Professor Katiraie   Calculus I   Spring 2008   Test V Form A (chapters 1 -- 5)

Name ___________________________________   Total Possible Points = 140

☺☺☺ **Plus 10 Points Extra Credit** ☺☺☺

1) Find the equation of the tangent line to the curve $y = 2x\cos x$,
at the point $(\pi, -2\pi)$. (10 Points)

2) A particle starts at the origin and moves along the parabola $y = x^2$ such that its distance from the origin increases at 5 units per second. How fast is its x-coordinate changing as it passes through the point (5 , 25)? (10 points)

3) The angle of elevation of the Sun is decreasing at a rate of 0.25 rad/hour. How fast is the shadow cast by a 400–foot–tall building increasing when the angle of elevation of the Sun is $\frac{\pi}{6}$ (10 points)

3) A plane flying horizontally at an altitude of 5 km and a speed of 400 km/h passed directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 7 km away from the station. (10 points)

4) If 2000 sq. cm of material is available to make a box with a square base and an open top, find the largest possible volume of the box. (10 points)

5) Find the points on the ellipse $6x^2 + y^2 = 8$ that are farthest away from the point (2,0) (10 points)

9) Given $f^{/}(x) = 2\sqrt{x} \bullet (6-5x)$ and f(1) = 7; Find $f(x)$ (5 points)

10) Given $f^{//}(x) = 5x^{-2}$, $x > 0$, $f(2) = 3$, $f(4) = 0$
Find $f(x)$ (5 points)

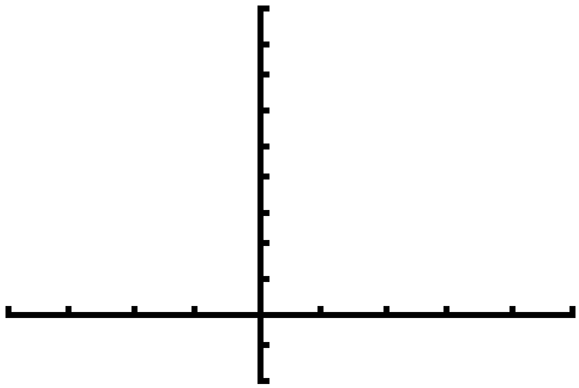
11) If $\int_0^3 f(x)dx = 21$, $\int_0^6 g(x)dx = 4$, and $\int_0^3 g(x)dx = 7$

a) Find the value of $\int_3^0 f(x) * g(x)dx$ (5 points)

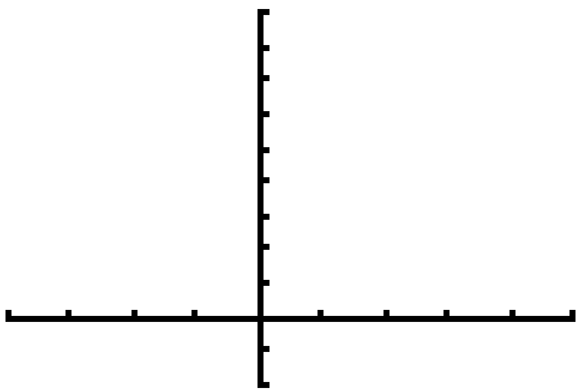
b) Find the value of $\int_3^0 (f(x) - g(x))dx$ (5 points)

12) Given the function $f(x) = 1 + x^2$, $-3 \le x \le 1$
Estimate the area under the graph of f(x) using 4
(hint: n = 4)approximating rectangles and taking the sample points to be:

a) Right endpoints (Draw the appropriate rectangles and find the area) (5 points)



b) Midpoints (Draw the appropriate rectangles and find the area) (5 points)



13) Given $\int_a^b x\,dx = 30$ and $\int_b^a 5\,dx = -30$ Find a and b. (10 points)

14) Water flows from the bottom of a storage tank at a rate of

$r(t) = 200 - 4t \quad \dfrac{\text{liters}}{\text{minute}}$, Where $0 \le t \le 50$ minutes (10 points)

a) Find the amount of water that flows from the tank initially (at time t = 0).

b) Find the amount of water that flows from the tank during the first 25 minutes.

15) Find the area enclosed by the following curves: (10 points)

$y = 2x + x^2$

and

$y = 2x + 9$

16) Let $f(x) = \left(\int_{2x}^{10} \sqrt{t}\,dt \right) + 100$

Find the value of $f^{/}(10000)$ (5 points)

17) The velocity of a particle moving along a line is $t^2 - 3t - 4$ meters per second.

Find the acceleration of the particle when the velocity of the particle is zero. (5 points)

18) Find the value of the integral $\int_C^D \frac{3x^2 - 5}{x} dx$ (5 points)

(Assume C >0 and D > 0, and leave your answer in terms of C and D)

19) Determine by differentiation whether the following formula is true or false (Must Show Procedure)

$$\int \frac{du}{u^2 + a^2} = \frac{1}{2a} \ln\left|\frac{2u+a}{u-a}\right| + C$$ (5 points)

Bonus Question:

20) Let $f(x) = \frac{1}{2} \int_{2x}^{5x} \frac{u+2}{u-1}\, du$

Find the value of $f'(0)$

(5 points)

21) A closed box with square base is to be built to house an ant colony. The bottom and top of the box will be made of material costing $1 per square foot, and all four sides are to be constructed of glass costing $5 per square foot. What are the dimensions of the box of greatest volume that can be constructed for $65?
(Round your answers to two decimal places) (5 points)