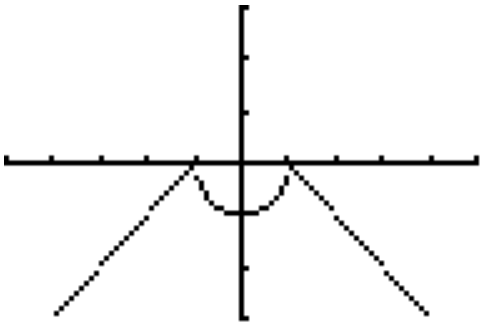
Professor Katiraie Calculus I Spring 2008 Test II Form A (chapters 1 and 2)

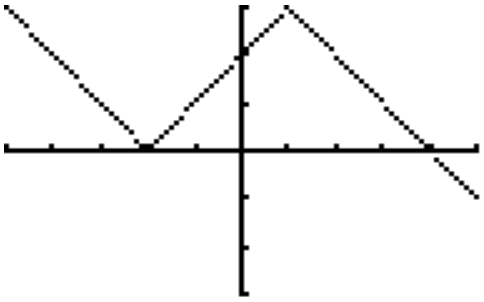
Name:_____ Total Possible Points = 140
(Plus 10 pts Extra Credit ☺)

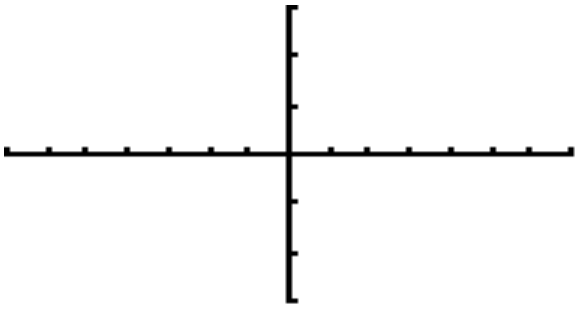
1) Find a formula that describes the following function: (8 Points)



2) The graph of $y = f(x)$ is given below; Sketch a graph of $y = -\frac{1}{2} f(x-2)$

(8 points)





3) Given the function: $f(x) = \frac{2}{x-1}$ (8 pts)

Find the following $\frac{f(x+h) - f(x)}{h}$

(Χλεαρλψ στατε εαχη στεπ οφ τηε προχεσσ).

4) Find the inverse of the following functions. (10 points)
(Must Show All the Appropriate Steps)

a) $y=\sqrt[3]{x-2}-5$	b) $f(x)=\frac{1}{4}\log(7x)$

5) Solve the following algebraically: (6 points)

a) $\left(\frac{1}{5}\right)^{2-x}=25$

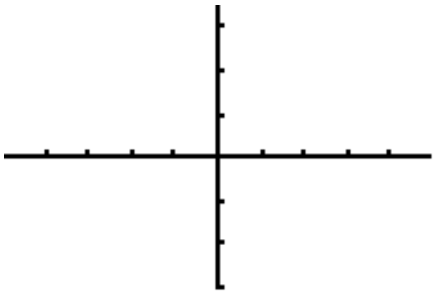
b) $e^{x^2}\bullet\frac{1}{e^6}=\left(e^{5x}\right)$

c) If $3^x=\frac{1}{49}$, what does 3^{-2x} equal?

6a) Sketch the graph of the following function:

$$f(x)=\begin{cases}2x & x\le -2\\ 2-2x & -2 < x < 0\\ e^{x}+2 & x\ge 0\end{cases}$$

(5 Points)



6b) Discuss **(with reasons)** where the function f(x) is **discontinuous** and why. (5 Points)

7) If an arrow is shot upward on the planet X with a velocity of 70 m/s, its height in meters after t seconds is given by $h(t)=70t-2t^{2}$ (10 Points)

a) Find the average velocity over the given time intervals:

i) [2 , 2.5]

j) [2 , 2.1]

k) [2 , 2.01]

l) [2 , 2.001]

b) Find the instantaneous velocity after two seconds.

8) $f(x)=\begin{cases} x^3+2 & x\le -2 \\ x^2+x+1 & -2<x<1 \\ x^4+3 & x\ge 1 \end{cases}$ (10 Points)

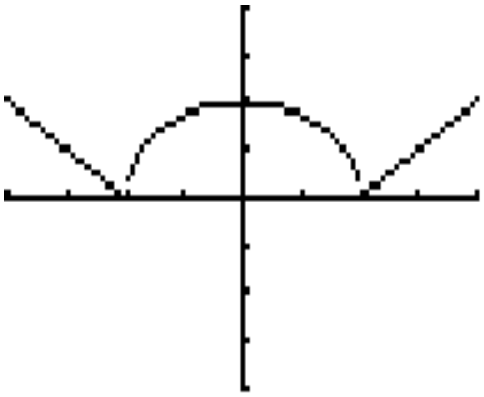
Find the following limits (give reasons, if the limit does not exist)

a) $\lim_{x\to -2} f(x)$ *b*) $\lim_{x\to -1} f(x)$

c) $\lim_{x\to 1^+} f(x)$ *d*) $\lim_{x\to 4} f(x)$

9) For the function whose graph is shown below, answer the following equations:

(9 Points)



a) At what number "a" does $\lim_{x\to a} f(x)$ **not** exist?

b) At what numbers "a" does $\lim_{x\to a} f(x)$ exists, yet $f(x)$ is **not continuous**?

c) At what numbers "a" $f(x)$ is continuous, but is **not differentiable**?

10) Given $f(x)=\begin{cases} 2x^3+16 & x\le -1 \\ x^2+bx+c & -1<x<1 \\ 3x^4-47 & x\ge 1 \end{cases}$ determine the values for b and c so that $f(x)$ is continuous everywhere. (10 Points)

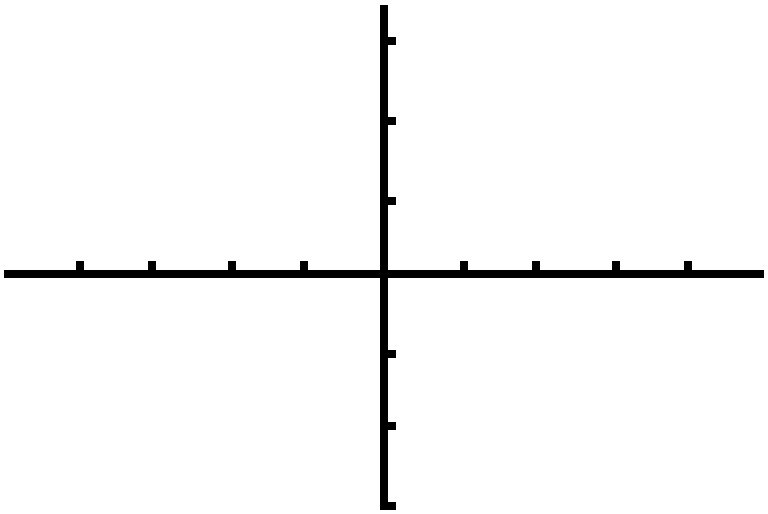
11) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (9 Points)

$\lim_{x\to\infty} f(x)=\lim_{x\to -\infty} f(x)=3$

$\lim_{x\to -2^+} f(x)=\lim_{x\to 1^-} f(x)=-\infty$

$\lim_{x\to -2^-} f(x)=\lim_{x\to 1^+} f(x)=\infty$

$f(0)=-2$



12) Find the following limits: (12 Points)

a) $\lim_{t \to 13} \frac{\sqrt{t+3}-4}{t-13}$

b) $\lim_{x \to -8} \frac{8+x}{\frac{1}{8}+\frac{1}{x}}$

c) $\lim_{x \to \infty} (\sqrt{x^2+2x}-x)$

d) $\lim_{t \to 0} \frac{1}{t} - \frac{1}{t^2+t}$

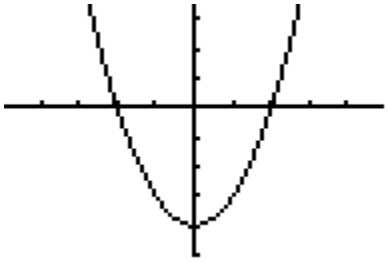
13) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (2 , 3) and (4 , -5). Find the following: (12 Points)

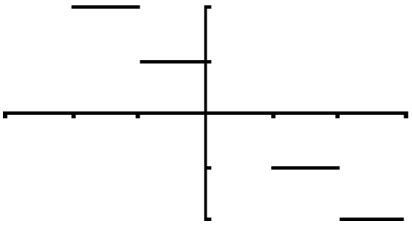
a) Find $f'(3)$

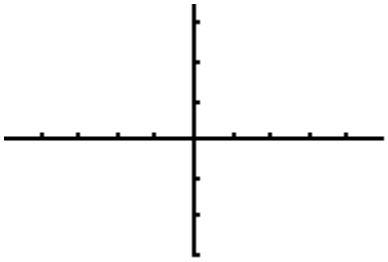
b) Find $f(3)$

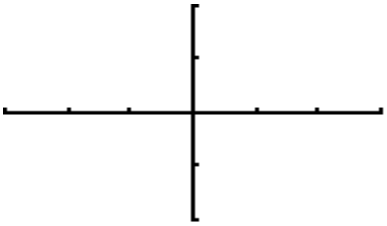
c) Find an equation of the line tangent to f at $x = 3$

14) Given the graph of $y = f'(x)$, sketch the graph of $y = f(x)$ (12 Points)









15) Find the following limit $\lim_{x \to \infty} \frac{\cos x}{x^4}$ (6 Points)

(Hint: Use the Squeeze Theorem)

16) Given $f(x)=\frac{1}{\sqrt{x-3}}$ (Extra Credits 5 Points)

Find the $f'(x)$ using either of the two definitions discussed in class.

17) Given $f(x)=\sqrt{1+2x}$ (Extra Credits 5 Points)

Find the $f'(x)$ using either of the two definitions discussed in class