Professor Katiraie Calculus I Spring 2008 Test III Form A (chapters 1 -- 3)

Name (☺ 1 Point)_____ Total Possible Points = 140
(Plus 10 pts Extra Credit ☺)

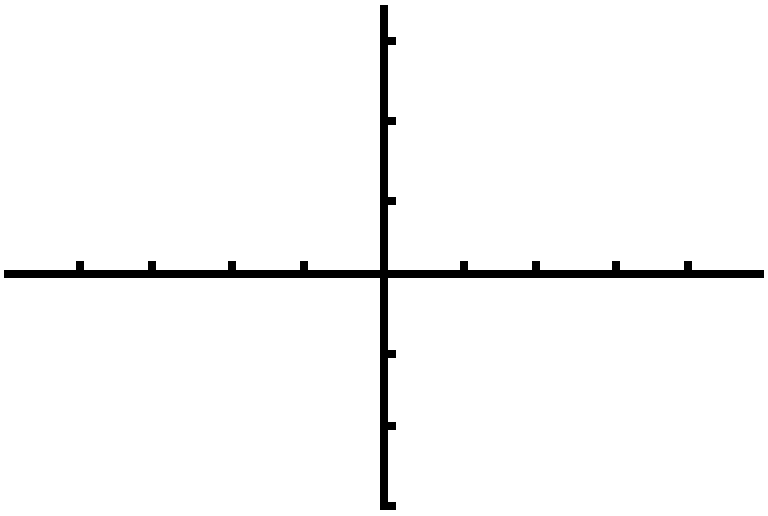
1) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (10 Points)

$$\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = -1$$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to 2^-} f(x) = \infty$$

$$\lim_{x\to -3^-} f(x) = \lim_{x\to 2^+} f(x) = -\infty$$

$$f(0) = 3$$



2) Given $f(x) = \begin{cases} 2x^3 + 7 & x \le -1 \\ x^2 + bx + c & -1 < x < 1 \\ x^4 - 10 & x \ge 1 \end{cases}$ determine the values for b and c so that $f(x)$ is continuous everywhere. (8 Points)

3) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (2 , 5) and (4 , -5). Find the following: (9 Points)

a) Find $f'(3)$

b) Find an equation of the line tangent to f at $x = 3$

c) Find $f(3)$

4) Find the following limits: (12 Points)

a) $\lim_{t \to 3} \frac{\sqrt{t+6}-3}{t-3}$

b) $\lim_{x \to -7} \frac{\frac{7+x}{1}}{\frac{1}{7}+\frac{1}{x}}$

c) $\lim_{t \to 0} \frac{\sqrt{t^2+25}-5}{t^2}$

d) $\lim_{t \to 0} \frac{2}{t^2+t} - \frac{2}{t}$

5) Find the derivative of the following functions: **(4 Points each)**

(Do Not Simplify)

a) $y = \sqrt{x} + \frac{1}{\sqrt[5]{x^7}}$	b) $y = e^{\tan(2\theta)}$
c) $y = \cos^5(4x)$	d) $y = \sin((5x)^3)$
e) $y = \sqrt{x + \sqrt{x}}$	f) $y = \left(\frac{3x-7}{x^2-1}\right)^7$
g) $y = 10^{\sec \pi\theta}$	h) $y = \sin(\cot\sqrt{1+x^2})$
i) $y = \sin^{-1}(x^2 + 2x + 1)$	j) $y = \ln(x^2 + 2x + 1)$

6) Find the equation of the tangent line to the curve $\sqrt{x}+\sqrt{y}=7$, at the point (9,16). (10 Points)

7) Find the derivative of the function $y^x=(x)^{\cos x}$ (10 points)

Compute y' in terms of x, and y. **(Hint: Use Natural Logarithms)**

8) Suppose that $h(x)=g(x)f(x)$, and $F(x)=g(f(x))$, where $f(5)=4,\ g(5)=2,\ g'(5)=-1,\ f'(5)=-2,\ g'(4)=-5$

a) Find $h'(5)$ (5 Points)

b) Find $F'(5)$. (5 Points)

9) Find all values of x so that the graph of $f(x) = x - 2\sin x$ will have a horizontal tangent?

(5 Points)

10) Find the equation of the tangent line to the curve $y = x\cos x$, at the point $(\pi, -\pi)$. (5 Points)

11) A particle moves on a vertical line so that its coordinate at time t is

$$s(t) = t^3 - 12t^2 + 3 \quad t \geq 0$$

where S(t) is measured in meters and t is measured in seconds. (10 Points)

a) When is the particle moving upward?

b) Find the distance that the particle travels in the time interval $7 \leq t \leq 9$ seconds.

c) When is the particle speeding up?

12) Consider the circle $x^4 + y^4 = 1$. (5 Points)

At what point(s) is the slope of the tangent line equal to 1?

13) Let $y = e^{\frac{x}{5}}$.

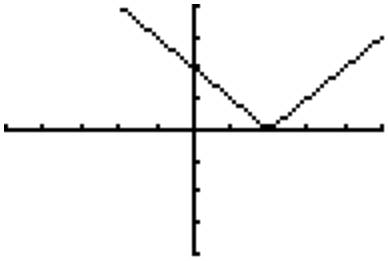
a) Find the differential dy. (3 Points)

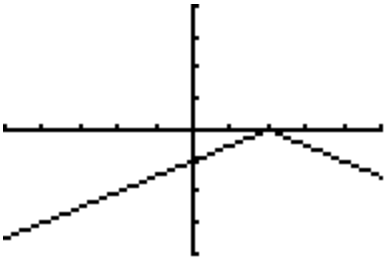
b) Evaluate dy if $x = 0$, and $dx = 0.3$ (2 Points)

Extra Credits

14) Given that $v(x) = \frac{f(x)}{g(x)}$, and $w(x) = f(x)g(x)$ (5 Points)

And graphs of f(x) and g(x)





Find the following:

$v'(0)$	$w'(1)$

15a) Find the linearization of $f(x) = \sqrt[3]{1+x}$ at a = 0. (3 points)

15b) Use the above to give an approximate value for $\sqrt[3]{0.95}$. (2 Points)