Final Exam Review – MA181

I Clearly and completely state:

1. Continuity at $x = a$: 1) $f(a)$ exists
2) $\lim_{x \to a} f(x) = f(a)$

2) Definition of the derivative:
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3) Critical nos. occur when x is in the Domain of $f(x)$ and $f'(x) = 0$ or does not exist. Make a chart of intervals on the x-axis and find the sign of $f'(x)$ in each interval.

4) If $f' > 0$, f is increasing, $f' < 0$, f is decreasing

5) If $f'' > 0$ graph is concave upward.
$f'' < 0$ " " " downward.

6) If a graph has a point of inflection, the concavity changes there, and $f'' = 0$

7) Fundamental Theorem of Calculus:
Suppose $f(x)$ is continuous on $[a, b]$

1.) If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$

2.) $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f ; $F' = f$

Part II - Review Problems

1. Evaluate each limit:

a) $\lim\limits_{x \to 4} \dfrac{x-4}{(x-4)(x+3)} = \lim\limits_{x \to 4} \dfrac{1}{x+3} = \dfrac{1}{7}$

b) $\lim\limits_{x \to \infty} \dfrac{3x-2}{9x-7} = \dfrac{3}{9}$

c) $\lim\limits_{x \to -\infty} \dfrac{2x^3}{x^2+1} \to -\infty$

d) $\lim\limits_{x \to 3^-} \dfrac{2x}{9-x^2} \to +\infty$

2. Vertical Asymptotes:

$y = \dfrac{2x}{(x+6)(x-1)}$ $\quad \lim\limits_{x \to 6^+} (y) \to \infty \quad x = -6$

$\lim\limits_{x \to 1^+} (y) \to \infty \quad x = 1$

Horizontal Asymptotes:

$\lim\limits_{x \to \infty} (y) = 0 \quad y = 0$

3. $f(x) = \begin{cases} x^2 - 8 & \text{if } x \le a \\ \dfrac{x^2-a^2}{x-a} & \text{if } x > a \end{cases}$ $\quad \dfrac{(x+a)(x-a)}{x-a} = x+a$

If $f(x)$ is continuous $\quad x^2 - 8 = x + a \quad$ at $x = a$

$f(x) = x^2 - 8, \; f(a) = a^2 - 8$

$a^2 - 8 = a + a, \quad a^2 - 2a - 8 = 0 \quad (a-4)(a+2) = 0$

$a = 4, a = -2$

#1

(a) $\lim_{x\to 4} \frac{x-4}{x^2-x-12} = \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(x+3)} \Rightarrow \frac{1}{x+3} = \frac{1}{7}$

(b) $\lim_{x\to\infty} \frac{3x-2}{9x-7} = \lim_{x\to\infty} \frac{\frac{3x}{x}-\cancel{\frac{2}{x}}}{\frac{9x}{x}-\cancel{\frac{7}{x}}} = \frac{3}{9} = \frac{1}{3}$

c) $\lim_{x\to-\infty} \frac{2x^3}{x^2+1} = \lim_{x\to-\infty} \frac{\frac{2x^3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \lim_{x\to-\infty} \frac{2x}{1+\cancel{\frac{1}{x^2}}} = 2(-\infty) = -\infty$

d) $\lim_{x\to 3^-} \frac{2x}{9-x^2} = +\infty$ By evaluating $f(2.9999) = \frac{2(2.9999)}{9-(2.9999)^2} = 9999.83$

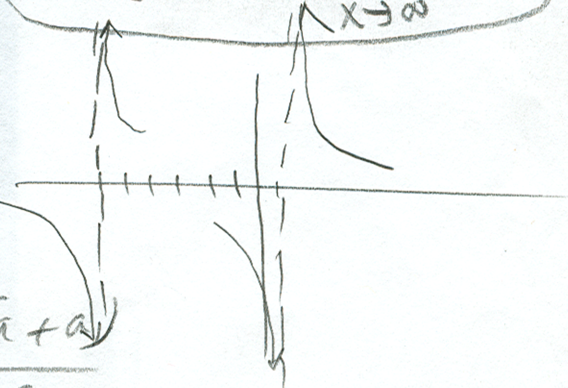
#2

$y = \frac{2x}{x^2+5x-6} = \frac{2x}{(x+6)(x-1)}$

V.A: $x=-6$; $x=1$

B/C $\lim_{x\to-6^-}(y) = \infty$; $\lim_{x\to 1^+}(y) = \infty$

H.A $y=0$ B/C $\lim_{x\to\infty}(y) = 0$



#3

$f(x) = \begin{cases} x^2-8 & \text{if } x \le a \\ \frac{x^2-a^2}{x-a} & \text{if } x > a \end{cases}$

$a^2-8 = \frac{a^2-a^2}{a-a} = \frac{\cancel{(a-a)}(a+a)}{\cancel{(a-a)}}$

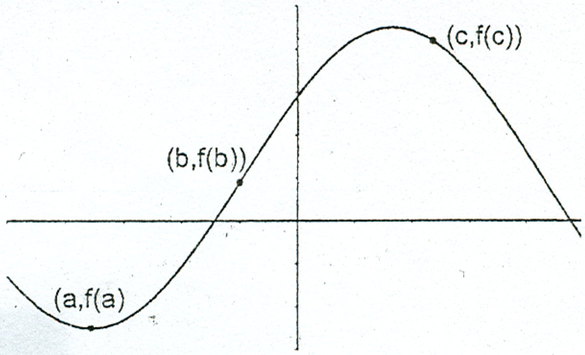
$a^2-8 = 2a$

$a^2-2a-8=0$

$(a-4)(a+2)=0$

$a=4 \quad a=-2$

4. Use the graph of f(x), given to the right, to decide if each of the following is positive, negative or zero.



(a)	$f(a)$ n	(b)	$f(b)$ p	(c)	$f(c)$ p
(d)	$f'(a)$ 0	(e)	$f'(b)$ p	(f)	$f'(c)$ n
(g)	$f''(a)$ p	(h)	$f''(b)$ 0	(i)	$f''(c)$ n

5. Find the equation of the line tangent to the curve:

$$y = \sqrt{9-4x} \qquad x=-4,\ y=5$$

$$y' = \frac{-4}{2\sqrt{9-4x}} \qquad y'(-4) = \frac{-2}{5}$$

$$y - 5 = -\tfrac{2}{5}(x+4)$$

$$y = -\tfrac{2}{5}x - \tfrac{8}{5} + 5 \quad \text{or} \quad 2x + 5y = 17$$

6. a) $y = (2x^3+5x^2-6x-4)^5$

$$\frac{dy}{dx} = y' = 5(2x^3+5x^2-6x-4)^4(6x^2+10x-6)$$

b) $y = \tan^3(5x)$

$$\frac{dy}{dx} = y' = 3\tan^2(5x)\cdot 5 \cdot \sec^2(5x)$$

$$= 15\tan^2(5x)\cdot\sec^2(5x)$$

c) $y = \sqrt{\frac{x}{3x+1}}$, $y' = \frac{\sqrt{3x+1}}{2\sqrt{x}}\left[\frac{(3x+1)(1)-x(3)}{(3x+1)^2}\right]$

$$\frac{dy}{dx} = y' = \frac{1}{2\sqrt{x}\,(3x+1)^{3/2}}$$

d) $y = (x^2+4)^2(2x^3-1)^3$

$$y' = (x^2+4)^2\left[3(2x^3-1)^2(6x^2)\right] + (2x^3-1)^3\left[2(x^2+4)(2x)\right]$$

$$= 2x(x^2+4)(2x^3-1)^2\left(9x(x^2+4) + 2(2x^3-1)\right)$$

$$= 2x(x^2+4)(2x^3-1)^2(9x^3+36x+4x^3-2)$$

$$= 2x(x^2+4)(2x^3-1)^2(13x^3+36x-2)$$

#4 a) Negative b) Positive c) Positive

d) Zero e) Positive f) Negative

g) Positive h) Zero i) Negative

#5 $y=\sqrt{9-4x}=(9-4x)^{1/2}$

$$y'=\frac{1}{2}(9-4x)^{-1/2}(-4)=\frac{-2}{\sqrt{9-4x}}$$

$$m=y'\Big|_{x=-4,\,y=5}=\frac{-2}{\sqrt{25}}=\frac{-2}{5}$$

$$y-5=\frac{-2}{5}(x+4)\implies y=\frac{-2}{5}x-\frac{8}{5}+5=\boxed{\frac{-2}{5}x+\frac{17}{5} \text{ OR } 2x+5y=17}$$

#6 a) $y'=5(2x^3+5x^2-6x-4)^4(6x^2+10x-6)$

b) $y'=3\tan^2(5x)\sec^2(5x)(5)=15\tan^2(5x)\sec^2(5x)$

c) $$y'=\frac{1}{2}\left[\frac{x}{3x+1}\right]^{-\frac{1}{2}}\left[\frac{1(3x+1)-3x}{(3x+1)^2}\right]=\frac{1}{2}\left[\frac{3x+1}{x}\right]^{\frac{1}{2}}\left[\frac{1}{(3x+1)^2}\right]$$

$$=\frac{1}{2\sqrt{x}\,(3x+1)^{3/2}}$$

d) $$y'=2(x^2+4)2x(2x^3-1)^3+3(2x^3-1)^2(6x^2)(x^2+4)^2$$

$$=4x(x^2+4)(2x^3-1)^3+18x^2(2x^3-1)^2(x^2+4)^2$$

$$=2x(x^2+4)(2x^3-1)^2\left[2(2x^3-1)+9x(x^2+4)\right]$$

$$=2x(x^2+4)(2x^3-1)^2(4x^3-2+9x^3+36x)=2x(x^2+4)(2x^3-1)^2(13x^3+36x-2)$$

#6.e) $4x^2y + x^2 - 6y^4 = 5$

$8xy + 4x^2y' + 2x - 24y^3y' = 0$

$y'[4x^2 - 24y^3] = -2x - 8xy$

$$y' = \frac{-2x - 8xy}{4x^2 - 24y^3} = \frac{-2(x + 4xy)}{4(x^2 - 6y^3)} = \boxed{\frac{-(x + 4xy)}{2(x^2 - 6y^3)}}$$

#6 f) $y' = 2e^{2x}\sin x + e^{2x}\cos x = e^{2x}(2\sin x + \cos x)$

#6g) $$y' = \frac{1}{\frac{(5x+2)^4}{\sqrt{x^2+5}}} \cdot \frac{20(5x+2)^3\sqrt{x^2+5} - \frac{1}{2}(x^2+5)^{-1/2} \cdot 2x(5x+2)^4}{(x^2+5)}$$

$$y' = \frac{20(5x+2)^3\sqrt{x^2+5} - \frac{2x}{2}(x^2+5)^{-\frac{1}{2}}(5x+2)^4}{(5x+2)^4(x^2+5)^{1/2}} = \frac{20}{(5x+2)} - \frac{x}{(x^2+5)}$$

$$= \boxed{\frac{15x^2 - 2x + 100}{(5x+2)(x^2+5)}}$$

#6h) $$y' = \frac{1}{(x^3)^2 + 1} \cdot 3x^2 = \boxed{\frac{3x^2}{x^6 + 1}}$$

6. e) $4x^2y + x^2 - 6y^4 = 5$

$$4x^2 \cdot \frac{dy}{dx} + 8xy + 2x - 24y^3\frac{dy}{dx} = 0$$

$$(4x^2 - 24y^3)\frac{dy}{dx} = -(8xy + 2x)$$

$$dy/dx = \frac{-2(4xy + x)}{2(2x^2 - 12y^3)} = -\frac{4xy + x}{2x^2 - 12y^3}$$

f) $y = e^{2x} \cdot \sin x$

$$y' = e^{2x}[\cos x] + \sin x\,[2e^{2x}]$$

$$= e^{2x}(\cos x + 2\sin x)$$

g) $y = \ln \dfrac{(5x+2)^4}{\sqrt{x^2+5}}$

$$y' = \frac{\sqrt{x^2+5}}{(5x+2)^4}\left[\frac{\sqrt{x^2+5}\,[4(5x+2)^3 \cdot 5] - \frac{(5x+2)^4(2x)}{2\sqrt{x^2+5}}}{(\sqrt{x^2+5})^2}\right]$$

$$= \frac{\sqrt{x^2+5}}{(5x+2)^4}\left[\frac{\frac{20(\sqrt{x^2+5})^2(5x+2)^3 - x(5x+2)^4}{\sqrt{x^2+5}}}{(x^2+5)}\right]$$

$$= \frac{\sqrt{x^2+5}\,(5x+2)^3\,[20(x^2+5) - x(5x+2)]}{(5x+2)^4 \cdot \sqrt{x^2+5}\,(x^2+5)}$$

$$= \frac{15x^2 - 2x + 100}{(5x+2)(x^2+5)}$$

or: $y = 4\ln(5x+2) - \frac{1}{2}\ln(x^2+5)$

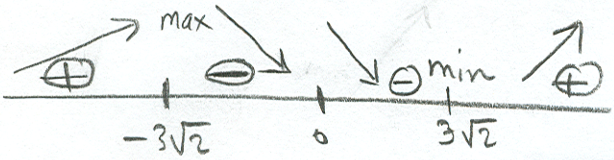
$$y' = \frac{4 \cdot 5}{5x+2} - \frac{2x}{2(x^2+5)}$$

$$= \frac{20}{5x+2} - \frac{x}{x^2+5} = \frac{20(x^2+5) - x(5x+2)}{(5x-2)(x^2+5)}$$

$$= \frac{15x^2 - 2x + 100}{(5x+2)(x^2+5)}$$

#7 $y = x^5 - 30x^3 + 6$

$y' = 5x^4 - 90x^2 \Rightarrow 0 \Rightarrow 5x^2[x^2 - 18] = 0$

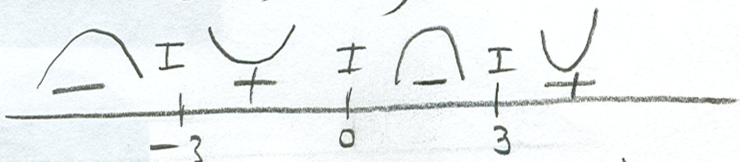


$\boxed{X = 0}$ $\quad X^2 = (9)(2)$

$\boxed{X = \pm 3\sqrt{2}}$

Rel. Max $(-3\sqrt{2}, 922.4)$ $\quad$ Relative Min $(3\sqrt{2}, -910.4)$

$y'' = 20x^3 - 180x$



$(-3, 573)$ $\quad (0, 6)$ $\quad (3, -561)$

$f''(0) = 0 \Rightarrow$

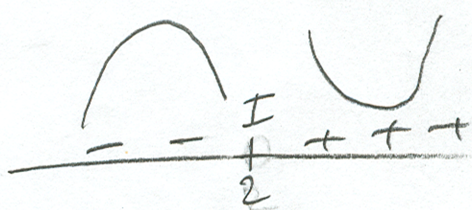
$0 = 20x(x^2 - 9) \Rightarrow x = 0 \quad x = \pm 3$

$f''(3\sqrt{2}) = +763.7 \Rightarrow (3\sqrt{2}, -910) = \text{Rel. Min}$

$f''(-3\sqrt{2}) = -763.7 \Rightarrow (-3\sqrt{2}, 922.41) = \text{Rel. Max}$

#8 $y' = 3x^2 - 12x + 9$

$y'' = 6x - 12 = 6(x - 2)$



Concave down $\quad (-\infty, 2)$ or $X < -2$

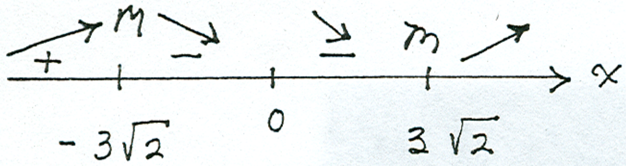
Concave up $\quad (2, \infty)$ or $X > 2$

7) $y = x^5 - 30x^3 + 6$

$y' = 5x^4 - 90x^2 = 5x^2(x^2 - 18)$

$0 = 5x^2(x^2 - 18)$

$x = 0 \qquad x = \pm\sqrt{18} = \pm 3\sqrt{2}$

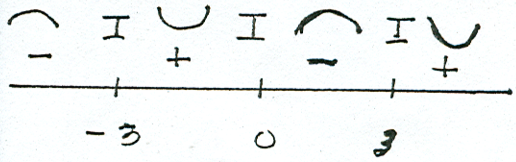


$M: (-3\sqrt{2}, 922.4) \quad m: (3\sqrt{2}, -910.4)$

$y'' = 20x^3 - 180x$

$0 = 20x(x^2 - 9)$

$x = 0, \quad x = \pm 3$



$(-3, 593) \quad (0, 6) \quad (3, -561)$

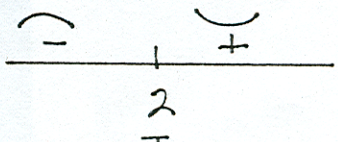
8) $y = x^3 - 6x^2 + 9x$

$y' = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

$y'' = 6x - 12 = 6(x - 2)$

$0 = 6(x - 2)$

$x = 2$



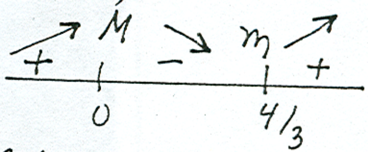
C.D. $x < 2$, C.U. $x > 2$

9) $f(x) = x^3 - 2x^2$ on $[-1, 1]$

$f'(x) = 3x^2 - 4x = x(3x - 4)$

$0 = x(3x - 4)$

$x = 0, \quad x = 4/3$

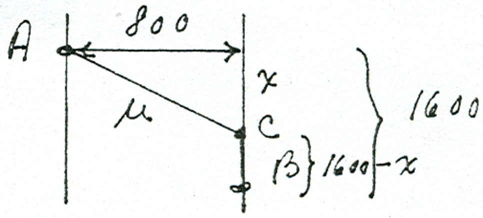


$\underbrace{f(-1) = -3}_{\text{absolute minimum}} \quad f(1) = -1 \quad \underbrace{f(0) = 0}_{\text{absolute maximum}}$

10) $C(x) = 5u + 3(1600 - x)$

$= 5\sqrt{800^2 + x^2} + 3(1600 - x)$

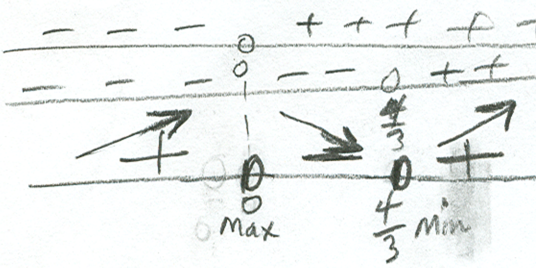
$C'(x) = \dfrac{5 \cdot 2x}{2\sqrt{800^2 + x^2}} - 3$



$x = 600 \quad C(x) = \$8{,}000$

#9 $f(x) = x^3 - 2x^2 \; [-1, 1]$

$f'(x) = 3x^2 - 4x$



$3x^2 - 4x = 0 \Rightarrow x(3x-4) = 0$

$\boxed{x = 0}$ $\boxed{x = \frac{4}{3}}$

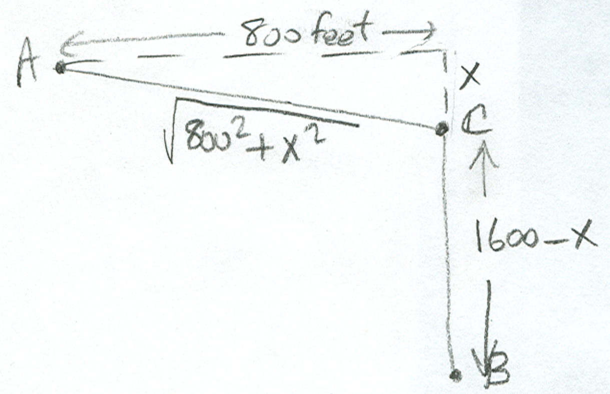
Also; $f''(x) = 6x - 4$

$f''(0) = -4 \Rightarrow$ (0, 0) is a maximum of f(x)

And at $x = -1$ $y = -3$ → minimum of f(x)

at $x = 1$ $y = -1$

#10



$C(x) = \frac{\$5}{\text{foot}}(800^2 + x^2)^{\frac{1}{2}} \cancel{\text{foot}} + \frac{\$3}{\text{foot}}(1600 - x)\cancel{\text{foot}}$

$C'(x) = \frac{5}{\cancel{2}}(800^2 + x^2)^{-\frac{1}{2}} \cdot \cancel{2}x - 3 = 0 \Rightarrow \frac{5x}{(800^2 + x^2)^{\frac{1}{2}}} = 3$

$\frac{25x^2}{800^2 + x^2} = \frac{9}{1} \Rightarrow \frac{16}{9}x^2 = 800^2 \Rightarrow x^2 = 360000$

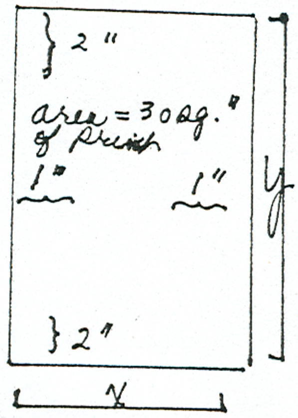
$25x^2 = 9(800^2 + x^2)$

$25x^2 = 9(800^2) + 9x^2 \Rightarrow 16x^2 = 9(800)^2$

$x = 600$ feet $C(600) = \$8000$

Old

11. A page is to contain 30 square inches of print. The margins at the top and the bottom of the page are each 2 inches wide. The margins on each side are only 1 inch wide. Find the imensions of the page so that the least paper is used.



Printed area $= 30 = (x-2)(y-4)$

$$30 = xy - 4x - 2y + 8$$

$$xy - 2y = 30 - 8 + 4x$$

$$y = \frac{22+4x}{x-2}$$

Area of page: $A = xy = x\left(\frac{22+4x}{x-2}\right)$

$$A'(x) = x\left[\frac{(x-2)[4]-(22+4x)[1]}{(x-2)^2}\right] + \left(\frac{22+4x}{x-2}\right)$$

$$= x\left[\frac{4x-8-22-4x}{(x-2)^2}\right] + \frac{(22+4x)(x-2)}{(x-2)^2}$$

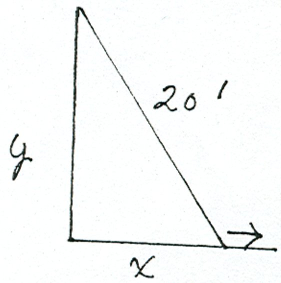
$$0 = \frac{-30x+22x-44+4x^2-8x}{(x-2)^2}$$

$$0 = 4x^2-16x-44 = 4(x^2-4x-11)$$

Using the binomial theorem: $x = 2+\sqrt{15}$ ☺

$$y = \frac{22+4(2+\sqrt{15})}{(2+\sqrt{15})-2} = \frac{22+8+4\sqrt{15}}{\sqrt{15}}\left[\frac{\sqrt{15}}{\sqrt{15}}\right] = 4+2\sqrt{15}$$ or $2x$!

12. A 20-foot ladder is propped against a wall. The foot of the ladder is sliding away from the wall at the rate of 6 ft. per minute. At what rate is the top of the ladder moving down the wall when the foot of the ladder is 16 ft. from the wall?



$$x^2 + y^2 = 20^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

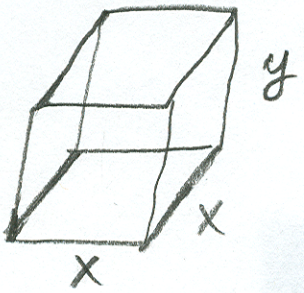
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\left.\frac{dy}{dt}\right|_{x=16'} = -\frac{16}{12}(6) = -8 \text{ ft/min}$$

$$(16)^2 + y^2 = 20^2$$

$$y = \sqrt{400-256} = 12$$

#11



$$SA = \overset{x^2}{Top} + \overset{x^2}{Bottom} + \overset{4xy}{4\ sides}$$

$$C(x) = 5x^2 + 1x^2 + 4xy$$

$$C(x) = 6x^2 + 4xy$$

$$6x^2 + 4xy = 72$$

$$y = \frac{72 - 6x^2}{4x} = 18 \cdot \frac{1}{x} - \frac{3}{2}x$$

$$V(x) = x^2 \cdot y = x^2\left[\frac{18}{x} - \frac{3}{2}x\right] = 18x - \frac{3}{2}x^3$$

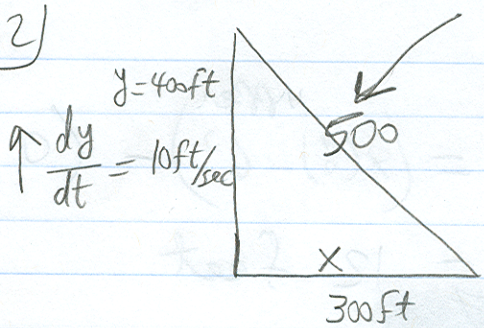
$$V'(x) = 18 - \frac{9}{2}x^2 = 0 \Rightarrow x^2 = 4 \Rightarrow x = 2 \text{ feet}$$

$y = 6$ feet

size = 2 feet × 2 feet × 6 feet

12

$400^2 + 300^2 = c^2 \Rightarrow \boxed{c = 500\,ft}$



a) $\dfrac{dc}{dt} = ?$

$$x^2 + y^2 = c^2$$

$$\cancel{2x\frac{dx}{dt}} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$$

0

But $\dfrac{dx}{dt} = 0$ because $x = 300ft$ i.e. always constant

$$\frac{dc}{dt} = \frac{y}{c}\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{400ft}{500ft} * 10ft/sec = \boxed{8.00\ ft/sec}$$

$$\tan\theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{y'x - x'y}{x^2}\right)$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{400}{300}\right)^2}\left(\frac{10(300) - 0}{(300)^2}\right) = \boxed{0.012\ \frac{radians}{sec}}$$

Another Method

$$y = x\tan\theta \Rightarrow y' = x\sec^2\theta\frac{d\theta}{dt} + \cancel{(x')}\tan\theta$$

0

$$10 = 300\left(\frac{500}{300}\right)^2\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = 0.012\ radians/sec$$

a) $\int_0^6 (8-2t)\,dt$

$8t - \frac{\cancel{2}t^2}{\cancel{2}} \Big|_0^6 = \left(8(6) - 6^2\right) - (0)$ upper

$S(6) - S(0) \Rightarrow = 12$ feet

b) $v(t) = 8-2t \qquad 0 \leq t \leq 6$

$8-2t = 0 \Rightarrow -2t = -8 \Rightarrow t = \frac{-8}{-2} = 4.$

$2t = 8 \Rightarrow t = 4$ sec

$$\left|\int_0^4 v(t)\,dt\right| + \left|\int_4^6 v(t)\,dt\right|$$

$$\left|\int_0^4 (8-2t)\,dt\right| + \left|\int_4^6 (8-2t)\,dt\right|$$

$$(8t-t^2)\Big|_0^4 + (8t-t^2)\Big|_4^6$$

$$= \left|\left(8(4)-4^2\right)-(0)\right| + \left|\left(8(6)-6^2\right) - \left(8(4)-4^2\right)\right|$$

$$16 + |12 - 16|$$

$$16 + 4 = 20 \text{ feet}$$

13. Evaluate each definite or indefinite integral:

(a) $\int_{-1}^{2}(3x^2 - x + 2)\,dx$ (b) $\int \frac{2x^2 + 5x}{x^3}\,dx$ (c) $\int (2\cos t - 5\sec^2 t)\,dt$

a) $= x^3 - \frac{x^2}{2} + 2x \Big]_{-1}^{2} = \left(2^3 - \frac{2^2}{2} + 2(2)\right) - \left((-1)^3 - \frac{(-1)^2}{2} - 2\right)$

$= 8 - 2 + 4 + 1 + \frac{1}{2} + 2$

$= 13\frac{1}{2} = \frac{27}{2} = 13.5$

b) $\int (2x^{-1} + 5x^{-2})\,dx = 2\ln|x| - \frac{5}{x} + C$

c) $\int (2\cos t - 5\sec^2 t)\,dt = 2\sin t - 5\tan t + C$

14. An object moves according to the equation $s = t(2t-1)^3$, where s is the position of the object in feet and t is time in seconds. Find the acceleration when t = 1.

$s'(t) = v(t) = t\left[3(2t-1)^2 \cdot 2\right] + (2t-1)^3 = (2t-1)^2(8t-1)$ $s'' = v' = a(t)$

15. Use Newton's Method to approximate the root of cosx + x = 2 to four decimal places.

$= 4(2t-1)(12t-1)$

2.9883 I graphed $y_1 = \cos(x) + x - 2$ and traced to the x-int. x is near 3. Then run Newton Prog. $x \approx 2.9883$

$a(1) = 36$ ft/sec²

16. For the functions $f(x) = \frac{1}{1-x^2}$ and $g(x) = \frac{x^2}{1-x^2}$, $g'(x) = \frac{(1-x^2)(2x) - x^2(-2x)}{(1-x^2)^2}$

(a) Show that these functions have the same derivative. $= \frac{2x}{(1-x^2)^2} = f'(x)$

(b) What does this imply about the relationship between these functions? That is, are they the same function? If not, in what way do they differ? You may find it helpful to graph the functions. $g(x): 1 - x^2 \overline{)\,x^2\,}$ $\frac{x^2 - 1}{1}$ $= 1 - f(x)$ or $f(x) = g(x) + 1$

17. The graph of the derivative of a certain function f appears below. $m = f'(x)$

(a) Suppose f(1) = 5, Find an equation of the line tangent to the graph of f at (1,5). $m(1) = f'(1) = 3/2$

(b) Suppose f(-1) = -2. Could f(3) = -6? Why or why not? On $[-1,3]$ $f' > 0$ f is increasing $y - 5 = \frac{3}{2}(x-1)$

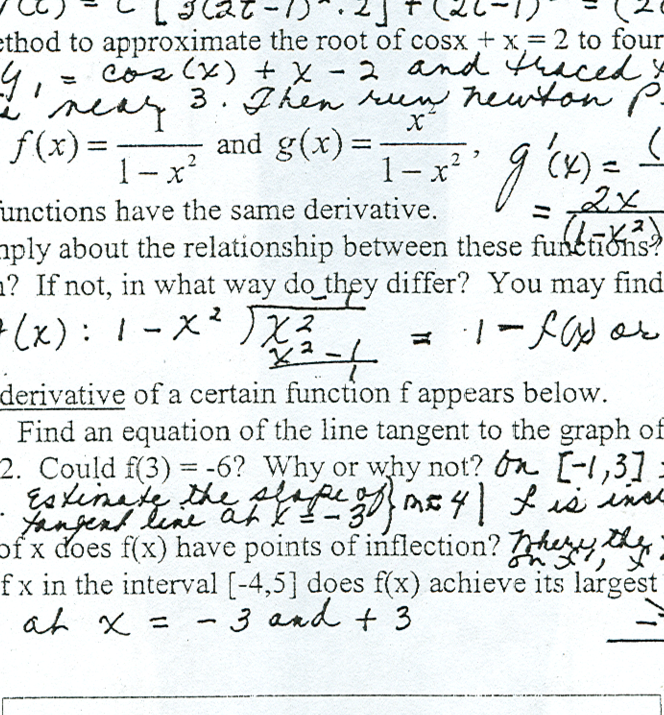
(c) Estimate $f''(-3)$. Estimate the slope of tangent line at $x = -3$ $m \approx 4$ $y = \frac{3}{2}x + \frac{7}{2}$

(d) At which values of x does f(x) have points of inflection? Where the tangent line is horizontal on f', $f'' = 0$ $x = -1$, $x = 3$

(e) At which value of x in the interval [-4,5] does f(x) achieve its largest value? Its smallest value? $f' = 0$ at $x = -3$ and $+3$

↘ m ↗ ↗ M

-3 3 x



Minimum Value of f occurs at $x = -3$

Maximum Value of f occurs at $x = 5$

18. Two curves are said to cross at right angles if their tangent lines are perpendicular at the crossing point. The technical word for "crossing at right angles" is *orthogonal*. Show that the curves y = sin 2x and y = -sin(x/2) are orthogonal at the origin. Draw both graphs and both tangent lines in a square viewing window to confirm your results.

Let $f(x) = \sin(2x)$

$f'(x) = 2\cos(2x)$

$f'(0) = 2\cos(2\cdot 0)$

$= 2(1) = 2$

Let $g(x) = -\sin(x/2)$

$g'(x) = 1/2(-\cos(x/2))$

$g'(0) = -1/2 \cos(0/2)$

$= -1/2\,(1) = -1/2$

Lines which are perpendicular have slopes which are negative reciprocals of each other: 2 & -1/2 are negative reciprocals.

19. Explain the difference between the average rate of change in a function and the instantaneous rate of change. What's the difference in the way you compute these quantities? Illustrate by considering the average rate of change of the function $f(t) = t^3 + 5t$ on the interval [1.9, 2.1] and the instantaneous rate of change of the same function at t = 2.

Average Rate of Change: $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

Instantaneous Rate of Change: The derivative

$f(t) = t^3 + 5t$ $[1.9, 2.1]$ Let $x = 1.9$ and $\Delta x = .2$

$ARC = \dfrac{f(2.1) - f(1.9)}{.2} = \dfrac{19.761 - 16.359}{.2} = 17.01$

$IRC = f'(t) = 3t^2 + 5$. $f'(2) = 3(2)^2 + 5 = 17$

20. Some values of a function f and its derivative f' are tabulated below:

c) $j'(x) = 4x^3 \cdot f'(x^4)$

$j'(-1) = -4\,(3)$

$= -12 < 0$

j is decreasing

x	-3	-2	-1	0	1	2	3
f(x)	4	-1	1	3	-2	2	5
$f'(x)$	0	1	2	-1	3	-2	-3

d) $k(x) = [F(x)]^2$

$k'(x) = 2F(x) \cdot f($

$k''(x) = 2F(x) \cdot f'($

$+ 2[f(x)]$

$k''(0) = 2(3)(-1) +$

$2(3)^2$

$= -6 + 18 =$

$12 > 0$ and k is c.

(a) Let g(x) = 2f(x + 3). Evaluate g'(-2)

$g'(x) = 2f'(x+3)$

$g'(-2) = 2f'(-2+3) = 2f'(1) = 6$

(b) Let $h(x) = \dfrac{\ln x}{f(x)}$. Is x = 1 a critical number of h? Justify your answer.

$h'(x) = \{f(x)\cdot\frac{1}{x} - \ln(x) \cdot f'(x)\} \div [f(x)]^2$ $h'(1) = -1/2 \neq 0$ no

(c) Let $j(x) = f(x^4)$. Is j increasing or decreasing at x = -1? Justify your answer.

(d) Suppose that F is an antiderivative of f and that F(0) = 3. If $k(x) = (F(x))^2$, is k concave

21. Suppose that N(t), the number of bacteria is a culture at time t is given by

$N(t) = 25 + te^{-t/20}$, where N is measured in millions of bacteria and t is in hours.

(a) At what time during the interval [0,100] is the number of bacteria the smallest? What is the minimum number? $N'(t) = -\frac{1}{20} t e^{-t/20} + e^{-t/20} = e^{-t/20}(1 - \frac{1}{20} t)$

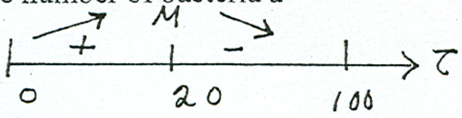
$N(0) = 25$

(b) At what time during the interval [0,100] is the number of bacteria the largest? What is the maximum number? at 20 hours $N(20) = 32.36$ million

$N'(t) = 0$ when $t = 20$

(c) At what time during the interval [0,100] is the rate of change of the number of bacteria a minimum? $N''(t) = \frac{1}{(20)^2} t e^{-t/20} - \frac{1}{10} e^{-t/20}$

$N''(t) = 0$ when $t = 40$



minimum $\rightarrow N(0) = 25$ million

$N(100) = 25 + 100e^{-100/20} = 25.674$m.

22. Find a function y = f(x) with the following properties:

(i) $f''(x) = 6x$

(ii) Its graph passes through the point (0,1) and has a horizontal tangent there.

If $f''(x) = 6x$, $f'(x) = 3x^2 + C_1$ horizontal tangent at $x = 0$ $f'(0) = 0 = C_1$

$f(x) = x^3 + C_2$

$f(0) = 0 + C_2 = 1 \Rightarrow C_2 = 1$

Therefore, $f(x) = x^3 + 1$

23. The acceleration of gravity near the surface of Mars is 3.72m/sec^2. If a rock is blasted straight up from the surface with an initial velocity of 93 m/sec, how high does it go?

$a = -3.72 \text{m/sec}^2$

$v = -3.72t + 93$ when $v = 0$, $3.72t = 93$

$t = 25$ sec.

$s = -\frac{3.72}{2} t^2 + 93t + 0$

$s(25) = 1162.5$m

24. Find $\int_2^4 x \ln x \, dx$ in two ways.

(a) Use left and right sums with n = 50 and average your results. av. (6.209..., 6.787...) ≈ 6.70!

(b) One of the three functions F, G and H below is an antiderivative of the function x ln x. Decide which function it is and then use the Fundamental Theorem of Calculus to evaluate the given integral.

F = x ln x - x, G = x^2(2ln x - 1)/4, H = (x^{2}ln x)/2

$F'(x) = x \cdot \frac{1}{x} + \ln x - 1 = 1 + \ln x - 1 = \ln x$

* $G'(x) = \frac{x^2}{4}\left[\frac{2}{x}\right] + \frac{2x}{4}(2\ln x - 1) = \frac{2x}{4} - \frac{2x}{4} + \frac{4x\ln x}{4} = x \ln x$

$H'(x) = \frac{1}{2}x^2 \cdot \frac{1}{x} + \ln x\,[x] = \frac{1}{2}x + x\ln x$

$\int_2^4 x \ln x \, dx = G(x)\Big]_2^4 = G(4) - G(2) = \frac{4^2}{4}(2\ln(4) - 1) - \frac{2^2}{4}(2\ln(2) -$

$= 4 \cdot 2\ln 4 - 4 - 2\ln 2 + 1 = \ln 4^8 - \ln 2^2 -$ [illegible]

$= \ln(4^8/2)$ [illegible]

25. Given the curve $x^2y - 5xy^3 + 6 = 0$,

(a) Find the slope and equation of the tangent line to this curve at (3,1).

(b) Use a linear approximation to estimate the y-value of a point on this curve with x-value 3.2.

a) $x^2 \cdot \frac{dy}{dx} + 2xy - 5x \cdot 3y^2 \frac{dy}{dx} - 5y^3 = 0$

$(x^2 - 15xy^2)\frac{dy}{dx} = 5y^3 - 2xy$

$\frac{dy}{dx} = \frac{5y^3 - 2xy}{x^2 - 15xy^2}$

$m|_{(3,1)} = \frac{5(1)^3 - 2(3)(1)}{(3)^2 - 15(3)(1)^2} = \frac{-1}{-36} = \frac{1}{36}$

$y - 1 = \frac{1}{36}(x-3) \Rightarrow y = \frac{1}{36}x + \frac{33}{36}$

or $x - 36y + 33 = 0$

b) $y(3.2) \approx \frac{1}{36}(3.2) + \frac{33}{36} = 1.005\overline{5}$

26. Let $f(x) = x^2 + \frac{a}{x}$.

(a) What value of a makes f have a local minimum at x = 2?

(b) What value of a makes f have a point of inflection at x = 1?

(c) Is there any value of a for which f can have a local maximum? Why or why not?

a) $f'(x) = 2x - \frac{a}{x^2}$

$0 = 2x - \frac{a}{x^2}$

$0 = 2x^3 - a$

$x^3 = a/2$ If $x = 2$, $a = 2 \cdot 2^3 = 16$

b) $f''(x) = 2 + \frac{2a}{x^3}$

$0 = 2 + \frac{2a}{x^3}$

$0 = 2x^3 + 2a$

$a = -x^3$ If $x = -1$, $a = (-1)^3 = -1$

c) If $a = 2x^3$, then $x = \sqrt[3]{a/2}$, which is one critical no.
This will yield a minimum value of f $\lim f(x) = \infty$

27. You are designing right circular cylindrical cans with volumes of 1000 cubic centimeters. The manufacturer of these cans will take waste into account. There is no waste in cutting the aluminum for the sides, but the tops and bottoms of radius r will be cut from squares that measure 2r cm on a side. The total amount of aluminum used up by each can will therefore be $A = 8r^2 + 2\pi rh$. Find the values of r and h which will minimize the amount of aluminum used.

$A = 2(2r)^2 + 2\pi r h$ $\qquad V = 1{,}000 = \pi r^2 h$

$h = \dfrac{1{,}000}{\pi r^2}$

$A = 8r^2 + 2\pi r\left(\dfrac{1{,}000}{\pi r^2}\right)$

$A = 8r^2 + \dfrac{2{,}000}{r}$

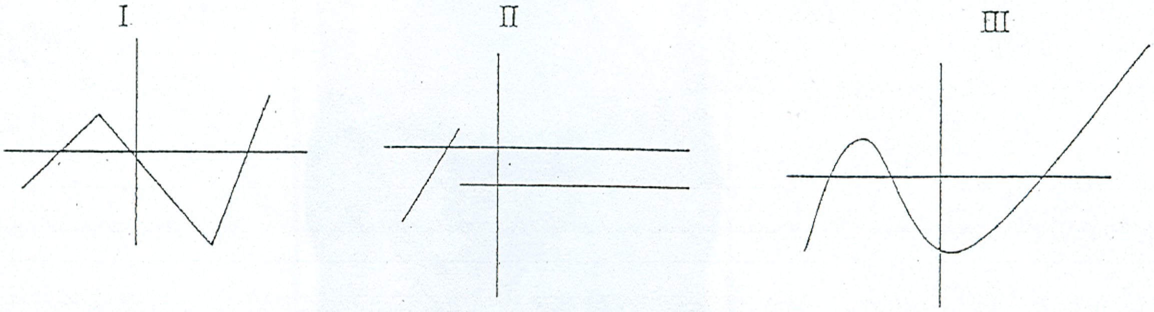
$A'(r) = 16r - \dfrac{2{,}000}{r^2}$

$0 = 16r^3 - 2{,}000$ and $r = 5$ cm

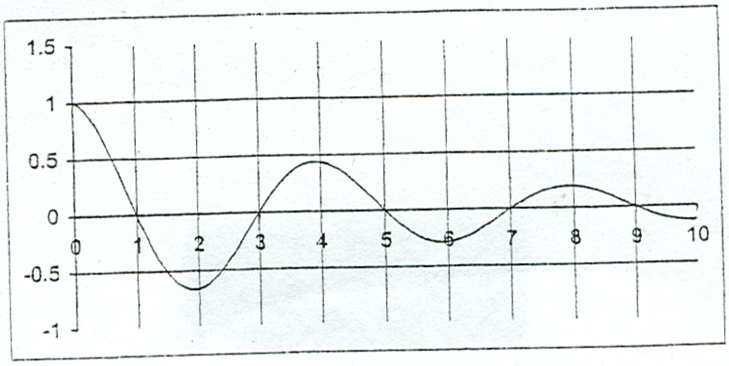
$h = \dfrac{1{,}000}{\pi(5)^2} = \dfrac{40}{\pi}$ cm

28. Which of the graphs below suggest a function y = f(x) that is I & III
(a) continuous for all real x?
(b) Differentiable for all real x? b) III
(c) Both (a) and (b)? c) III
(d) Neither (a) nor (b)? d) II

Explain in each case.

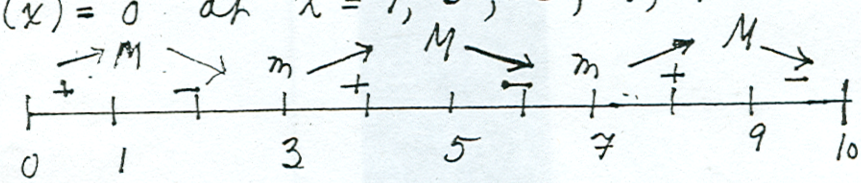


29. Let g(x) be defined by $g(x) = \int_0^x f(t)\,dt$. The graph of f is shown below.



(a) Does g(x) have any local maxima within the interval [0,10]? If so, where are they located?

$f(x) = 0$ at $x = 1, 3, 5, 7, 9$



Local maxima occur where $x = 1, 5, 9$

(b) At what value of x does g(x) attain its absolute minimum value on the interval [0,10]? 3 - [illegible]

(c) On which subinterval(s) of [0,10], if any, is the graph of g(x) concave up? Justify your answer. Where $g'' = f'$ is positive the graph of g is c.u. (2,4) and (6,8) f is inc.

(d) Approximate the values of g(1) and g(3).

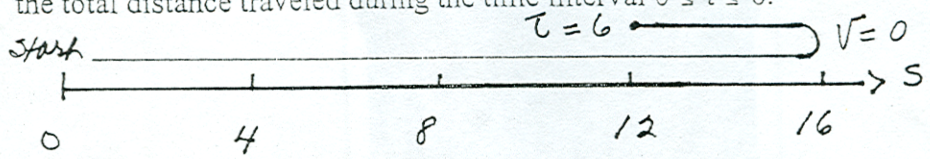
$g(1) \approx .6$ $g(3) \approx -.2$ or $-.3$

30. The velocity of an object moving on a horizontal line is given by v(t) = 8 - 2t (in ft/sec). Find

If $v(t) = 8 - 2t$, $s(t) = 8t - t^2 + s_0$, let $s_0 = 0$
When $v = 0$, $t = 4$ $s(4) = 8(4) - 4^2 = 16$

(a) the displacement during the time interval $0 \le t \le 6$.
(b) the total distance traveled during the time interval $0 \le t \le 6$.

a)



position of object

The Velocity is 0 at t = 4 sec. The object is at 16' and then turns left. At 6 sec. the position is 12'

b) The object has traveled 16' to the right and then 4' to the left. Total distance = 20'