Calculus I Professor: Fred Katiraie Quiz Eight

Name:______________________________

1) Find the general indefinite integral.

a) $\int \frac{\sin x}{1-\sin^2 x} dx$ hint: $\sin^2 x + \cos^2 x = 1$ (2 point)

$$= \int \frac{\sin x}{\cos^2 x} dx = \int \frac{\sin x}{\cos x} \frac{1}{\cos x} dx = \int \tan x \sec x \, dx = \sec x + C$$

b) $\int \left((1 - \frac{1}{\sqrt{x}})(1 + \frac{1}{\sqrt{x}}) \right) dx$ (1 point)

$$= \int \left(1 - \frac{1}{x}\right) dx = x - \ln|x| + c$$

c) $\int \frac{t^3 + 2t^2}{\sqrt{t}} dt$ (1 point)

$$\frac{t^3 + 2t^2}{\sqrt{t}} = t^{3-\frac{1}{2}} + 2t^{2-\frac{1}{2}} = t^{\frac{5}{2}} + 2t^{\frac{3}{2}}$$

$$= \int \left(t^{\frac{5}{2}} + 2t^{3/2}\right) dt = \frac{2}{7} t^{7/2} + 2 \cdot \frac{2}{5} t^{\frac{5}{2}} + c$$

$$= \frac{2}{7} t^{7/2} + \frac{4}{5} t^{\frac{5}{2}} + c$$

d) $\int \pi^3 dx$ $= \pi^3 x + c$ (1 point)

e) $\int \left(\sec^2 t + t^2 + 2\right) dt$ (1 point)

$$= \tan t + \frac{t^3}{3} + 2t + c$$

f) $\int \frac{\sin 2x}{\sin x} dx$ hint: sin2x = 2 sinx cosx (2 point)

$$= \int \frac{2 \cancel{\sin x} \cos x}{\cancel{\sin x}} dx = 2 \sin x + c$$

2) Given the velocity function (in meters per second) for a particle along a line is
$v(t) = t^2 - 2t - 8$, $1 \le t \le 6$

a) Find the displacement of the particle during the above interval. (2 points)

$$\int_1^6 (t^2 - 2t - 8)\, dt = -3\frac{1}{3} \text{ meters}$$

So, the particle is $3\frac{1}{3}$ meters left of starting Point.

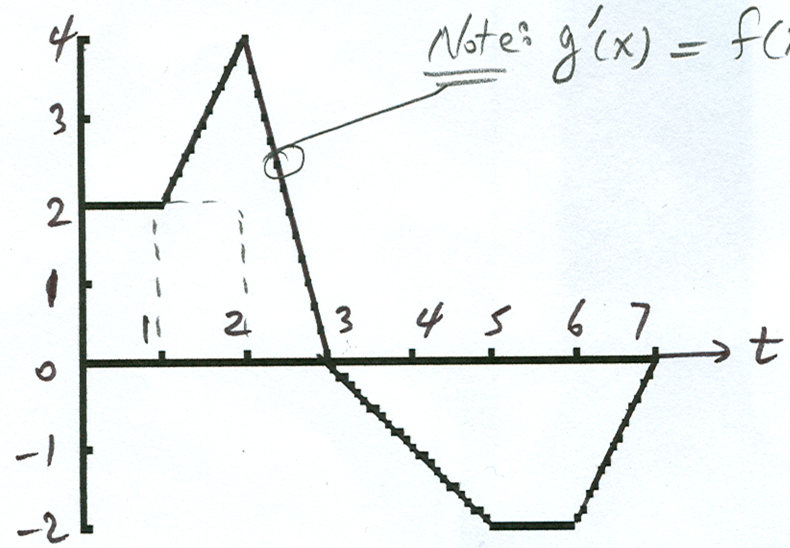
b) Find the distance traveled by the particle during the above interval. (2 points)

$$\left|\int_1^4 (t^2 - 2t - 8)dt\right| + \left|\int_4^6 (t^2 - 2t - 8)dt\right|$$

$$= 18 + 14.67 = 32.67 \text{ meters}$$

3) Let $g(x) = \int_0^x f(t)\, dt$, where f(t) is the function whose graph is shown below.

(2 points Each)



Note: $g'(x) = f(x)$

$g'(x)$ is Positive

a) Evaluate g(6)

$(1)(2) + (1)(2) + \frac{1}{2}(1)(2) + \frac{1}{2}(1)(4) + \frac{1}{2}(2)(-2) + 1(-2) = 3$

b) On what interval is g increasing? $(0, 3)$

c) Where does g have a maximum value?

at $t = 3$ OR $x = 3$

d) On what interval is g concave downward?

Concave up where $g''(x) > 0$; $(1, 2)$ and $(6, 7)$

Concave down when $g''(x) < 0$; $(2, 3) \cup (3, 5)$