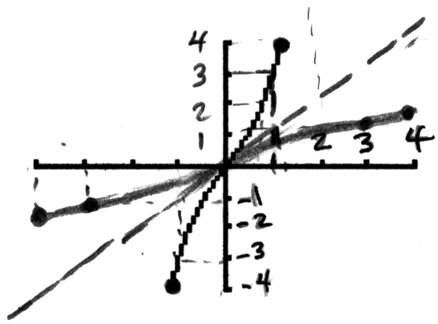
Professor Katiraie Calculus I Fall 2006 Test II (chapters 1 and 2)

Name: Solution Total Possible Points = 140
(Plus 10 pts Extra Credit)

(10 Points)



1) The graph of g is given.

a) State the value of g(1) $= 3$

b) Why is g one-to-one? g(x) is one-to-one Because it passes the horizontal line test

c) Estimate the value of $g^{-1}(2)$? $= 0.7$ OR 0.8

d) Estimate the domain of $g^{-1}(x)$ $[-4, 4]$

e) Sketch the graph of $g^{-1}(x)$ ✓

$(1000, \$9000)$
$(1500, \$12000)$

(10 Points) 2)A small-appliance manufacturer finds that it costs \$9000 to produce 1000 toaster ovens a week and \$12000 to produce 1500 toaster ovens a week.

a) Express the cost as a function of the number of the toaster ovens produced, assuming that it is linear.

$m = 6$

$9000 = 6(1000) + b$

$b = 3000$

$C(x) = 6x + 3000$

b) What is the slope of the graph and what does it represent?

Slope $= 6$

Each additional toaster costs \$6 to produce.

c) What is the y-intercept of the graph and what does it represent?

y-intercept is \$3000, and that is the fixed cost.

$f^{-1}(x)$

$f(x)$

x	y
-53	-2
-5	-1
1	0
7	1
55	2
307	3

(10 Points) 3) Let f be a one-to-one function whose inverse function is given by the formula:

$$f^{-1}(x) = x^5 + 2x^3 + 3x + 1$$

a) Compute $f^{-1}(-1) = -5$

b) Compute $f(1) = 0$

c) Compute the value of x such that $f(x) = 1$ $\Rightarrow x = 7$

d) Compute the value of y such that $f^{-1}(y) = 1$ $\Rightarrow y = 0$

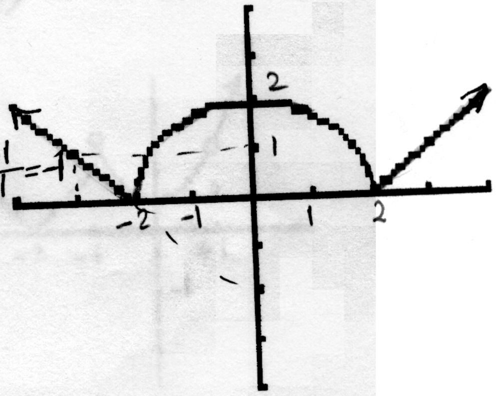
(10 Points) 4) Find a formula that describes the following function:

$(-3,1)$

$(-2,0)$

$m = \frac{0-1}{-2--3} = \frac{-1}{1} = -1$

$y =$



$$f(x) = \begin{cases} -x-2 & x \leq -2 \\ \sqrt{4-x^2} & -2 < x < 2 \\ x-2 & x \geq 2 \end{cases}$$

5) If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after t seconds is given by $h(t) = 58t - 0.83t^2$ (10 Points)

a) Find the average velocity over the given time intervals:

t / x	$h(t)$ / y
1	57.17
2	112.68
1.5	85.1325
1.1	62.7957
1.01	57.73

i) [1,2] $v_{avg} = \frac{112.68 - 57.17}{2-1} = 55.51$ m/sec

j) [1,1.5] $v_{avg} = \frac{85.1325 - 57.17}{1.5-1} = 55.925$ m/sec

k) [1,1.1] $v_{avg} = \frac{62.7957 - 57.17}{1.1-1} = 55.957$ m/sec

l) [1,1.01] $v_{avg} = \frac{57.73 - 57.17}{1.01-1} = 56$ m/sec

b) Find the instantaneous velocity after one second.

v at 1 sec $= 56$ m/sec

6) $f(x)=\begin{cases} x^3+2; x\le -1 \\ x^2+x [redacted]; -1<x<1 \\ x^4+2; x\ge 1 \end{cases}$ find the following limits (10 Points)

a) $\lim_{x\to -1^-} f(x) = (-1)^3+2 = -1+2 = 1$

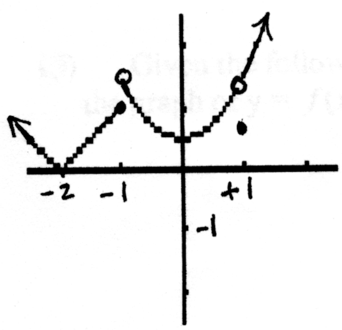
b) $\lim_{x\to -1} f(x) = DNE$ B/c $\lim_{x\to -1^+} f(x) = (-1)^2 + (-1) = 0$

c) $\lim_{x\to 1^+} f(x) = (1)^4+2 = 3$

d) $\lim_{x\to 1} f(x) = DNE$ B/c $\lim_{x\to 1^-} f(x) = (1)^2+1+1 = 2$

7) For the function whose graph is shown below, answer the following equations:

(10 Points)



a) At what number "a" $\lim_{x\to a} f(x)$ does not exist? at $\boxed{a=-1}$

B/c $\lim_{x\to -1^-} f(x) = 1$ $\lim_{x\to -1^+} f(x) = 1.5$

b) At what numbers "a" $\lim_{x\to a} f(x)$ **exists,** yet $f(x)$ is **not continuous**? at $\boxed{a=1}$

B/c $\lim_{x\to 1} f(x) \neq f(1)$

c) At what numbers "a" $f(x)$ is **not differentiable**?

at $a=-2$, $a=-1$, and $a=1$

8) Use the intermediate Value Theorem to show that there is a root of the equation $x^3+2x^2-42=0$ on the interval (0,3). (5 Points)

Observe that $f(x)$ is continuous over $[0,3]$

and $f(0)=-42$ and $f(3)=3$ and $f(0)\neq f(3)$

then by IVT $\exists\ c\in(0,3)$ s.t $f(c)=0$

9) Given $f(x) = \begin{cases} 2x^3 + 16; x \le -2 \\ x^2 + bx + c; -2 < x < 2 \\ 3x^4 - 48; x \ge 2 \end{cases}$ determine the values for b and c

so that $f(x)$ is continuous everywhere. (10 Points)

$2(-2)^3 + 16 = 0$ & $3(2)^4 - 48 = 0$

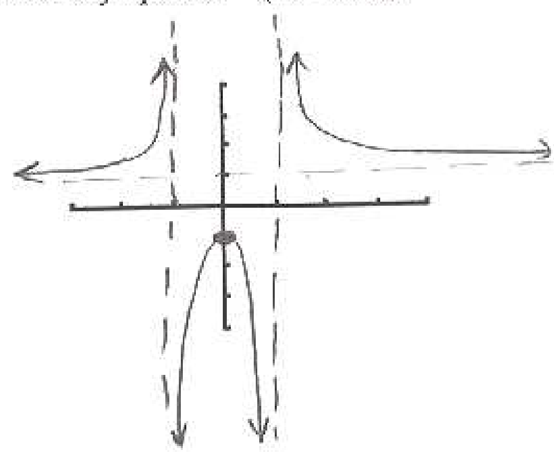
$(-2)^2 + b(-2) + c = 0 \Rightarrow \begin{cases} -2b + c = -4 \\ 2b + c = -4 \end{cases}$

$2^2 + b(2) + c = 0 \Rightarrow$

$2c = -8$ $\quad -2b - 4 = -4$

$\boxed{c = -4}$ and $\boxed{b = 0}$

10) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. Label all horizontal and vertical asymptotes. (10 Points)



$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 1$

$\lim_{x \to -1^+} f(x) = \lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to -1^-} f(x) = \lim_{x \to 1^+} f(x) = \infty$

$f(0) = -1$

11) If $f(x) = \sqrt{x-3}$ (10 Points)

Find the $f'(x)$ using either of the two definitions discussed in class.

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h-3} - \sqrt{x-3}}{h} \cdot \frac{\sqrt{x+h-3} + \sqrt{x-3}}{\sqrt{x+h-3} + \sqrt{x-3}} = \frac{\cancel{x}+\cancel{h}-3-(\cancel{x}-3)}{\cancel{h}(\sqrt{x+h-3}+\sqrt{x-3})}$

$f'(x) = \lim_{h \to 0} \frac{1}{\sqrt{x+h-3} + \sqrt{x-3}} = \boxed{\frac{1}{2\sqrt{x-3}}}$

Find the equation of the tangent line to the curve $f(x) = \sqrt{x-3}$ at the point (4, 1)

$f'(4) = \frac{1}{2(1)} = \frac{1}{2}$ $\quad y - 1 = \frac{1}{2}(x-4) \Rightarrow \boxed{y = \frac{1}{2}x - 1}$

12) Find the following limits (15 Points)

a) $\lim_{x\to 2^+} \frac{1}{x-2} = +\infty$

b) $\lim_{x\to\infty} \frac{1-2x^2}{x^2+x} \begin{matrix} \div x^2 \\ \div x^2 \end{matrix} = \lim_{x\to\infty} \dfrac{\frac{1}{x^2} - \frac{2x^2}{x^2}}{\frac{x^2}{x^2} + \frac{x}{x^2}} = \frac{-2}{1} = \boxed{-2}$

c) $\lim_{x\to\infty} (\sqrt{x^2+2x} - x) \cdot \dfrac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} = \lim_{x\to\infty} \dfrac{\cancel{x^2}+2x - \cancel{x^2}}{\sqrt{x^2+2x}+x}$

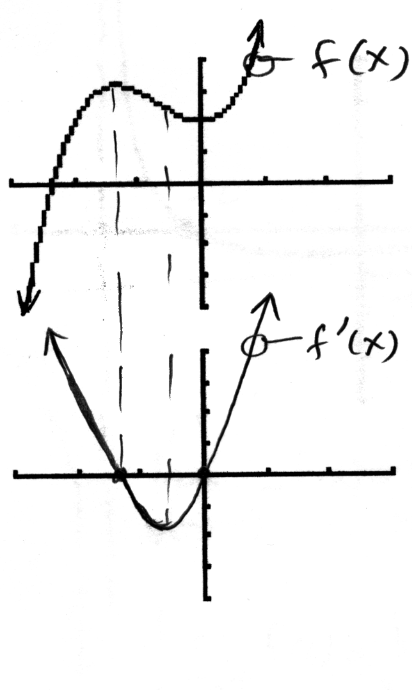
Now Divide Top & Bottom by x, we get

$\lim_{x\to\infty} \dfrac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2}} + \frac{x}{x}} = \frac{2}{2} = \boxed{1}$

d) $\lim_{x\to\infty} \sin x \Rightarrow$ DNE

e) $f(x) = \lim_{x\to\infty} e^{-x} = \lim_{x\to\infty} \frac{1}{e^x} = \frac{1}{\infty} = \boxed{0}$

13) Given the graph of $y = f(x)$, sketch the graph of $y = f'(x)$ (5 Points)



(10 Points)

14) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (-2,3) and (4,-1). Find the following:

a) find $f'(3) = \dfrac{-1-3}{4--2} = \dfrac{-4}{6} = \boxed{\dfrac{-2}{3}}$

b) Find an equation of the line tangent to f at $x = 3$

$y - 3 = \frac{-2}{3}(x - -2) \Longrightarrow y = \frac{-2}{3}x - \frac{4}{3} + 3 = \boxed{\frac{-2}{3}x + \frac{5}{3}}$

c) Find $f(3)$

$f(3) = \boxed{-\dfrac{1}{3}}$

(5 Points)

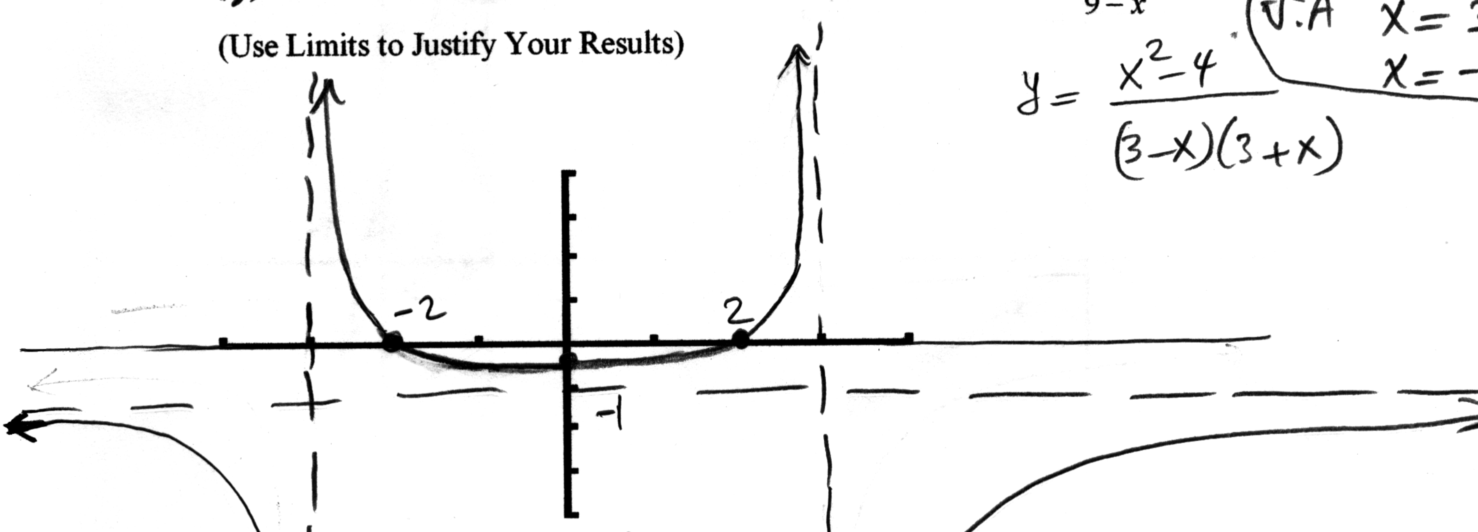
15) Find the vertical and horizontal asymptote(s) of the curve $y = \dfrac{x^2-4}{9-x^2}$

(Use Limits to Justify Your Results)

$\boxed{HA \quad y = -1}$

$\boxed{V.A \quad x = 3 \quad x = -3}$

$y = \dfrac{x^2-4}{(3-x)(3+x)}$



$\lim\limits_{x \to -3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = -\infty$

$\lim\limits_{x \to \pm\infty} f(x) = -1$

$\lim\limits_{x \to -3^+} f(x) = \lim\limits_{x \to 3^-} f(x) = \infty$

(Extra Credits ☺ 5 Points)

16) If $\quad 3x \le f(x) \le x^3 + 2 \quad$ for $\; 0 \le x \le 2$

Evaluate $\lim_{x \to 1} f(x)$

$$\lim_{x \to 1} 3x \le \lim_{x \to 1} f(x) \le \lim_{x \to 1} x^3 + 2$$

But $\lim_{x \to 1} 3x = 3 \quad$ and $\quad \lim_{x \to 1} x^3 + 2 = 3$

Thus by Squeeze theorem $\boxed{\lim_{x \to 1} f(x) = 3}$

(Extra Credits ☺ 5 Points)

17) Given the graphs of $y = f'(x)$, sketch the graphs of $y = f\ (x)$