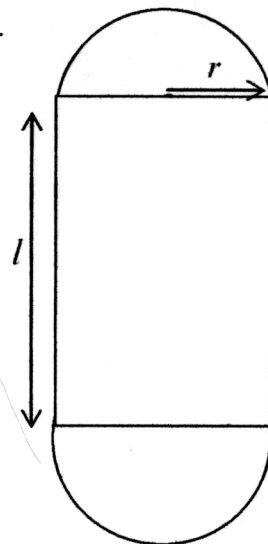
$f(x+h) = \frac{1}{x+h+1}$

[4 pts]

3. Given $f(x) = \frac{1}{x+1}$; find $\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$

$$= \frac{x+1 - (x+h+1)}{(x+h+1)(x+1)} \cdot \frac{1}{h} = \frac{-h}{(x+h+1)(x+1)h} = \frac{-1}{(x+h+1)(x+1)}$$

4. [4 pts] A field has the shape of a rectangle with a semicircle at each end. The length of the rectangular portion of the field is l , and the radius of each semicircle is r . If the outside perimeter of the field is 250 meters, express the area of the field as a function of r , and simplify your answer.



$$2\pi r + 2l = 250$$

$$\pi r + l = 125 \Rightarrow l = 125 - \pi r$$

$$= \pi r^2 + (l)(2r)$$

$$= \pi r^2 + (125 - \pi r)(2r)$$

$$= \pi r^2 + 250r - 2\pi r^2 = 250r - \pi r^2 \text{ (m}^2\text{)}$$

5. [4 pts] This problem is copied from the homework exercises, #15, section 1.2 of your textbook. Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 113 chirps per minute at 70°F and 173 chirps per minute at 80°F .

(a) Find a linear equation that models the temperature T as a function of the number of chirps per minute N .

$$\begin{matrix} N & T \\ (113, & 70) \end{matrix}$$

$$m = \frac{80 - 70}{173 - 113} = \frac{10}{60} = \frac{1}{6}$$

$$(173, 80)$$

$$y - 70 = \frac{1}{6}(x - 113) \Rightarrow y = \frac{1}{6}x + \frac{307}{6}$$

$$T = \frac{1}{6}N + \frac{307}{6}$$

(b) What is the slope of the graph? What does it represent in terms of the story?

slope = $\frac{1}{6}$ it means that for every 1°F temperature increase the no. of chirps increases by 6.

(c) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

$$T = \frac{1}{6}(150) + \frac{307}{6} = 76.167^\circ\text{F}$$