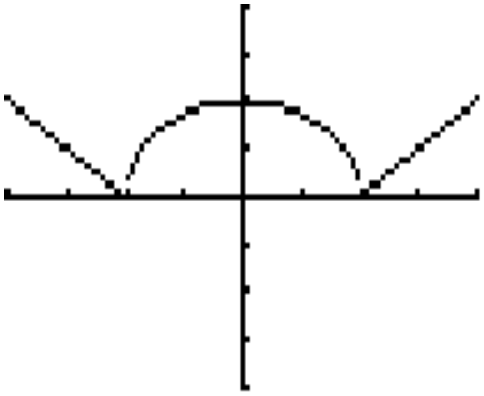
Professor Katiraie Calculus I Fall 2006 Test III (chapters 1, 2, and 3)

Name:_____Total Possible Points = 140 (Plus 10 pts Extra Credits ☺)

Show All Your Work, No Procedure = No Points

(10 Points) 1) Find a formula that describes the following function:



2) Use the intermediate Value Theorem to show that there is a root of the equation $x^3 + 2x^2 - 42 = 0$ on the interval (0,3). **(5 Points)**

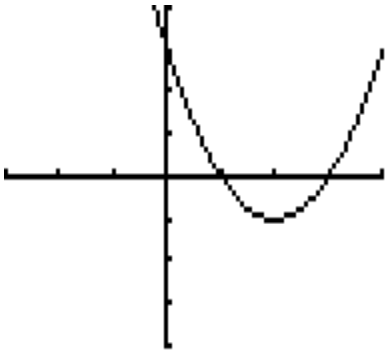
3) Find the following limits **Algebraically** **(5 Points)**

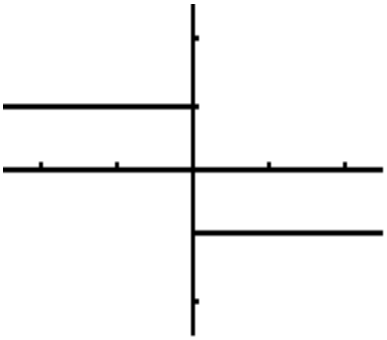
a) $\lim_{x\to\infty} \frac{1-2x^2}{x^2+x}$

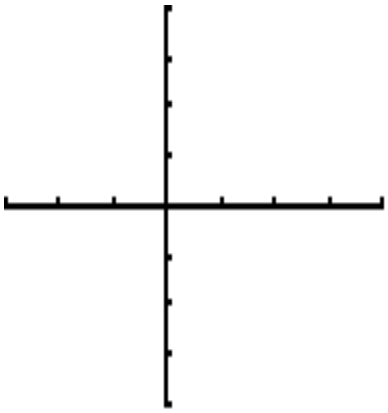
b) $\lim_{x\to\infty} (\sqrt{x^2+2x} - x)$

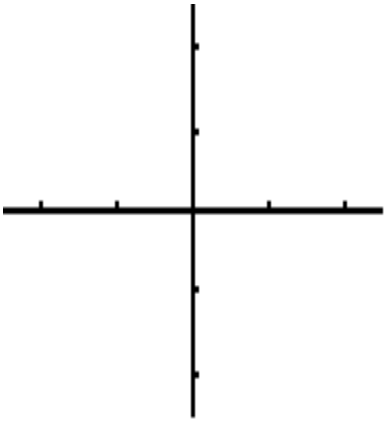
(10 Points)

4) Given the graphs of $y = f'(x)$, sketch the graphs of $y = f\ (x)$

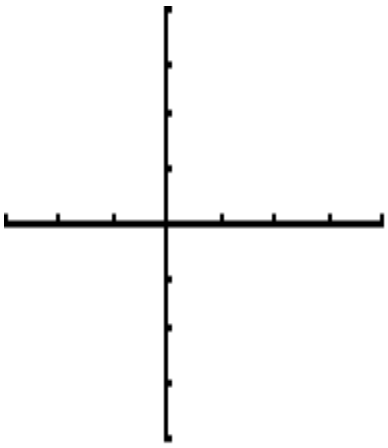








5) Sketch the graph of a function that satisfies all of the following conditions:**(10 points)**



$f'(-1) = f'(1) = 0$

$f'(x) < 0$ if $|x| < 1$

$f'(x) > 0$ if $|x| > 1$

$f(-1) = 4$

$f(1) = 0$

$f''(x) < 0$ if $x < 0$

$f''(x) > 0$ if $x > 0$

6) Find the derivative of the following functions: **(4 Points each)**

(Do Not Simplify)

a) $y = 4\pi^2 + \left(\frac{1}{2}x\right)^5$

b) $y = t^2 - \frac{1}{\sqrt[4]{t^3}}$

c) $y = \sin^7(x^2 + 2x + 1)$

d) $y = \frac{\sec 2\theta}{1 + \tan 2\theta}$

e) $y = \log_5(1 + 2x)$

f) $y = \sin^{-1}(x^2 + 2x + 1)$

g) $y = 17^{\cot \pi\theta}$

8)**(2 Points each)** a) Find the first derivative of $g(x) = x\sin(x)$.

b) Find the second derivative of $g(x) = x\sin(x)$

c) Evaluate the second derivative of g(x) at x = $\frac{\pi}{6}$. (In other words, find $g''\left(\frac{\pi}{6}\right)$

9) Given $h(x) = \sqrt[3]{1+3x}$ (4 points each)

a) Find a linearization of $h(x) = \sqrt[3]{1+3x}$ at $a = 0$

b) Use your answer to estimate $\sqrt[3]{1.03}$

(8 Points) 10) Find the equation of the tangent line to the curve $\sqrt{x} + \sqrt{y} = 9$ at the point (16 , 25).

(7 points) **11)** Find the derivative of the function $y = (x)^{\cos x}$

Compute $y^{/}$ in terms of x (Hint: Use Natural Logarithms)

(8 Points) **12)**

Suppose that $h(x) = f(x)g(x)$, and $F(x) = f(g(x))$, where

$f^{/}(5) = 11$

$w^{//}(5) = 8$

$f(2) = 3$

$g(2) = 5$ a) Find $F^{/}(2)$ b) Find $h^{/}(2)$.

$g^{/}(2) = 4$

$w^{/}(5) = -2$

$f^{/}(2) = -2$

(4 Points) 13) Given the following ellipse $x^2 + 2y^2 = 1.$

a) At what point(s) is the slope of the tangent line equal to 1?

b) At what point(s) is the slope of the tangent line equal to 0?

(4 Points)

14) The mass of part of a wire is $m = x(2+\sqrt{x})$ kilograms, where x is measured in meters from one end of the wire. Find the linear density of the wire when $x = 16$ meters. (Hint: linear density $\rho = \frac{dm}{dx}$). **(8 Points)**

(9 Points) 15) Let $y = e^{\frac{x}{5}}$.

a) Find the differential dy.

b) Evaluate dy if $x = 0$, and $dx = 0.3$

c) Evaluate Δy if $x = 0$, and $dx = 0.3$ (Hint: $\Delta y = f(x+\Delta x) - f(x)$)

(8 Points) 16) Find the **equation of the tangent line** to the parametric curve $x = t^2 + 3, \quad y = 2t^3 - t$ at the point corresponding to $t = 2$.

EXTRA CREDIT PROBLEMS

(5 Points) 16) Find all values of x so that the graph of $g(x) = \sqrt{3}\,x + 2\sin(x)$ will have a horizontal tangent?

(5 Points) **17)** Show that the following curves are orthogonal (i.e Perpendicular)

$2x^2 + y^2 = 3$

$x = y^2$