Calculus I   Test III   Professor: Fred Katiraie   Spring 2006

Name:_____ $\text{Score} = \dfrac{}{140}$

Please Show Your Work for Full Credit, Include Units Whenever Possible.
Justify all your answers

1)   Given $f(x) = \begin{cases} x\ \sin\left(\dfrac{1}{x}\right) & \text{if}\quad x \neq 0 \\ 0 & \text{if}\quad x = 0 \end{cases}$

a)   Is $f(x)$ continuous at $x = 0$?   (must use definition of continuity)

(5 points)

b)   Is $f(x)$ differentiable at $x = 0$?   (Justify your answer)   (5 points)

2) Sketch the graph of a function that satisfies all of the following conditions: (10 points)

$f'(-1) = f'(1) = 0$

$f'(x) < 0$ if $|x| < 1$

$f'(x) > 0$ if $|x| > 1$

$f(-1) = 4$

$f(1) = 0$

$f''(x) < 0$ if $x < 0$

$f''(x) > 0$ if $x > 0$

3) Each figure below shows the graphs of a function, its first derivative, and its second derivative. Identify which is which. (10 points)

4) Each figure below shows the graphs of a function, its first derivative, and its second derivative. Identify which is which. (10 points)

5) Find the derivative of the following functions: **(4 Points each)**

(Do Not Simplify)

a) $y = \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^5}}$

b) $y = e^{\sec 2\theta}$

c) $y = \sin^5(3x^2 + 5x + 1)$

d) $y = \csc((7x)^3)$

e) $y = \sqrt{5x + \sqrt{x + 5}}$

f) $y = \left(\dfrac{x - 7}{x^2 + 1}\right)^9$

g) $y = 17^{\cot \pi \theta}$

h) $y = \sin^{-1}(x^2 + 2x + 1)$

6) Given $f(x) = e^{x^2 - 3x + 2}$

a) Find the first derivative of $f(x) = e^{x^2 - 3x + 2}$.

b) Find the second derivative of $f(x) = e^{x^2 - 3x + 2}$

c) Evaluate the second derivative of f (x) at x = 2. (In other words, find $f''(2)$.)

7) Given $h(x) = \sqrt{1 - x}$

a) Find a linearization of $h(x) = \sqrt{1 - x}$ at $a = 0$

b) Use your answer to estimate $\sqrt{0.99}$

(8 Points) 8) Find the equation of the tangent line to the curve $\dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{y}} = \dfrac{7}{12}$

at the point (9,16).

(7 points) 9) Let $y = f(x)$ be implicitly defined as $\qquad x^{\sin y} = y^{\cos x}$

Compute y' in terms of $x,$ and y. (Hint: Use Natural Logarithms)

(6 Points) 10) Suppose that $h(x) = \dfrac{g(x)}{w(x)}$, and $F(x) = g(g(x))$, where

$g''(2) = -1$

$w''(5) = 8$

$w(5) = 4$

$g(5) = 2 \qquad$ a) Find $h'(5)$ $\hspace{5cm}$ b) Find $F'(5)$.

$g'(5) = -1$

$w'(5) = -2$

$g'(2) = -5$

(6 Points) 11) Consider the circle $x^4 + y^4 = 1$.

a) At what point(s) is the slope of the tangent line equal to 1?

b) At what point(s) is the slope of the tangent line equal to 0?

Show All Your Work,   **No Procedure = No Points**

(6 Points)
12) The position of a particle is given by the equation
$S = f(t) = t^3 - 6t^2 + 9t \qquad t \geq 0$,

      where "t" is measured in seconds and "S" is in meters.

a) When is the particle at rest?

b) When is the particle moving forward?

(6 Points)
13) Based on the above $S = f(t) = t^3 - 6t^2 + 9t \qquad t \geq 0$
where "t" is measured in seconds and "S" is in meters.

a) Find the total distance traveled by the particle during the first **five** seconds?   **(2 points)**

b) When is the particle speeding up?   **(2 points)**

c) When is the particle slowing down?   **(2 points)**

(7 Points) 14) Find all values of x so that the graph of $f(x) = \sqrt{3}\, x - 2\cos x$
will have a horizontal tangent?

(7 Points) 15) Find the slope of the tangent line to the curve $x^3 + y^3 = 6xy$

at the point $(3, 3)$.

EXTRA CREDIT PROBLEMS

(Extra Credit 6 Points)   **16)** Given the curve $y = e^x$

a) Find an equation of the tangent to the curve $y = e^x$ that is parallel to the line
$$x - y - 1 = 0$$

b) Find an equation of the tangent to the curve $y = e^x$ that passes through the origin.

(Extra Credit 2 points)   **17)**   Consider the curve given by $x = t^2 + 3, \quad y = 2t^3 - t$.

Find $\dfrac{dy}{dx}$ at the point corresponding to $t = 2$.

(Extra Credit 5 points) 18) Show that the following curves are orthogonal (i.e Perpendicular)

$$3x^2 + 2x - 3y^2 = 1$$
$$6xy + 2y = 0$$