Professor Katiraie   Calculus I   Fall 2006   Test IV (Chapters 3, and 4)

Name:____________________(1 Point) Total Possible Points = 140 (Plus 10 pts Extra Credits ☺)

Show All Your Work,   **No Procedure = No Points**

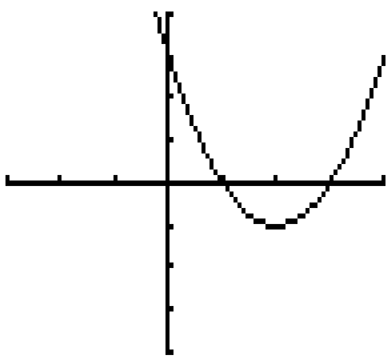
(7 points) **1)** Find the derivative of the function $y = (2x)^{\cos x}$

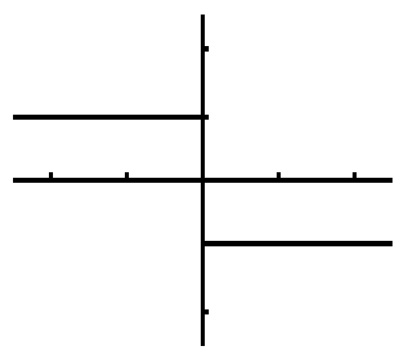
Compute $y^{/}$ in terms of x (Hint: Use Natural Logarithms)

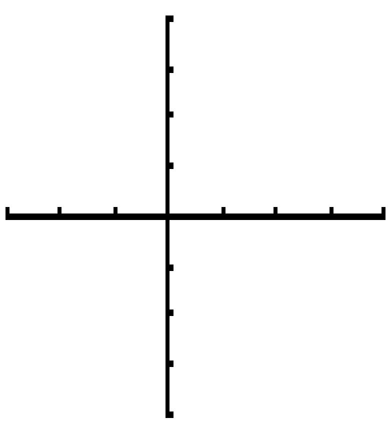
(6 Points)

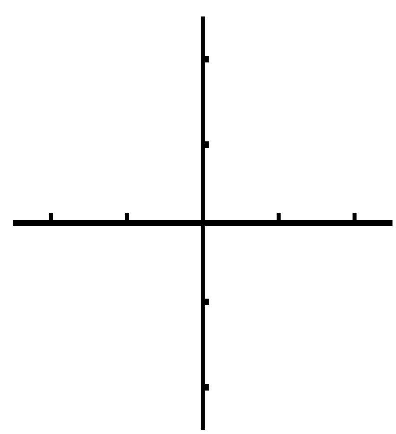
2) Given the graphs of $y = f^{/}(x)$,

Sketch the graphs of $y = f(x)$









3) Given the following ellipse $2x^2 + y^2 = 1$.

a) At what point(s) is the slope of the tangent line equal to 1? **(5 Points)**

b) At what point(s) is the slope of the tangent line equal to 0? **(5 Points)**

4) Show that the following curves are orthogonal (i.e Perpendicular)

$$2x^2 + y^2 = 3$$
$$x = y^2$$

(9 Points)

5) Find the **equation of the tangent line** to the parametric curve
$x = t^2 + 3, \quad y = 2t^3 - t$ at the point corresponding to $t = 2$. **(8 Points)**

For Problems 6 - 8: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.

6) At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 5:00 PM.? (10 points)

7) A man starts walking north at 4 ft/s from a point P. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft due east of P. At what rate are the people moving apart 15 minutes after the woman starts walking? (10 points)

8) A container in the shape of an inverted right circular cone has a radius of 7.00 inches at the top and a height of 9.00 inches. At the instant when the water in the container is 5.00 inches deep, the surface level is falling at the rate of - 0.700 in./sec. Find the rate at which water is being drained? (10 points)

For problems 9 - 11: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.

9) If 1200 sq. cm of material is available to make a box with a square base and an open top, find the largest possible volume of the box. (10 points)

10) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point (1,0) (10 points)

11) A rectangular field is to be enclosed on four sides with a fence. Fencing costs $8 per foot for two opposite sites, and $7 per foot for the other two sides. Find the dimensions of the field of area 730 sq. ft. that would be the cheapest to enclose. (10 points)

12) Analytically find the exact value of all critical numbers of the following functions. (In other words, find the x-coordinates of the critical points.) (10 points)

a) $y = x^{\frac{4}{5}}(x-4)^2$

b) $y = x^{\frac{2}{3}}(x^2-4)$

13) Given that the function $f(x) = x^3 + ax^2 + bx + c$ has critical numbers at $x = -3$, and $x = 2$, find a and b. (10 points)

14) Find the extreme values of the function $f(x) = \dfrac{4x}{x^2+1}$ and where they occur

(Please Justify Your Answer Using Calculus Methods Discussed in Class) (10 points)

15) A company has cost function $C(x) = 84 + 1.26x - 0.01x^2 + 0.00007x^3$ and demand function $p(x) = 3.5 - 0.01x$, where x is the number of staplers and $p(x)$ is in dollars.

a) How many units should the company make to maximize its profit? (5 points)

b) How much is the maximum profit? (5 points)

16a) Given $f'(x)=\sqrt{x}\bullet(6+5x)$ and f(1) = 10; Find $f(x)$ (5 points)

16b) Given $f''(x)=x^{-2}$, $x>0$, $f(1)=0$, $f(2)=0$; Find $f(x)$ (5 points)

16) A pumpkin pie is thrown upward with a speed of 48 ft/sec from the edge of a cliff 432 feet above the ground.
(Assume gravity of earth is -32)

a) Find the pie's height above the ground t seconds later. (4 points)

b) When does the pie reach its maximum height? (3 points)

c) When does the pie hit the ground? (3 points)

Extra Credit Problems

17) Use Newton's Method to find all roots of the equation $\sin x = x^2 - 3x + 1$ correct to six decimal places (5 points)

18) The angle of elevation of the Sun is decreasing at a rate of 0.25 rad/hour. How fast is the shadow cast by a 400–foot–tall building increasing when the angle of elevation of the Sun is $\frac{\pi}{6}$ (5 points)