

**Calculus I    Test II    Professor: Fred Katiraie    Spring 2006**

Name: \_\_\_\_\_

Total Possible Points =140

**Please Show Your Work for Full Credit, Include Units Whenever Possible.**

**Justify all your answers**

1) (20 Points) Let  $f$  be a one-to-one function whose inverse function is given by the formula:  $f^{-1}(x) = x^5 - 3x^3 + 5x + 2$

a) Compute  $f^{-1}(-1)$

b) Compute  $f(1)$

c) Compute the value of  $x$  such that  $f(x) = 1$

d) Compute the value of  $y$  such that  $f^{-1}(y)=1$

2) Find an expression for the function whose graph consists of the line segment from the point  $(-2,2)$  to the point  $(-1,0)$  together with the top half of the circle with the center at the origin and radius 1. (10 points)

(10 Points) 3) A small-appliance manufacturer finds that it costs \$9000 to produce 1000 toaster ovens a week and \$12000 to produce 1500 toaster ovens a week.

a) Express the cost as a function of the number of the toaster ovens produced, assuming that it is linear.

b) What is the slope of the graph and what does it represent?

c) What is the y-intercept of the graph and what does it represent?

4) A ball is thrown into the air with a velocity of 40 feet per second, its height in feet after  $t$  seconds is given by  $y = 40t - 16t^2$  (4 Pts Each)

a) Find the average velocity for the time period beginning when  $t = 1$  and **lasting**

i) 0.05 s

j) 0.01 s

b) Find the instantaneous velocity when  $t = 1$  (2 Pts)

5) Given  $f(x) = \sqrt{1+2x}$

a) Find the domain of  $f(x)$  (2 pts)

b) Use the definition of the a derivative to find  $f'(x)$  (10 Pts)

c) Find the domain of  $f'(x)$  (3 pts)

6) Find the following limits algebraically: (Justify your Answer) (5 Pts Each)

a)  $\lim_{t \rightarrow 0} \frac{1}{t} - \frac{1}{t^2 + t}$

6b) Find the following limits algebraically  $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 25} - 5}{t^2}$

7) Find the following limit:  $\lim_{t \rightarrow 0} x^4 \cos\left(\frac{1}{x^2}\right) = 0$

(Hint: Use the Squeeze Theorem)

(5 Pts)

8) Given  $f(x) = x^3 + 3x^2 + 1$

a) Graph the function

(1 point)

b) Graph  $f'(x)$

(4 points)

- 9) The graph of  $f(x)$  is shown below. State, with reasons, the numbers at which  $f(x)$  is not differentiable. (5 points)

10) Given  $f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

- a) Is  $f(x)$  continuous at  $x = 0$ ? (must use definition of continuity)

(5 points)

- b) Is  $f(x)$  differentiable at  $x = 0$ ? (Justify your answer)

(5 points)

11) Sketch the graph of a function that satisfies all of the following conditions: (10 points)

$$f'(-1) = f'(1) = 0$$

$$f'(x) < 0 \quad \text{if } |x| < 1$$

$$f'(x) > 0 \quad \text{if } |x| > 1$$

$$f(-1) = 4$$

$$f(1) = 0$$

$$f''(x) < 0 \quad \text{if } x < 0$$

$$f''(x) > 0 \quad \text{if } x > 0$$

12) Sketch the graph of the following function

And use it to determine all the values of “a” for which  $\lim_{x \rightarrow a} f(x)$  exists. (5 Pts)

$$f(x) = \begin{cases} 3-x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ (x-3)^2 & \text{if } x \geq 1 \end{cases}$$

13) Each figure below shows the graphs of a function, its first derivative, and its second derivative. Identify which is which. (10 points)

14) Each figure below shows the graphs of a function, its first derivative, and its second derivative. Identify which is which. (10 points)

15) Given  $g(x) = x \ln x$  and  $g'(x) = \ln x + 1$

Find an equation of the tangent line at  $x = e$  (5 points)