

Name \_\_\_\_\_

Total Possible Points = 140

☺☺☺ **Plus 10 Points Extra Credit** ☺☺☺

- 1) If the diagonal of a square decreases at the rate of 2 inch/second, how fast is the area changing when the side of the square is 15 inches? (10 points)

- 2) A particle starts at the origin and moves along the parabola  $y = x^2$  such that its distance from the origin increases at 5 units per second. How fast is its x-coordinate changing as it passes through the point (5 , 25)? (10 points)

- 3) The angle of elevation of the Sun is decreasing at a rate of 0.2 rad/hour. How fast is the shadow cast by a 400-foot-tall building increasing when the angle of elevation of the Sun is  $\frac{\pi}{3}$  (10 points)

4) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$10 per foot for two opposite sides, and \$5 per foot for the other two sides. Find the dimensions of the field of area 730 sq. ft. that would be the cheapest to enclose.

(10 points)

5) Analytically find the exact value of all critical numbers of the following functions. (In other words, find the x-coordinates of the critical points.)

(10 points)

$$y = x^{\frac{4}{5}}(x - 4)^2$$

6) A company has cost function  $C(x) = 100 - 14x + x^2$  and demand function  $p(x) = 25 - x$ , where  $x$  is the number of staplers and  $p(x)$  is in dollars.

a. How many units should the company make to maximize its profit? (5 points)

b. How much is the maximum profit? (3 points)

c. What price would produce maximum profit? (3 points)

7) Given  $f'(x) = 2\sqrt{x} \cdot (6 - 5x)$  and  $f(1) = 7$ ; Find  $f(x)$  (5 points)

8) Given  $f''(x) = 5x^{-2}$ ,  $x > 0$ ,  $f(2) = 3$ ,  $f(4) = 0$   
Find  $f(x)$  (5 points)

9) If  $\int_0^3 f(x)dx = 21$ ,  $\int_0^6 g(x)dx = 4$ , and  $\int_0^3 g(x)dx = 7$

a) Find the value of  $\int_3^0 f(x) * g(x)dx$  (5 points)

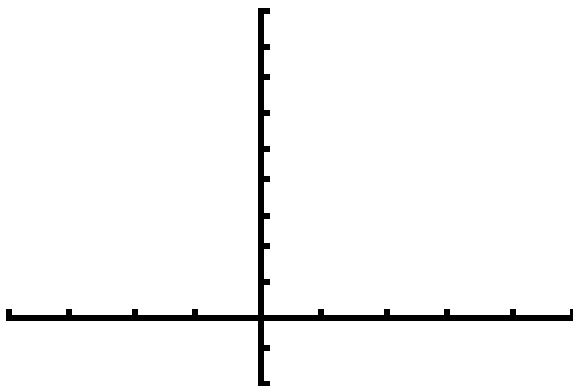
b) Find the value of  $\int_3^0 (f(x) - g(x))dx$  (5 points)

10) Given the function  $f(x) = 2 + x^2$ ,  $-2 \leq x \leq 2$

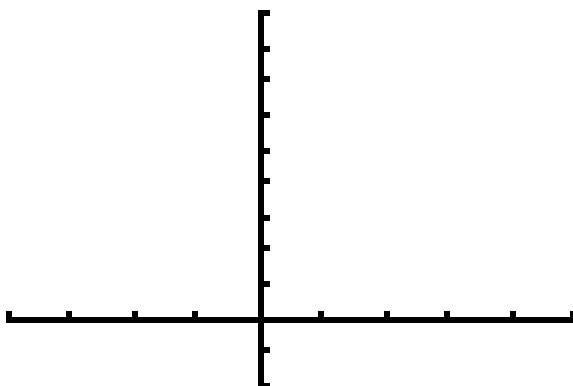
Estimate the area under the graph of f(x) using 4

(hint: n = 4) approximating rectangles and taking the sample points to be:

a) Right endpoints (Draw the appropriate rectangles and find the area) (5 points)



b) Midpoints (Draw the appropriate rectangles and find the area) (5 points)



11) Given  $\int_a^b x dx = 36$  and  $\int_b^a 5 dx = -15$  Find a and b. (10 points)

12) Water flows from the bottom of a storage tank at a rate of  $r(t) = 200 - 4t$   $\frac{\text{liters}}{\text{minute}}$ , Where  $0 \leq t \leq 50$  minutes (10 points)

a) Find the amount of water that flows from the tank initially (at time  $t = 0$ ).

b) Find the amount of water that flows from the tank during the first 25 minutes.

13) Find the area enclosed by the following curves: (10 points)

$$y = 2x + x^2$$

and

$$y = 2x + 9$$

14) Let  $f(x) = \left( \int_{2x}^{10} \sqrt{t} dt \right) + 100$

Find the value of  $f'(10000)$

(5 points)

15) The velocity of a particle moving along a line is  $t^2 - 3t - 4$  meters per second.

Find the velocity of the particle when the acceleration of the particle is zero.

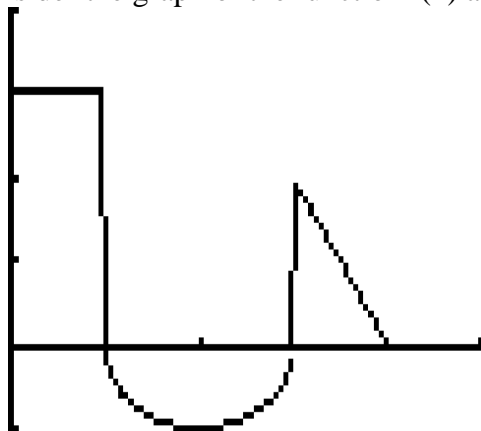
(5 points)

16) Find the value of the integral  $\int_C^D \frac{3x^2 - 5}{x} dx$

(5 points)

(Assume  $C > 0$  and  $D > 0$ , and leave your answer in terms of  $C$  and  $D$ )

17) Consider the graph of the function  $f(x)$  and



Using geometry compute the following:

(5 Points)

a)  $\int_{1/2}^3 f(x) dx$

b)  $\int_0^2 f(x) dx$

c)  $\int_1^4 f(x) dx$

d)  $\int_4^0 f(x) dx$

e)  $\int_4^4 f(x) dx$

**Bonus Question:**

18) Let  $f(x) = \frac{1}{2} \int_{2^x}^{5^x} \frac{u+2}{u-1} du$

Find the value of  $f'(0)$

(5 points)

**Bonus Question:**

19) A closed box with square base is to be built to house an ant colony. The bottom and top of the box will be made of material costing \$1 per square foot, and all four sides are to be constructed of glass costing \$5 per square foot. What are the dimensions of the box of greatest volume that can be constructed for \$65?

(Round your answers to two decimal places)

(5 points)