Professor Katiraie Calculus I Spring 2008 Test II Form C (chapters 1 and 2)

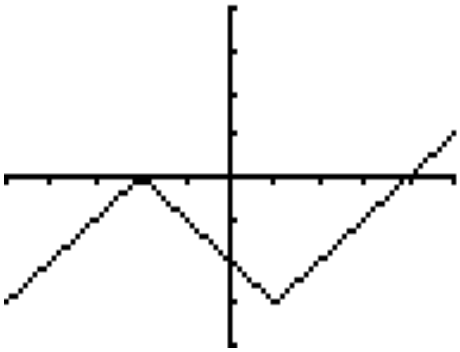
Name:_____ Total Possible Points = 140
(Plus 10 pts Extra Credit ☺)

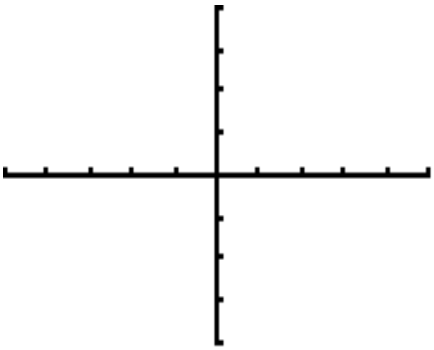
1) (8 pts) Given the function: $f(x)=\frac{1}{x-2}$ Find the following $\frac{f(x+h)-f(x)}{h}$

(Χλεαρλψ στατε εαχη στεπ οφ τηε προχεσσ).

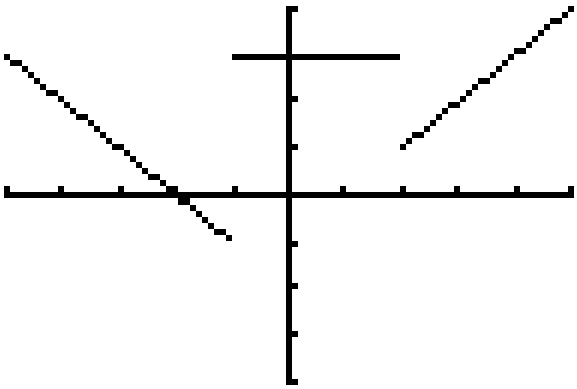
2) The graph of $y=f(x)$ is given below; Sketch a graph of $y=-\frac{1}{3}f(x-2)$

(8 points)





3) Find a formula that describes the following function:
(8 Points)



4) Solve the following algebraically: (6 points)

a) $\left(\frac{1}{3}\right)^{2-x} = 27$

b) $e^{x^2} \bullet \frac{1}{e^{-6}} = \left(e^{-5x}\right)$

c) If $3^x = \frac{1}{10}$, what does 3^{-4x} equal?

5. Find the inverse of the following functions. (10 points)
(Must Show All the Appropriate Steps)

a) $y = \sqrt[4]{x+7} - 6$	b) $f(x) = -\frac{1}{6}\ln(7x)$

6) If an arrow is shot upward on the planet X with a velocity of 60 m/s, its height in meters after t seconds is given by $h(t) = 60t - 2t^2$ (10 Points)

a) Find the average velocity over the given time intervals:

i) [2 , 2.5]

j) [2 , 2.1]

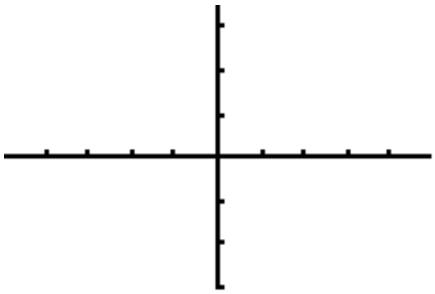
k) [2 , 2.01]

l) [2 , 2.001]

b) Find the instantaneous velocity after two seconds.

7a) Sketch the graph of the following function:

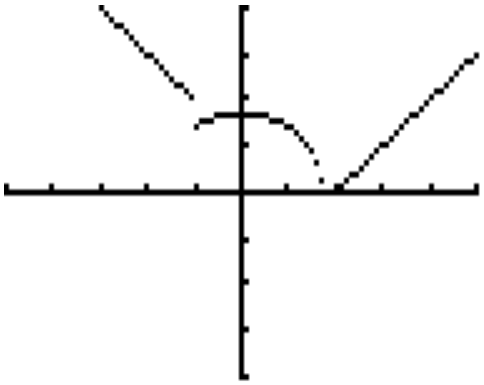
$$f(x) = \begin{cases} x & x \le -1 \\ -x - 3 & -1 < x < 0 \\ e^x + 2 & x \ge 0 \end{cases}$$ (5 Points)



7b) Discuss **(with reasons)** where the function f(x) is **discontinuous** and why. (5 Points)

8) For the function whose graph is shown below, answer the following equations:

(9 Points)



a) At what number "a" $\lim_{x \to a} f(x)$ does **not** exist?

b) At what numbers "a" $\lim_{x \to a} f(x)$ exists, yet $f(x)$ is **not continuous**?

c) At what numbers "a" $f(x)$ is continuous, but is **not differentiable**?

9) $$f(x) = \begin{cases} x^3 + 2 & x \le -2 \\ x^2 + x + 1 & -2 < x < 1 \\ x^4 + 3 & x \ge 1 \end{cases}$$ (10 Points)

Find the following limits (give reasons, if the limit does not exist)

$a) \lim_{x \to 1} f(x)$ $b) \lim_{x \to -1} f(x)$

$c) \lim_{x \to -2^+} f(x)$ $d) \lim_{x \to -3} f(x)$

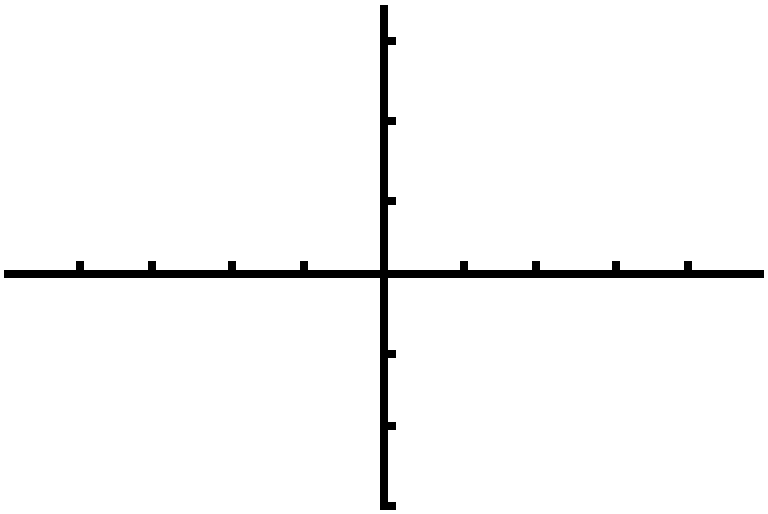
10) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (9 Points)

$$\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = -1$$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to 2^-} f(x) = \infty$$

$$\lim_{x\to -3^-} f(x) = \lim_{x\to 2^+} f(x) = -\infty$$

$$f(0) = 3$$



11) Given $f(x) = \begin{cases} 2x^3 + 7 & x \le -1 \\ x^2 + bx + c & -1 < x < 1 \\ x^4 - 10 & x \ge 1 \end{cases}$ determine the values for b and c so that $f(x)$ is continuous everywhere. (10 Points)

12) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (2 , 5) and (4 , -5). Find the following: (12 Points)

a) Find $f'(3)$

b) Find $f(3)$

c) Find an equation of the line tangent to f at $x = 3$

13) Find the following limits: (12 Points)

a) $\lim_{t \to 3} \frac{\sqrt{t+6} - 3}{t-3}$

b) $\lim_{x \to -7} \frac{\frac{7+x}{7}}{\frac{1}{7} + \frac{1}{x}}$

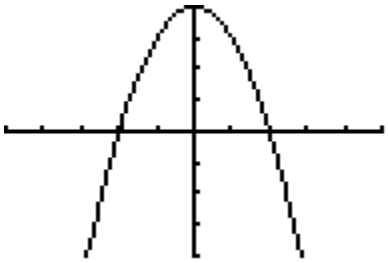
c) $\lim_{t \to 0} \frac{\sqrt{t^2+25} - 5}{t^2}$

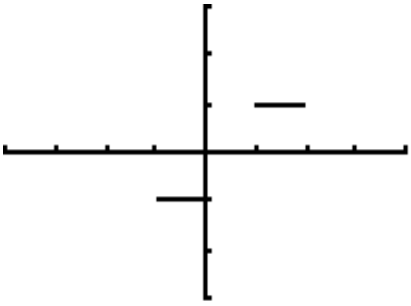
d) $\lim_{t \to 0} \frac{2}{t^2+t} - \frac{2}{t}$

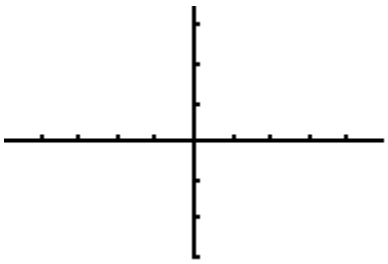
14) Find the following limit $\lim_{t\to 0} x^4 cos\left(\frac{1}{x^2}\right)$ (6 Points)

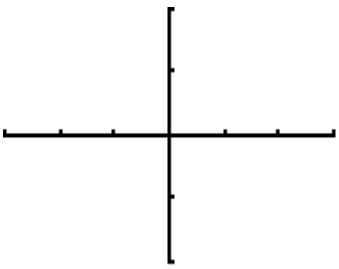
(Hint: Use the Squeeze Theorem)

15) Given the graph of $y = f'(x)$, sketch the graph of $y = f(x)$ (12 Points)









16) Given $f(x)=\sqrt{1+3x}$ (Extra Credits 5 Points)

Find the $f'(x)$ using either of the two definitions discussed in class

17) Given $f(x)=\frac{1}{\sqrt{x+1}}$ (Extra Credits 5 Points)

Find the $f'(x)$ using either of the two definitions discussed in class.