~~MA181~~ Professor Katiraie Fall 2005 Quiz Three

Name Key

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Algebraically find the limit, if it exists.

1) $\lim\limits_{x \to -\infty} \dfrac{2x^3 + 2x^2}{x - 5x^2}$

$$\lim_{x\to-\infty} \frac{\frac{2x^3}{x^2} + \frac{2x^2}{x^2}}{\frac{x}{x^2} - \frac{5x^2}{x^2}} = \frac{-\infty + 2}{-5} = \boxed{+\infty}$$

2) $\lim\limits_{x \to \infty} \dfrac{3x^{-1} + 2x^{-3}}{4x^{-2} + x^{-5}}$

$$= \frac{\frac{3}{x} + \frac{2}{x^3}}{\frac{4}{x^2} + \frac{1}{x^5}} = \frac{\frac{3x^2+2}{x^3}}{\frac{4x^3+1}{x^5}} = \frac{3x^2+2}{\cancel{x^3}} \cdot \frac{\cancel{x^5}^{\,2}}{4x^3+1}$$

$$= \frac{3x^4+2x^2}{4x^3+1} = \frac{\frac{3x^4}{x^3} + \frac{2x^2}{x^3}}{\frac{4x^3}{x^3} + \frac{1}{x^3}} = \lim_{x\to\infty} \frac{3x + \frac{2}{x}}{4 + \frac{1}{x^3}} = \boxed{+\infty}$$

3) $\lim\limits_{t \to 9} \dfrac{-3 - \sqrt{t}}{9 - t} \cdot \dfrac{-3+\sqrt{t}}{-3+\sqrt{t}}$

$$= \frac{(-3)^2 - (\sqrt{t})^2}{(9-t)(-3+\sqrt{t})} = \lim_{t\to 9} \frac{\cancel{9-t}}{(\cancel{9-t})(-3+\sqrt{t})} = \frac{1}{0} = \boxed{\text{DNE}}$$

By Using the Squeeze Theorem show that the limit is equal to Zero.

4) $\lim\limits_{x \to -\infty} \dfrac{\cos 2x}{x^2}$

$$-1 \le \cos 2x \le 1$$

$$\frac{-1}{x^2} \le \frac{\cos 2x}{x^2} \le \frac{1}{x^2}$$

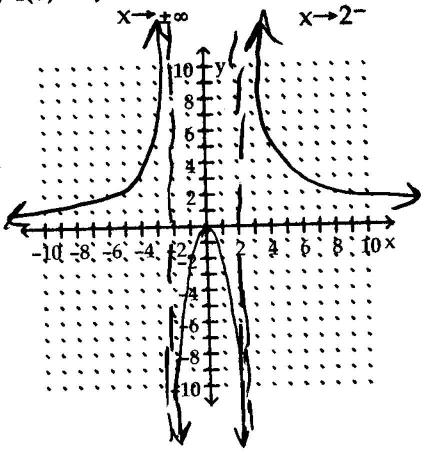
$$\lim_{x\to-\infty} \frac{-1}{x^2} \le \lim_{x\to-\infty} \frac{\cos 2x}{x^2} \le \lim_{x\to-\infty} \frac{1}{x^2}$$

$$0 \le \lim_{x\to-\infty} \frac{\cos 2x}{x^2} \le 0$$

So, $\boxed{\lim\limits_{x\to-\infty} \dfrac{\cos 2x}{x^2} = 0}$

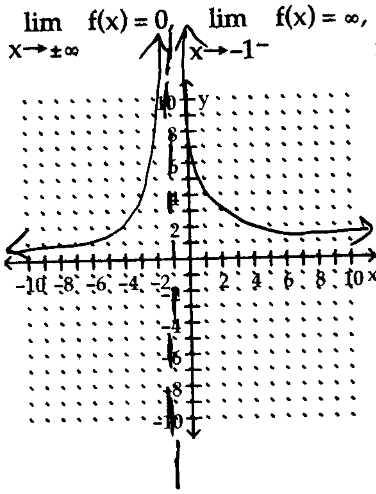
ch the graph of a function y = f(x) the satisfies the given conditions.

5) $f(0) = 0$, $\lim\limits_{x \to \pm\infty} f(x) = 0$, $\lim\limits_{x \to 2^-} f(x) =$ $\lim\limits_{x \to -2^+} f(x) = -\infty$, $\lim\limits_{x \to 2^+} f(x) = \infty$, $\lim\limits_{x \to -2^-} f(x) = \infty$.



Find a function that satisfies the given conditions and sketch its graph.

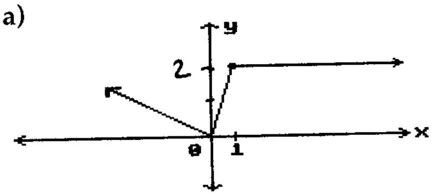
6) $\lim\limits_{x \to \pm\infty} f(x) = 0$, $\lim\limits_{x \to -1^-} f(x) = \infty$, $\lim\limits_{x \to -1^+} f(x) = \infty$.



$$y = f(x) = \frac{1}{(x+1)^2}$$

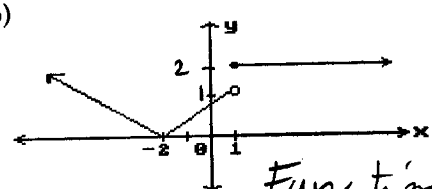
Find all points where the function is discontinuous.

7) a)



Continuous everywhere

b)



Function is discontinuous at $x=$

b/c $\lim\limits_{x \to 1^-} f(x) = 1$ and $\lim\limits_{x \to 1^+} f(x) = 2$

So $\lim\limits_{x \to 1} f(x) =$ DNE & $f(1) = 2$

since $\lim\limits_{x \to 1} f(x) \neq f(1)$

then $f(x)$ is discontinuous at $x = 1$