Professor Katiraie Calculus I Quiz 6 (Take Home)

Name________________________________ Due Thursday November 16th at 2:00 PM (NO EXCUSES)

"Students May Seek Help From the Tutors in the Math / Science Center"

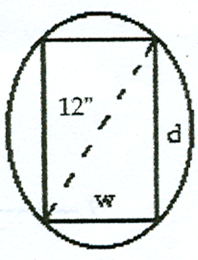
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

1) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 27 ft^3. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary. 1) ______________

2) From a thin piece of cardboard 40 in. by 40 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary. 2) ______________

3) The strength S of a rectangular wooden beam is proportional to its width times the square of its depth. Find the dimensions of the strongest beam that can be cut from a 12-in.-diameter cylindrical log. (Round answers to the nearest tenth.) 3) ______________



4) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 13 - \frac{x}{26}$. How many candy bars must be sold to maximize revenue? 4) ______________

5) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$8 per foot for two opposite sides, and \$7 per foot for the other two sides. Find the dimensions of the field of area 730 ft^2 that would be the cheapest to enclose. 5) ____________

6) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost: 6) ____________

$R(x) = 30x - 0.5x^2$
$C(x) = 4x + 2.$

7) At noon, ship A was 16 nautical miles due north of ship B. Ship A was sailing south at 16 knots (nautical miles per hour; a nautical mile is 2000 yards) and continued to do so all day. Ship B was sailing east at 5 knots and continued to do so all day. The visibility was 5 nautical miles. Did the ships ever sight each other? 7) ____________

8) How close does the curve $y = \sqrt{x}$ come to the point $\left(\frac{8}{3}, 0\right)$? (Hint: If you minimize the square of the distance, you can avoid square roots.) 8) ____________

9) Show that $g(x) = \dfrac{a + x}{\sqrt{b^2 + (a + x)^2}}$ is an increasing function of x. 9) ____________

10) How close does the semicircle $y = \sqrt{16 - x^2}$ come to the point $(1, \sqrt{2})$? 10) ____________

#1

$$x^2h = 27 \qquad h = \frac{27}{x^2}$$

$$SA = x^2 + 4xh = x^2 + 4(x)\left(\frac{27}{x^2}\right) = x^2 + 108x^{-1}$$

$$SA' = 2x - 108x^{-2} = 0$$

$$2x = \frac{108}{x^2} \Rightarrow 2x^3 = 108 \qquad x^3 = 54$$

$x = 3.78$ ft

$h = \frac{27}{3.78^2} = 1.9$ ft

So, the Box is 3.78 by 3.78 by 1.9 ft

3.8 ft by 3.8 ft by 1.9 ft

#2

$$V = x(40-2x)(40-2x) = x(1600 - 160x + 4x^2)$$

$$= 4x^3 - 160x^2 + 1600x$$

$$V' = 12x^2 - 320x + 1600 = 0$$

$x \approx 6.7$ inches

$40 - 2x = 40 - 2(6.7) \approx 26.7$ inches

26.7" by 26.7" by 6.7" $\qquad V = 4740.7$ inch^3

#3)

$S = KWd^2$ Assume $K=1$

$S = Wd^2$ and $12^2 = W^2 + d^2$

$d^2 = 12^2 - W^2$

$S = W(12^2 - W^2)$

$S = 144W - W^3$

$\frac{dS}{dW} = 144 - 3W^2$ and let $\frac{dS}{dW} = 0$ $+3W^2 = 144$

$W^2 = 48$

$W = 4\sqrt{3}$

✓ $W = 6.928$ inches

✓ $d = 9.8$ inches

#4)

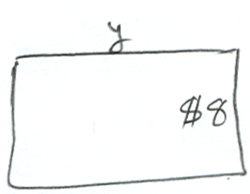
$P(x) = 13 - \frac{x}{26}$

$R(x) = x \cdot P = 13x - \frac{x^2}{26}$

$\frac{dR(x)}{dx} = 13 - \frac{2x}{26} = 0 \Rightarrow \frac{x}{13} = 13 \Rightarrow x = 169$ ✓

Thousand Candy

#5)



$xy = 730 \Rightarrow y = \frac{730}{x}$

$\text{Cost} = 2(8x) + 2(7y)$

$= 16x + 14y$

$C = 16x + 14\left(\frac{730}{x}\right) = 16x + 10220x^{-1}$

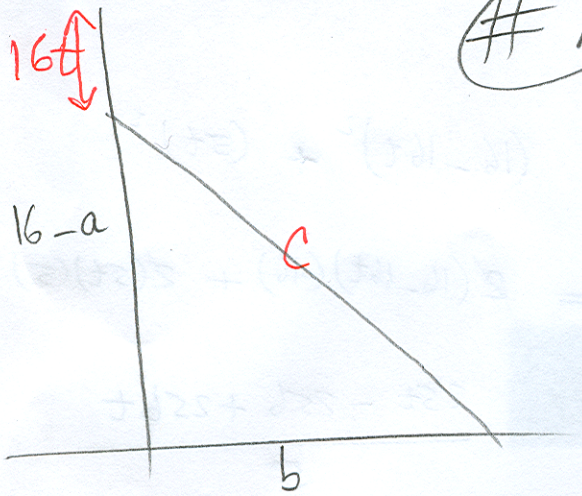
$\frac{dC}{dx} = 16 - 10220x^{-2} = 0$

$16 - \frac{10220}{x^2} = 0$ $16 = \frac{10220}{x^2}$

✓ $x = 25.3$ feet at \$8

✓ $y = \frac{730}{25.3} = 28.9$ feet at \$7

#7



$$(16-a)^2 + b^2 = c^2$$

$$(16-16t)^2 + (5t)^2 = c^2$$

$$(16-16t)^2 + (5t)^2 = 5^2$$

$$256 - 512t + 256t^2 + 25t^2 = 25$$

$$281t^2 - 512t + 231 = 0$$

$$t = \frac{512 \pm \sqrt{(-512)^2 - (4)(281)(231)}}{2(281)} \boxed{\begin{aligned} &= 1 \\ &= 0.822 \end{aligned}}$$

Yes, the ships will sight each other if

$0.822 \leq t \leq 1$ HR

#6) Profit = Rev. − Cost

$P' = R' - C'$ at Max Profit $R' = C'$

$30 - 1X = 4 \Rightarrow X = 30 - 4 = 26$ units

#8) $(X, \sqrt{X})$ $\left(\frac{8}{3}, 0\right)$

$$D^2 = (\sqrt{x} - 0)^2 + \left(X - \frac{8}{3}\right)^2 = X + X^2 - \frac{16}{3}X + \frac{64}{9}$$

$$= X^2 - \frac{13}{3}X + \frac{64}{9}$$

$$2d\frac{dD}{dX} = 2X - \frac{13}{3} = 0$$

$$2X = \frac{13}{3} \qquad X = \frac{13}{6} \text{ units}$$

$\therefore$ Minimum Distance $= \sqrt{\frac{13}{6} + \left(\frac{13}{6} - \frac{8}{3}\right)^2} = \sqrt{\frac{29}{12}}$ units

#9) $$g(x) = \frac{a + x}{\sqrt{b^2 + (a+x)^2}}$$

$$g'(x) = \frac{1\left(\sqrt{b^2 + (a+X)^2}\right) - \frac{1}{2}\left(b^2 + (a+x)^2\right)^{-\frac{1}{2}}(2)(a+x)(1)(a+x)}{\left(\sqrt{b^2 + (a+x)^2}\right)^2}$$

$$= \frac{\sqrt{b^2 + (a+x)^2} - \dfrac{(a+x)^2}{\sqrt{b^2 + (a+x)^2}}}{b^2 + (a+x)^2} = \frac{\dfrac{b^2 + (a+x)^2 - (a+x)^2}{\sqrt{b^2 + (a+X)^2}}}{b^2 + (a+x)^2}$$

$$g'(x) = \frac{b^2}{\left(b^2 + (a+x)^2\right)^{3/2}}$$

Since Numerator is positive and denominator is positive then $g'(x) > 0$

Hence $g(x)$ is increasing everywhere.

#10) $(x, \sqrt{16-x^2})$

$(1, \sqrt{2})$

$$D = \sqrt{(\sqrt{16-x^2}-\sqrt{2})^2 + (x-1)^2}$$

$$D^2 = (\sqrt{16-x^2}-\sqrt{2})^2 + (x-1)^2$$

$$2D\frac{dD}{dx} = 2(\sqrt{16-x^2}-\sqrt{2})'\left(\frac{1}{2}(16-x^2)^{-\frac{1}{2}}(-2x)\right) + 2(x-1)$$

$$= \frac{-2x(\sqrt{16-x^2}-\sqrt{2})}{(16-x^2)^{1/2}} + \frac{2(x-1)}{1}$$

$$= \frac{-2x(\sqrt{16-x^2}-\sqrt{2}) + (2x-2)(16-x^2)^{1/2}}{(16-x^2)^{1/2}}$$

$$2D\frac{dD}{dx} = \frac{\cancel{-2x\sqrt{16-x^2}} + 2x\sqrt{2} + \cancel{2x\sqrt{16-x^2}} - 2\sqrt{16-x^2}}{(16-x^2)^{1/2}} = 0$$

$$\Rightarrow \quad 2x\sqrt{2} - 2\sqrt{16-x^2} = 0$$

$$x\sqrt{2} = \sqrt{16-x^2}$$

$$2x^2 = 16-x^2$$

$$3x^2 = 16$$

$$x^2 = 5.33 \Rightarrow x = 2.3$$

$(2.3, 3.27)$

$(1, \sqrt{2})$

$D_{\text{minimum}} = 2.27$