Professor Katiraie Calculus I Spring 2008 Test IV Form B (chapters 1 -- 4)

Name ___________________________________ Total Possible Points = 140

☺☺☺ Plus 10 Points Extra Credit ☺☺☺

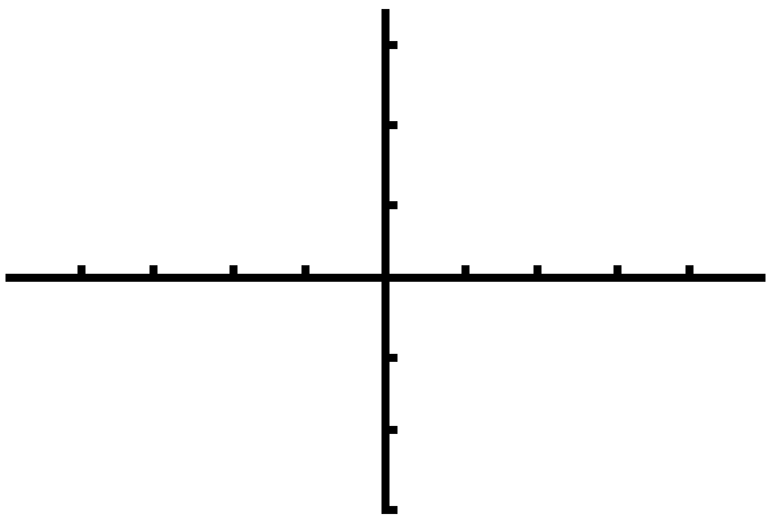
1) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (6 Points)

$$\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = -1$$

$$\lim_{x\to -2^+} f(x) = \infty \qquad \lim_{x\to 1^-} f(x) = -\infty$$

$$\lim_{x\to -2^-} f(x) = -\infty \qquad \lim_{x\to 1^+} f(x) = \infty$$

$$f(0) = 1.5$$



2) Find the equation of the tangent line to the curve $y = x\cos x + x\sin x$, at the point $(\pi, -\pi)$. (10 Points)

3) Find the derivative of the following functions: **(4 Points each)**

(Do Not Simplify)

a) $y=\sqrt{2x+\sqrt{3x}}$	b) $y=e^{\sec(2\theta)}$
c) $y=\sin^5(4x)$	d) $y=\sin^{-1}(x^2+2x+1)$
e) $y=10^{[\sin(3\theta)+\cos(2\theta)]}$	f) $y=\left(\dfrac{-3x-7}{2x^2-1}\right)^7$

For Problems 4 - 6: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.

4) A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point (4 , 2), its x-coordinate increases at a rate of 3 cm/sec. How fast is the distance from the particle to the origin changing at this instant? (10 points)

5) If the diagonal of a square decreases at the rate of 2 inch/second, how fast is the area changing when the side of the square is 15 inches? (10 points)

6) A plane flying horizontally at an altitude of 2.5 km and a speed of 400 km/h passed directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 4 km away from the station (10 points)

For problems 7 - 9: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.

7) If 1400 sq. cm of material is available to make a box with a square base and an open top, find the largest possible volume of the box. (10 points)

8) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point (2,0) (10 points)

9) A rectangular field is to be enclosed on four sides with a fence. Fencing costs $10 per foot for two opposite sites, and $5 per foot for the other two sides. Find the dimensions of the field of area 730 sq. ft. that would be the cheapest to enclose.

(10 points)

10) Analytically find the exact value of all critical numbers of the following functions.
(In other words, find the x-coordinates of the critical points.) (12 points)

$y = x^{\frac{4}{5}}(x-4)^2$

11) Find all value(s) of c (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = (x-3)^3$ on the interval $[0,1]$ (5 points)

12) Given that the function $f(x) = x^3 + ax^2 + bx$ has critical numbers at $x = 1$, and $x = -2$, find a and b. (5 points)

13) Find all the points of inflection of $f(x) = x^5 e^{-x}$ (5 points)
(Must Justify Your Answer)

14) A company has cost function $C(x) = 100 - 14x + x^2$ and demand function $p(x) = 18 - x$, where x is the number of staplers and $p(x)$ is in dollars.

a. How many units should the company make to maximize its profit? (5 points)

b. How much is the maximum profit? (3 points)

c. What price would produce maximum profit? (3 points)

15) Given $f^{\prime\prime}(x) = 2x^{-2}$, $x > 0$, $f(1) = 1$, $f(2) = 0$ Find $f(x)$ (12 points)

16) A pumpkin pie is thrown upward with a speed of 20 ft/sec from the edge of a cliff 150 feet above the ground.
(Assume gravity of earth is -32)

a) Find the pie's height above the ground t seconds later. (4 points)

b) When does the pie reach its maximum height? (3 points)

c) When does the pie hit the ground? (3 points)

Bonus Question:

18) Find the **equation of the tangent line** to the parametric curve $x = t^2 + 3, \quad y = 2t^3 - t$ at the point corresponding to $t = 2$.

(5 points)

19) The angle of elevation of the Sun is decreasing at a rate of 0.25 radians/hour. How fast is the shadow cast by a 300–foot–tall building increasing when the angle of elevation of the Sun is $\frac{\pi}{3}$

(5 points)