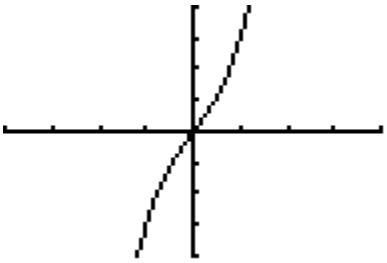
Professor Katiraie Calculus I Fall 2006 Test II (chapters 1 and 2)

Name:_____ Total Possible Points = 140
(Plus 10 pts Extra Credit)

(10 Points)



1) The graph of g is given.

a) State the value of g(1)

b) Why is g one-to-one?

c) Estimate the value of $g^{-1}(2)$?

d) Estimate the domain of $g^{-1}(x)$

e) Sketch the graph of $g^{-1}(x)$

(10 Points) 2)A small-appliance manufacturer finds that it costs $9000 to produce 1000 toaster ovens a week and $12000 to produce 1500 toaster ovens a week.

a) Express the cost as a function of the number of the toaster ovens produced, assuming that it is linear.

b) What is the slope of the graph and what does it represent?

c) What is the y-intercept of the graph and what does it represent?

(10 Points) 3) Let f be a one-to-one function whose inverse function is given by the formula:

$$f^{-1}(x) = x^5 + 2x^3 + 3x + 1$$

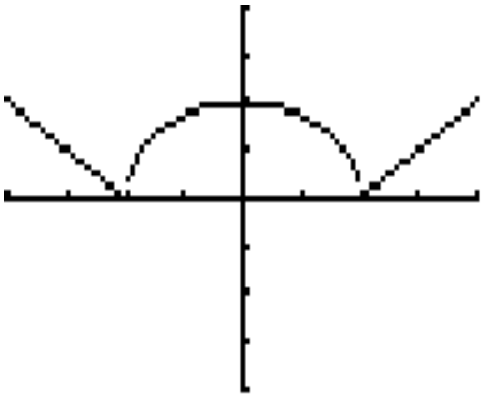
a) Compute $f^{-1}(-1)$

b) Compute $f(1)$

c) Compute the value of x such that $f(x) = 1$

d) Compute the value of y such that $f^{-1}(y) = 1$

(10 Points) 4) Find a formula that describes the following function:



5) If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after t seconds is given by $h(t) = 58t - 0.83t^2$ (10 Points)

a) Find the average velocity over the given time intervals:

i) [1,2]

j) [1,1.5]

k) [1,1.1]

l) [1,1.01]

b) Find the instantaneous velocity after one second.

6) $f(x)=\begin{cases} x^3+2 & \text{if } x\le -1 \\ x^2+x & \text{if } -1<x<1 \\ x^4+2 & \text{if } x\ge 1 \end{cases}$ Find the following limits (if they exit)

(10 Points)

a) $\lim_{x\to -1^-} f(x)$

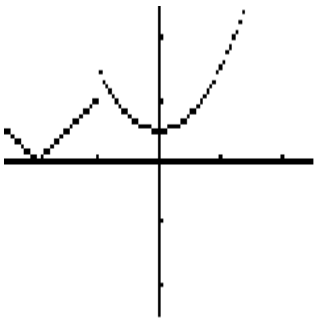
b) $\lim_{x\to -1} f(x)$

c) $\lim_{x\to 1^+} f(x)$

d) $\lim_{x\to 1} f(x)$

7) For the function whose graph is shown below, answer the following equations:

(10 Points)



a) At what number "a" $\lim_{x\to a} f(x)$ does **not** exist?

b) At what numbers "a" $\lim_{x\to a} f(x)$ **exists**, yet $f(x)$ is **not continuous**?

c) At what numbers "a" $f(x)$ is **not differentiable**?

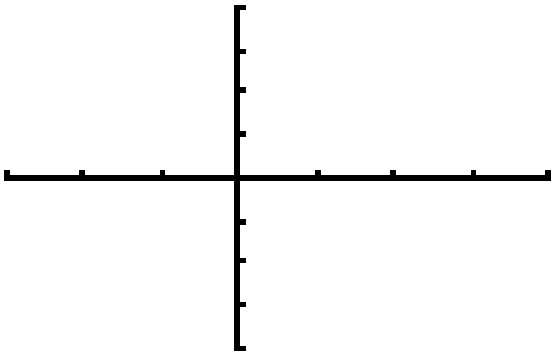
8) Use the intermediate Value Theorem to show that there is a root of the equation $x^3+2x^2-42=0$ on the interval (0,3). (5 Points)

9) Given $f(x)=\begin{cases}2x^3+16; x\le -2\\ x^2+bx+c; -2<x<2\\ 3x^4-48; x\ge 2\end{cases}$ determine the values for b and c

So that $f(x)$ is continuous everywhere. (10 Points)

10) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. Label all horizontal and vertical asymptotes. (10 Points)

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$$



$$\lim_{x\to -1^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$$
$$\lim_{x\to -1^-} f(x) = \lim_{x\to 1^+} f(x) = \infty$$
$$f(0) = -1$$

11) If $f(x)=\sqrt{x-3}$ (10 Points)

Find the $f'(x)$ using either of the two definitions discussed in class.

Find the equation of the tangent line to the curve $f(x)=\sqrt{x-3}$ at the point (4, 1)

12) Find the following limits (15 Points)

a) $\lim_{x\to 2^+} \frac{1}{x-2}$

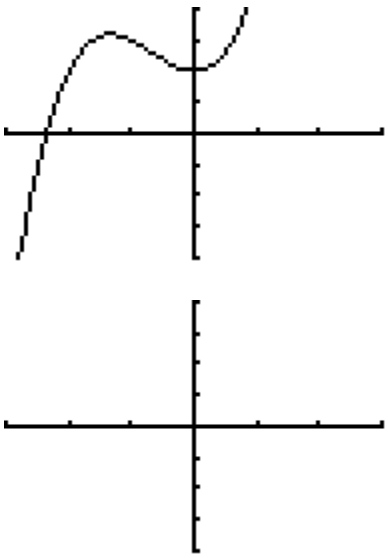
b) $\lim_{x\to\infty} \frac{1-2x^2}{x^2+x}$

c) $\lim_{x\to\infty} (\sqrt{x^2+2x}-x)$

d) $\lim_{x\to\infty} \sin x$

e) $f(x) = \lim_{x\to\infty} e^{-x}$

13) Given the graph of $y = f(x)$, sketch the graph of $y = f'(x)$ (5 Points)



(10 Points)

14) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (-2,3) and (4,-1). Find the following:

a) find $f'(3)$

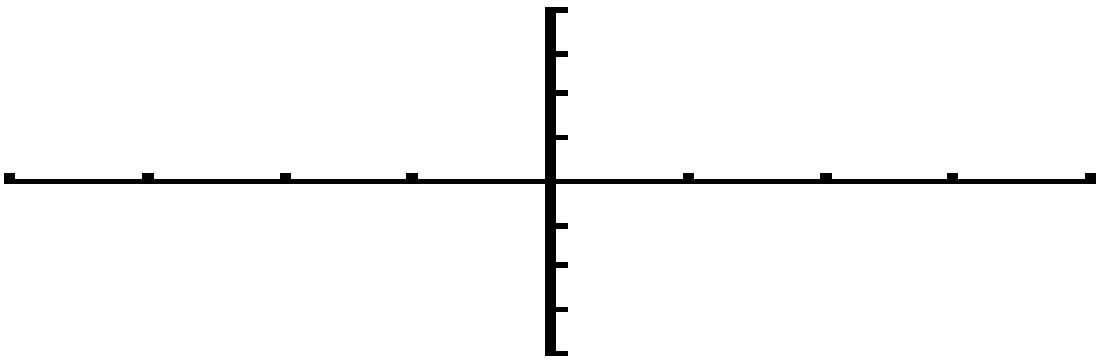
b) Find an equation of the line tangent to f at $x = 3$

c) Find $f(3)$

(5 Points)

15) Find the vertical and horizontal asymptote(s) of the curve $y = \dfrac{x^2 - 4}{9 - x^2}$

(Use Limits to Justify Your Results)



(Extra Credits ☺ 5 Points)

16) If $3x \le f(x) \le x^3 + 2$ for $0 \le x \le 2$

Evaluate $\lim_{x \to 1} f(x)$

(Extra Credits ☺ 5 Points)

17) Given the graphs of $y = f'(x)$, sketch the graphs of $y = f(x)$