MA181 Katiraie   Professor Katiraie Calculus I; Spring 08   Quiz Four Form A

Name:________________________________

1) Differentiate the following functions: (2 Points)

A) $f(x)=\dfrac{x^3+4x+3}{\sqrt{x}} = x^{\frac{5}{2}}+4x^{\frac{1}{2}}+3x^{-\frac{1}{2}}$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}}+2x^{-\frac{1}{2}}-\frac{3}{2}x^{-\frac{3}{2}}$

B) $y=\dfrac{-2\cos x}{1-\sin x}$ $\quad y' = \dfrac{2\sin x\,(1-\sin x)-(-\cos x)(-2\cos x)}{(1-\sin x)^2}$

$= \dfrac{2\sin x - 2\sin^2 x - 2\cos^2 x}{(1-\sin x)^2}$

$= \dfrac{2\sin x - 2(\sin^2 x+\cos^2 x)}{(1-\sin x)^2} = \dfrac{-2(1-\sin x)}{(1-\sin x)^2} = \boxed{\dfrac{-2}{1-\sin x}}$

2) If $f(x)=-2e^x g(x)+5x$, where $g(0)=4$ and $f'(0)=6$, find $g'(0)$. (2 Points)

$f'(x) = -2e^x g(x) + -2e^x g'(x) + 5$

$f'(0) = -2e^0 g(0) - 2e^0 g'(0) + 5$

$6 = -2(4) - 2g'(0) + 5$

$6 = -8 - 2g'(0) + 5$

$6 = -3 - 2g'(0)$

$9 = -2g'(0)$

$g'(0) = -\frac{9}{2}$

3) Prove that $\frac{d}{dx}(2\csc x) = -2\csc x\cot x$ (2 Points)

$\left(\dfrac{2}{\sin x}\right)' = \dfrac{0\cdot\sin x - 2\cos x}{\sin^2 x} = \dfrac{-2}{\sin x}\,\dfrac{\cos x}{\sin x}$

$= \boxed{-2\csc x\cot x}$

4) Find all values of x so that the graph of $f(x) = x - 2\sin x$ will have a horizontal tangent?

$f'(x) = 1 - 2\cos x$

(2 Points)

$1 - 2\cos x = 0$

$-2\cos x = -1$

$\cos x = \frac{1}{2} \Longrightarrow x = 60^\circ$ OR $\boxed{\frac{\pi}{3} + 2n\pi}$

$x = 300^\circ$ OR $\boxed{\frac{5\pi}{3} + 2n\pi}$

5) Find the equation of the tangent line to the curve $y = -2\cos x$ at the point $\left(\frac{\pi}{4}, -\sqrt{2}\right)$

$y' = +2\sin x$

(2 Points)

$y'\Big|_{\frac{\pi}{4}} = 2\sin\frac{\pi}{4} = 2\frac{\sqrt{2}}{2} = \sqrt{2}$

$y - y_1 = m(x - x_1)$

$y + \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

$\boxed{y = \sqrt{2}x - \sqrt{2}\frac{\pi}{4} - \sqrt{2}}$

6) Find the equation of the tangent line to the curve $y = -e^x \cos x$, at the point (0,-1).

(2 Points)

$y' = -e^x \cos x + -e^x(-\sin x) \Longrightarrow y'\Big|_{\text{when } x=0} = -e^0 \cos 0 + e^0 \sin 0 = -$

$y - y_1 = m(x - x_1) \Longrightarrow y + 1 = -1(x - 0) \Longrightarrow \boxed{y = -x - 1}$

7) Find the equation of the tangent line to the curve $y = x\cos x$, at the point $(\pi, -\pi)$

(2 Points)

$y' = 1\cos x - x\sin x$

$y' = 1\cos\pi - \pi\sin\pi = -1 \Longrightarrow \boxed{m = -1}$

$y - y_1 = m(x - x_1)$

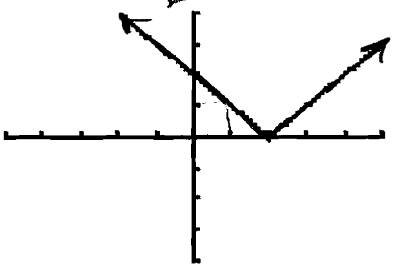
$y - \pi = -1(x - \pi)$

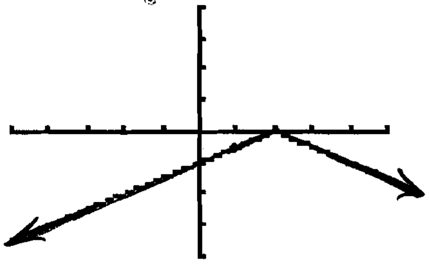
$y = -x + \pi - \pi$

$\boxed{y = -x}$

7) Given that $v(x)=\dfrac{f(x)}{g(x)}$, and $w(x)=f(x)g(x)$ (3 Points)

And graphs of f(x) and g(x)





Find the following:

$v'(0) = \left(\frac{f}{g}\right)' = \frac{f'g-g'f}{g^2} = \frac{(-1)(-1)-\frac{1}{2}(2)}{(-1)^2} = \frac{1-1}{1} = \boxed{0}$

$w'(2) = f'g + g'f$ = undefined (DOES NOT EXIST)

$w'(1) = f'g + g'f = (-1)\left(\frac{1}{2}\right) + \frac{1}{2}(1) = \boxed{0}$

(3 Points)

8) The position of a particle is given by the equation $S(t)=\dfrac{t^3}{3}-\dfrac{3t^2}{2}+2t$,

where "t" is measured in seconds and "S" is in meters.

a) When is the particle at rest?

$S'(t) = \frac{3t^2}{3} - \frac{6t}{2} + 2$

$= t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t=1 \; ; \; t=2$ seconds

b) When is the particle speeding up?

When velocity and acceleration have same signs

$1 < t < 1.5$ and $t > 2$ seconds

MA181 Katiraie   Professor Katiraie Calculus I; Spring 08   Quiz Four Form C
Tuesday Thursday Class

Name: Solution

1) Differentiate the following functions: (2 Points)

A) $f(x)=\dfrac{x^3+4\sqrt{x}+3}{\sqrt{x}} = x^{\frac{5}{2}}+4+3x^{-\frac{1}{2}}$

$f'(x)=\frac{5}{2}x^{\frac{3}{2}}-\frac{3}{2}x^{-\frac{3}{2}}$

B) $y=\dfrac{1-\sin x}{-2\cos x}$ $\quad y'=\dfrac{-\cos x(-2\cos x)-(2\sin x)(1-\sin x)}{(-2\cos x)^2}$

$=\dfrac{2\cos^2 x-2\sin x+2\sin^2 x}{(-2\cos x)^2}=\dfrac{2-2\sin x}{(-2\cos x)^2}$

2) If $f(x)=-2e^x g(x)-7x$
And $g(0)=4$ and $f'(0)=-6$, find $g'(0)$. (2 Points)

$f'(x)=-2e^x g(x)+-2e^x g'(x)-7$

$\Downarrow$

$f'(0)=-2e^0 g(0)-2e^0 g'(0)-7$

$-6=-2(4)-2g'(0)-7$

$-6=-15-2g'(0)$

$9=-2g'(0)$ $\quad \boxed{g'(0)=\frac{-9}{2}}$

3) Prove that $\frac{d}{dx}(10\sec x)=10\sec x\tan x$ (2 Points)

$\left(\dfrac{10}{\cos x}\right)'=\dfrac{0(\cos x)+10\sin x}{\cos^2 x}=\dfrac{10}{\cos x}\,\dfrac{\sin x}{\cos x}$

$\boxed{=10\sec x\tan x}$

4) Find **all values of** x so that the graph of $f(x)=\sqrt{3}\,x+2\cos x$ will have a horizontal tangent?

(2 Points)

$f'(x) = \sqrt{3} - 2\sin x = 0$

$-2\sin x = -\sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = 60^\circ$ and 120° OR $\boxed{\frac{\pi}{3} + 2n\pi}$ $\boxed{\frac{2\pi}{3} + 2n\pi}$

5) Find the **equation of the tangent line** to the curve $y=-2\cos x$ at the point $\left(\frac{5\pi}{6}, \sqrt{3}\right)$

(2 Points)

$y' = 2\sin x \Big|_{\text{at } x=\frac{5\pi}{6}}$

$m = 2\left(\frac{1}{2}\right) = 1$

$y - \sqrt{3} = 1\left(x - \frac{5\pi}{6}\right)$

$y = x - \frac{5\pi}{6} + \sqrt{3}$

6) Find the equation of the tangent line to the curve $y = -2e^x \cos x$, at the point (0, 2)

(2 Points)

$y' = -2e^x \cos x - 2e^x(-\sin x)\Big|_{\text{at } x=0} \Rightarrow \boxed{m = -2}$

$y - 2 = -2(x - 0)$

$y = -2x + 2$

7) Find the equation of the tangent line to the curve $y = x\sin x$, at the point $(\frac{\pi}{2}, \frac{\pi}{2})$

(2 Points)

$y' = 1\sin x + x\cos x\Big|_{\text{at } x=\frac{\pi}{2}} \Rightarrow \boxed{m = 1}$

$y - \frac{\pi}{2} = 1\left(x - \frac{\pi}{2}\right)$

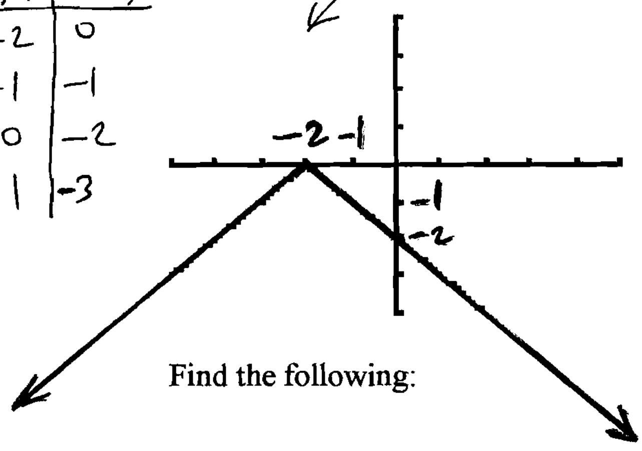
$y = x - \frac{\pi}{2} + \frac{\pi}{2}$

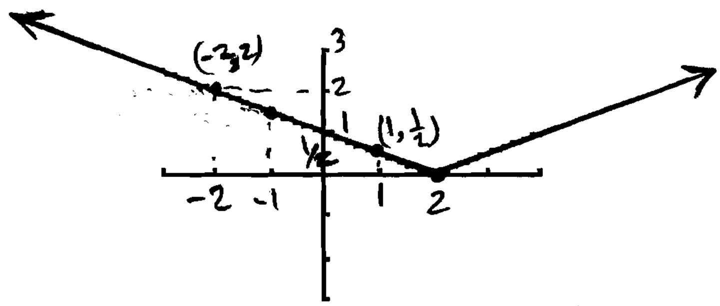
$y = x$

7) Given that $v(x)=\frac{f(x)}{g(x)}$, and $w(x)=f(x)g(x)$ (3 Points)

And graphs of f(x) and g(x)

X	f(X)
−2	0
−1	−1
0	−2
1	−3





X	g(X)
−2	2
−1	1.5
0	1
1	0.5
2	0

Find the following:

$v'(1) = \frac{f'(1)g(1) - f(1)g'(1)}{g^2} = \frac{(-1)(0.5) - (-3)(-\frac{1}{2})}{(\frac{1}{2})^2}$

$= \frac{-0.5 - 1.5}{(\frac{1}{4})} = \frac{-2}{1/4} = \boxed{-8}$

$w'(-1) = f'(-1)g(-1) + f(-1)g'(-1)$

$= (-1)(1.5) + (-1)(-\frac{1}{2}) = -1.5 + \frac{1}{2} = \boxed{-1}$

$w'(-2) = f'(-2)g(-2) + f(-2)g'(-2)$

Since $f'(-2)$ is Not Defined (B/c of sharp edge)

No Solution (DNE)

(3 Points)

8) The position of a particle is given by the equation $S(t)=\frac{t^3}{3}-3t^2+8t$

where "t" is measured in seconds and "S" is in meters.

a) When is the particle at rest? $S'(t) = \frac{3t^2}{3} - 6t + 8$

$t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$ $\quad t = 2, t = 4$ seconds

b) When is the particle speeding up?

$2 < t < 3$ and $t > 4$ seconds