Professor Katiraie Calculus I Summer 2006 Test I (chapters 1 and 2)

Name:____________________________________ Total Possible Points = 150
(Plus 14 pts Extra Credit ☺)

1) Given $f(x) = \dfrac{2}{x-1}$ find $\dfrac{f(x+h) - f(x)}{h}$ (7 Points)

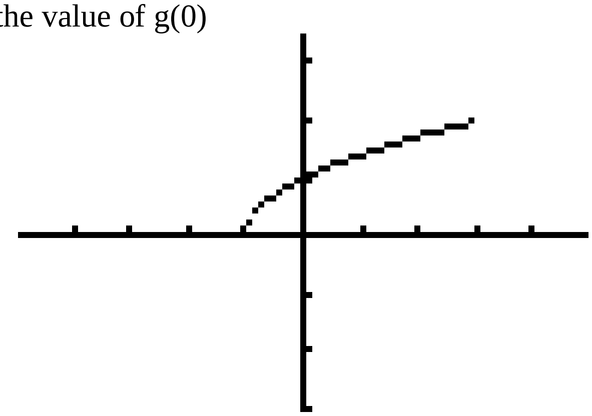
2) Find the Domain and Range of the following functions: (8 Points)

a) $f(x) = \sqrt{(4 - x^2)}$

b) $g(x) = \ln(\ln(x-6))$

3) The graph of g is given. (10 Points)

a) State the value of g(0)



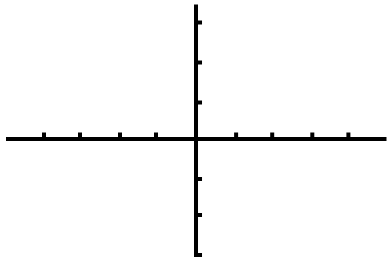
b) Why is g one-to-one?

c) Estimate the value of $g^{-1}(2)$?

d) Estimate the domain of $g^{-1}(x)$

e) Sketch the graph of $g^{-1}(x)$

4a) Sketch the graph of the following function: $f(x)=\begin{cases}2-2x & x<0\\ e^{x}+1 & x\geq 0\end{cases}$ (5 Points)



4b) Discuss **(with reasons)** where the function f(x) is **discontinuous** and why. (5 Points)

5) Determine (**algebraically**) whether f is even, odd, or neither even nor odd (10 Points)

a) $f(x)=3x^{5}-4x^{3}+3$

b) $f(x)=e^{-x^{2}}$

c) $f(x)=x+\sin(x)$

d) $f(x)=x^{4}+2x^{2}$

6) Solve the following equations algebraically. (10 points)
(Must Show All the Appropriate Steps)

a) $\log x+\log(x+15)=2$ □□

b) $\ln\left(3+x\right)-\ln\left(x-4\right)=\ln\left(2\right)$

7) If $f(x) = 5x + \ln(x+2)$, find $f^{-1}(-5)$ (5 Points)

8a) Sketch the curve represented by the parametric equation $x = \ln(t)$ $y = \sqrt{t}$ $1 \le t \le 5$
And indicate with an arrow the direction in which the curve is traced as t increases.
(10 Points)

8b) Eliminate the parameter to find a Cartesian equation of the curve.

8c) State the domain and range of the above graph.

9) Let f be a one-to-one function whose inverse function is given by the formula: (10 points)

$$f^{-1}(x) = x^5 + 3x^3 + 2x$$

a) Compute the value of y such that $f^{-1}(y) = 6$

b) Compute $f^{-1}(-2)$

c) Compute $f(330)$

d) Compute the value of x such that $f(x) = 1$

10) If an arrow is shot upward on the planet X with a velocity of 60 m/s, its height in meters after t seconds is given by $h(t) = 60t - 2t^2$ (10 Points)

a) Find the average velocity over the given time intervals:

i) [2 , 2.5]

j) [2 , 2.1]

k) [2 , 2.01]

l) [2 , 2.001]

b) Find the instantaneous velocity after two seconds.

11) $$f(x) = \begin{cases} x^3 + 2 & x \le -2 \\ x^2 + x + 1 & -2 < x < 1 \\ x^4 + 3 & x \ge 1 \end{cases}$$ (10 Points)

Find the following limits (give reasons, if the limit does not exist)

$a) \lim_{x \to -2} f(x)$

$b) \lim_{x \to -1} f(x)$

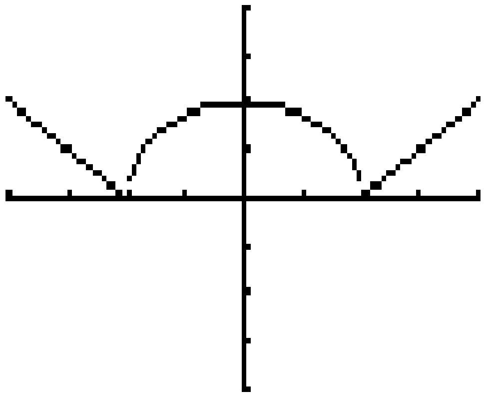
$c) \lim_{x \to 1^+} f(x)$

$d) \lim_{x \to 4} f(x)$

12) Find the equation of the exponential function of the form $y = Ca^x$ that passes through the points (0, 4) and (1, 8). (10 Points)

13) For the function whose graph is shown below, answer the following equations:

(10 Points)



a) At what number "a" does $\lim_{x \to a} f(x)$ **not** exist?

b) At what numbers "a" does $\lim_{x \to a} f(x)$ exists, yet $f(x)$ is **not continuous**?

c) At what numbers "a" $f(x)$ is continuous, but is **not differentiable**?

14) Given $f(x) = \begin{cases} 2x^3 + 16 & x \leq -1 \\ x^2 + bx + c & -1 < x < 1 \\ 3x^4 - 47 & x \geq 1 \end{cases}$ determine the values for b and c so that

$f(x)$ is continuous everywhere. (10 Points)

15) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (8 Points)

$$\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = 3$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to 1^+} f(x) = \infty$$

$$f(0) = -4$$

16) Find the following limits: (12 Points)

a) $\lim_{t\to 13} \dfrac{\sqrt{t+3}-4}{t-13}$

b) $\lim_{x\to -8} \dfrac{\frac{1}{8}+\frac{1}{x}}{8+x}$

c) $\lim_{x\to\infty} (\sqrt{x^2+2x}-x)$

d) $\lim_{x\to\infty} \dfrac{-x-2x^2+6}{3+4x+13x^2}$

(Extra Credit 3 Points)

17) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (2 , 3) and (4 , -5). Find the following:

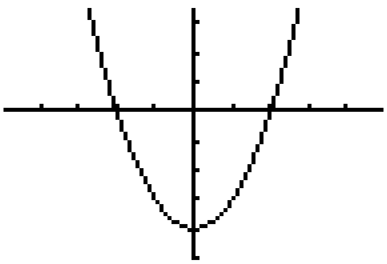
a) find $f'(3)$

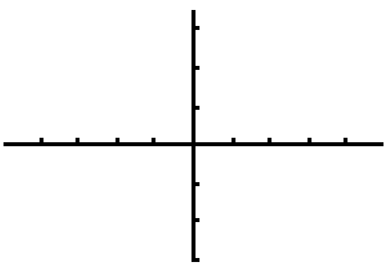
b) find $f(3)$

c) Find an equation of the line tangent to f at $x = 3$

(Extra Credit 3 Points)

18) Given the graph of $y = f'(x)$, sketch the graph of $y = f(x)$





(Extra Credit 3 Points)

19) Find the following limit $\lim_{x \to \infty} \frac{\cos x}{x^4}$

(Hint: Use the Squeeze Theorem)

20) (Extra Credit 5 Points)

Given $f(x) = \sqrt{x-3}$

Find the $f^{/}(x)$ using either of the two definitions discussed in class.