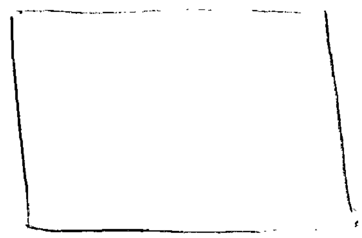
#1



$w = 3$ feet

$\frac{dw}{dt} = 1 \text{ ft/sec}$

$\frac{dL}{dt} = 2 \text{ ft/sec}$

$L = 5$ feet

$\frac{dA}{dt} = ?$

$A = L\,w$

$$\frac{dA}{dt} = \frac{dL}{dt} w + L \cdot \frac{dw}{dt}$$

$$\frac{dA}{dt} = (2 \text{ ft/sec})(3 \text{ feet}) + (5 \text{ ft})(1 \text{ f}$$

$$= 6 \frac{\text{ft}^2}{\text{sec}} + 5 \frac{\text{ft}^2}{\text{sec}} = 11 \frac{\text{ft}^2}{\text{sec}}$$

#2

$\frac{dL}{dt} = -1 \text{ ft/sec}$ $\qquad \frac{dA}{dt} = 0$

$\frac{dw}{dt} = ?$ $\qquad L = 10$ feet, $w = 5$ ft

$A = L\,w$

$$\frac{dA}{dt} = \frac{dL}{dt} w + L \cdot \frac{dw}{dt}$$

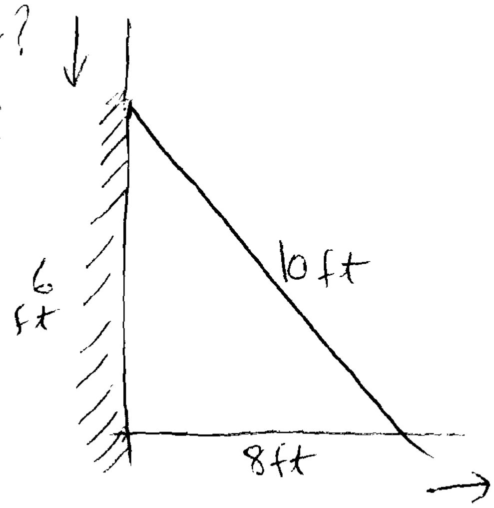
$\downarrow$

$$0 = (-1 \text{ ft/sec})(5 \text{ ft}) + (10 \text{ ft})\left(\frac{dw}{dt}\right)$$

$$\frac{dw}{dt} = \frac{5 \text{ ft}^2/\text{sec}}{10 \text{ ft}} = \frac{1}{2} \text{ ft/sec}$$

$\frac{dy}{dt} = ?$

#3



$\frac{dx}{dt} = 3\,ft/sec$

$$10^2 = 8^2 + y^2$$
$$100 = 64 + y^2$$
$$36 = y^2$$
$$\boxed{y = 6}$$

$$c^2 = a^2 + b^2$$

$$\cancel{2}c\frac{dc}{dt} = \cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt}$$

$$0 = 8\,ft\left(\frac{3\,ft}{sec}\right) + (6\,ft)\left(\frac{dy}{dt}\right)$$

$$0 = 24\,ft^2/sec + 6\,ft\,\frac{dy}{dt} \implies \frac{dy}{dt} = \frac{-24\,ft^2/sec}{6\,ft}$$

$$= -4\,ft/sec$$

Hence, the rate of the top of the ladder is sliding down the wa[ll] at 4 ft/sec.

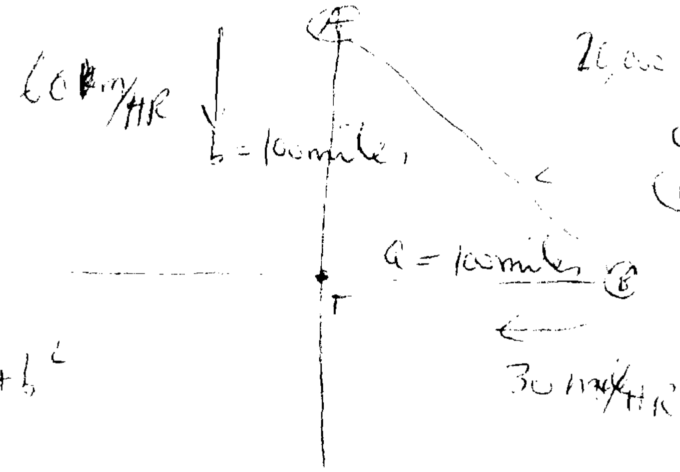
#4

$100^2 + 100^2 = c^2$

$20{,}000 = c^2$

$c = \sqrt{20000}$

$c = 100\sqrt{2}$



$c^2 = a^2 + b^2$

$$\cancel{2}c\,\frac{dc}{dt} = \cancel{2}a\,\frac{da}{dt} + \cancel{2}b\,\frac{db}{dt}$$

$$\frac{dc}{dt} = \frac{(100\text{ miles})\left(-30\ \frac{\text{miles}}{HR}\right) + (100\text{ miles})\left(-60\text{ mi}/HR\right)}{\cancel{100}\sqrt{2}\text{ miles}}$$

$$= \frac{-90}{\sqrt{2}}\ \text{miles}/HR$$

$$= \frac{-90}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{-\overset{45}{\cancel{90}}\sqrt{2}}{\cancel{2}} = -45\sqrt{2}\ \frac{\text{mile}}{HR}$$

Hence, the two cars are approaching each other at the rate of $45\sqrt{2}$ miles per hour

(#15) $f(x) = x^3 + ax^2 + bx \qquad x=1;\ x=3 \qquad$ C;#

$$f'(x) = 3x^2 + 2ax + b = 0$$

$$\begin{cases} 3(1)^2 + 2a + b = 0 \\ 3(3)^2 + 2(a)(3) + b = 0 \end{cases} \qquad \begin{array}{r} 3 + 2a + b = 0 \\ -1(27 + 6a + b = 0) \\ \hline \end{array}$$

$$\begin{array}{r} 3 + 2a + \cancel{b} = 0 \\ -27 - 6a - \cancel{b} = 0 \\ \hline -24 - 4a = 0 \\ 4a = -24 \end{array} \qquad a = -6$$

$$3 + 2(-6) + b = 0$$

$$3 - 12 + b = 0$$

$$-9 + b = 0$$

$$\boxed{b = 9}$$

MVT: If f is differentiable function on the interval $[a,b]$, then there exists a c between a, and such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

(#17) $f'(c) = \dfrac{f(\frac{1}{2}) - f(0)}{\frac{1}{2} - 0} = \dfrac{(\frac{1}{2})^2 - 0^2}{\frac{1}{2} - 0} = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \cdot \frac{2}{1} = -$

(#19) $f(x) = (x-2)^3 \quad [0,2]$

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{0 - (-2)^3}{2} = \frac{8}{2} = 4$$

$$f'(x) = 3(x-2)^2 = 4 \implies (x-2)^2 = \frac{4}{3} \implies x - 2 = \pm\frac{2}{\sqrt{3}}$$

$$x = 2 \pm \frac{2}{\sqrt{3}}$$

But $2 + \frac{2}{\sqrt{3}} = 3.155$ which is not $[0,2]$

thus $\boxed{c = 2 - \frac{2}{\sqrt{3}}}$

#20 $\quad f(x) = \dfrac{1}{1+x} \quad [0,1] \qquad\qquad f(x) = (1+x)^{-1}$

$$f'(x) = -1(1+x)^{-2} = \frac{-1}{(1+x)^2}$$

$$f'(c) = \frac{f(1) - f(0)}{1-0} \Rightarrow \frac{\frac{1}{2} - 1}{1} = \frac{-1/2}{1} = -\frac{1}{2}$$

$$\frac{-1}{(1+c)^2} = \frac{-1}{2} \implies (1+c)^2 = 2$$

$$(1+c) = \pm\sqrt{2}$$

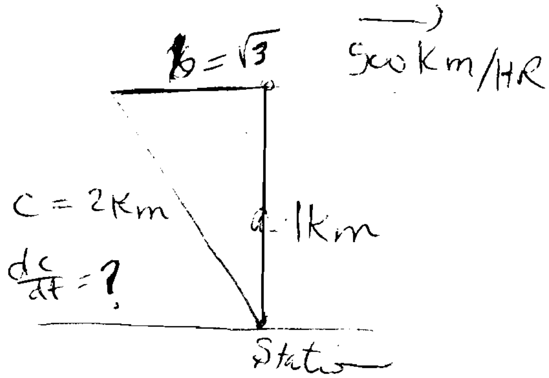
$$c = -1 \pm \sqrt{2} \begin{cases} \to 0.414 = -1+\sqrt{2} \\ \to \cancel{-2.414} \text{ outside of } [0,1] \end{cases}$$

$$1^2 + x^2 = 2^2$$

$$x^2 = 3$$

$$x = \sqrt{3}\ \text{km}$$

#5



$$c^2 = a^2 + b^2$$

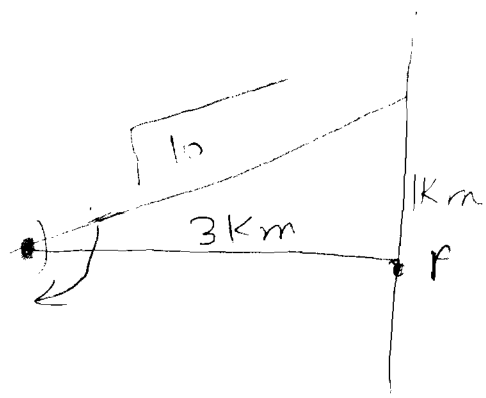
$$\cancel{2}c\frac{dc}{dt} = \cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt}$$

$$(2\text{Km})\left(\frac{dc}{dt}\right) = \cancel{(1\text{Km})(0)} + (\sqrt{3}\ \text{Km})(500\text{Km/HR})$$

$$\frac{dc}{dt} = \frac{(500\ \text{Km/HR})(\sqrt{3}\ \cancel{\text{Km}})}{(2\ \cancel{\text{Km}})}$$

#5 $\frac{dc}{dt} = 250\sqrt{3}\ \text{Km/HR}$

#6



$$\tan\theta = \frac{x}{3} \qquad x = 3\tan\theta$$

$$\frac{dx}{dt} = 3\sec^2\theta\,\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = 3\left(\frac{1}{\left(\frac{3}{\sqrt{10}}\right)^2}\right)\left(4\,\frac{rev}{min}\right)$$

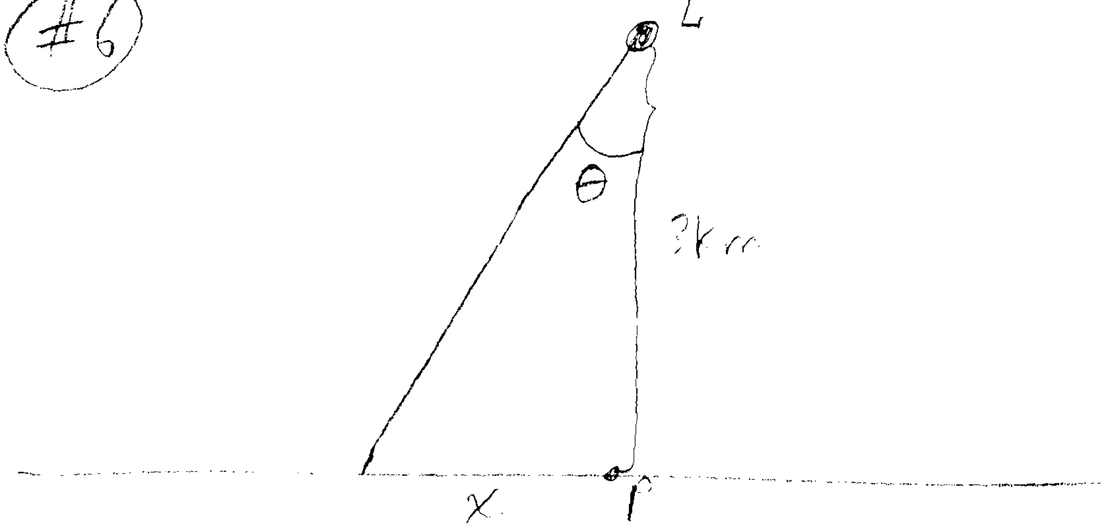
$$\frac{dx}{dt} = \frac{3}{\frac{9}{10}}\left(4\,\frac{rev}{min}\right)$$

$$\frac{dx}{dt} = \frac{120}{9}\ rev/min$$

$$= \frac{40}{3}\ rev/min * \frac{2\pi\ radians}{1\,\cancel{rev}}$$

$$= \frac{80\pi}{3}\ \frac{radians}{min}$$

#6



$$\tan\theta = \frac{x}{3}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{3}\frac{dx}{dt} \implies \frac{dx}{dt} = 3\sec^2\theta \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = 3\left(\frac{1}{\cos\theta}\right)^2\left(4\,\frac{rev}{min}\right)$$

$\Rightarrow$ where $x = 1\,\text{Km}$; $c = \sqrt{1+9} = \sqrt{10}\ K$

$$= 3\left(\frac{1}{3/\sqrt{10}}\right)^2 4\,\frac{rev}{min}$$

$$= \cancel{3}\cdot\frac{10}{\cancel{9}_{3}}(4)\frac{\cancel{rev}}{min}\cdot\frac{2\pi\ \text{radians}}{1\ \cancel{rev}}$$

$$= \boxed{\frac{80\pi}{3}\ \frac{\text{radians}}{\text{min}}}$$

(#7) $f(x) = x^2 - \frac{1}{2}$

$f'(x) = 2x = 0 \implies x = 0$

$f(0) = -\frac{1}{2}$

$f''(x) = 2$ Hence always concave up

Thus $-\frac{1}{2}$ is the minimum value of the function

(#8) $\underline{x = 0}$ Proof by previous Result.

(#9) $f(x) = x^3 - 3x + 27$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0$ $(-1)^3 - 3(-1) +$ $-1 + 3 + 27$

$x = 1$ $x = -1$ → Distance

$f(1) = 25$ $y = 29$

(#10) 4 Points $29 - 25 = 4$ Abs. Min $x = -10$ $y = -1$

(#11) $f(x) = x^3 - x^2 - x$ $-10 \leq x \leq 1$

$f'(x) = 3x^2 - 2x - 1 = 0$ $(3x+1)(x-1) = 0$

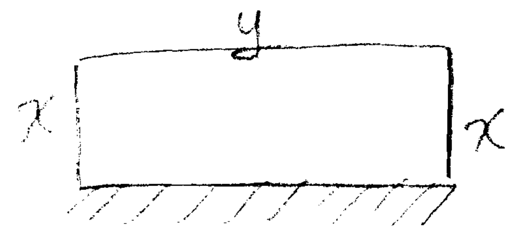
answer: $x = -\frac{1}{3}$, $y = 0.185$ Rel. Max Abs. Max $\boxed{x = 1 \quad y = -1}$ Rel. Min

$f''(x) = 6x - 2$

$f''(-\frac{1}{3}) = 6(-\frac{1}{3}) - 2 = -2 - 2 = -4$ Hence Concave down $\implies x = -\frac{1}{3}$ Rel. Max

$f''(1) = 6(1) - 2 = 4$ $\implies$ concave up $\implies x = 1$ Rel. min

#51



$$2x + y = 20 \text{ feet} \;;\; y = 20 - 2x$$

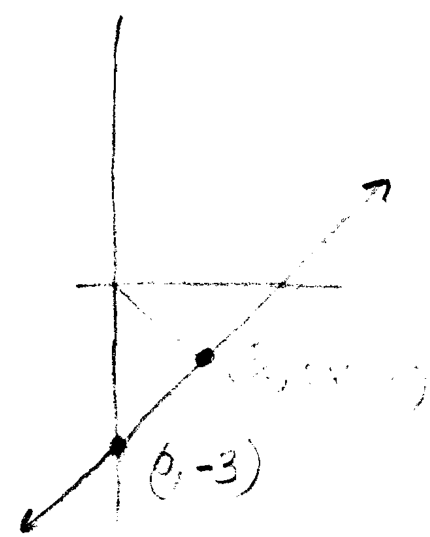
$$A = xy = x(20 - 2x) = 20x - 2x^2$$

$$\frac{dA}{dx} = 20 - 4x = 0 \Rightarrow x = \frac{-20}{-4} = 5 \text{ feet}$$

$$y = 20 - 2(5) = 10 \text{ f}$$

The maximum area is (5 feet)(10 feet) = $\overline{50 \text{ ft}}$

#52 $y = 2x - 3$



$$D = \sqrt{(x-0)^2 + (2x-3)^2}$$

$$D = \sqrt{x^2 + 4x^2 - 12x + 9}$$

$$D = \sqrt{5x^2 - 12x + 9}$$

$$\frac{dD}{dx} = \frac{1}{2}(5x^2 - 12x + 9)^{-1/2}(10x - 12) = 0$$

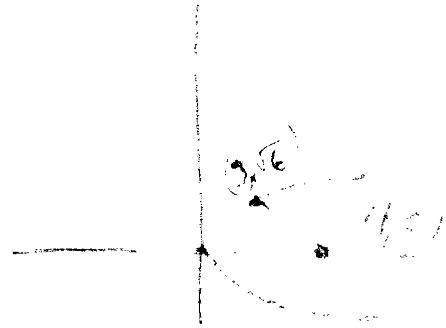
$$10x - 12 = 0 \Rightarrow x = \frac{12}{10} = 1.2$$

$$y = 2(x) - 3 = 2(1.2) - 3$$
$$= 2.4 - 3 = -0.6$$

Answer: Pt = $(1.2, -0.6)$

#5?

$(4,0)$



$y^2 = 2x$

$y = \pm\sqrt{2x}$

$D = \sqrt{(\sqrt{2x} - 0)^2 + (x-4)^2}$

$= \sqrt{2x + x^2 - 8x + 16} = \sqrt{x^2 - 6x + 16}$

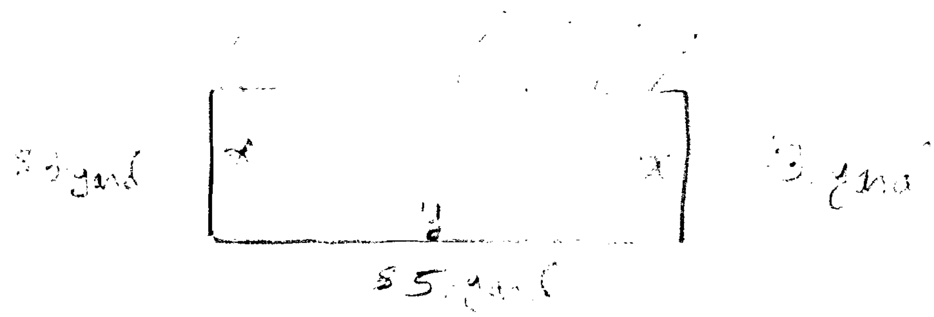
$\frac{dD}{dx} = \frac{1}{2}(x^2 - 6x + 16)^{-1/2}(2x - 6) = 0$

$\frac{2x - 6}{2\sqrt{x^2 - 6x + 16}} = 0$ $\quad x = 3$

$y = \sqrt{6}$

$D = (9 - 18 + 16)^{1/2} = \sqrt{7}$ $(3, \sqrt{6})$

5



$x \cdot y = 500$ and $y = \frac{500}{x}$

$C = 3x + 3x + 5y = 6x + 5 \cdot \frac{500}{x}$

$= 6x + 2500x^{-1}$

$\frac{dC}{dx} = 6 - \frac{3000}{x^2}$

$6x^2 = \ldots$

$x^2 = 500$

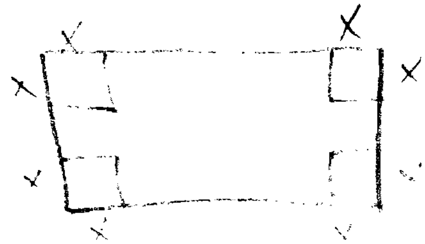
$x = 10\sqrt{5}$

$y = \frac{500}{x} = \frac{500}{10\sqrt{5}} = \frac{50}{\sqrt{5}}$

$y = \frac{50\sqrt{5}}{5} = 10\sqrt{5}$ yard

thus dimension is $10\sqrt{5}$ by $10\sqrt{5}$ yard

#56



$V = X(1[illegible] - 2x)(8 - [illegible])$

$V = [illegible]x [illegible](8 - [illegible])$

$\frac{dV}{dx} = 1[illegible] - [illegible](8 - [illegible]) - 2(1[illegible]x - [illegible])$ [illegible]

$0 = [illegible] - 32x + \cancel{[illegible]x^2} - \cancel{[illegible]x} + \cancel{[illegible]x}$ [illegible]

$12x^2 - 72x + 18[illegible]$

$6x^2 - [illegible] + [illegible]$

$3x^2 - 18x + [illegible]$ [illegible] $.147$ [illegible]

$x = [illegible]$

#57 $C(x) = 1000 - 10x + x^2$

Avg cost $= \dfrac{C(x)}{x} = \dfrac{1000 - 10x + x^2}{x}$

Avg cost$(100) = \dfrac{1000 - 10(100) + 100^2}{100} = \dfrac{[illegible]}{100} = \boxed{[illegible]}$

#58

[illegible]

Marginal cost $= C'(x) = -10 + 2x$

$C'(100) = -10 + 2(100) = -10 + 200 = \boxed{190}$

#59 $C(x) = 200 - 50x + x^2$ $\quad p(x) = 50 - x$

$R(x) = x \cdot p(x) = x(50 - x) = 50x - x^2$

$P(x) = R(x) - C(x) = (50x - x^2) - (200 - 50x + x^2)$

$= 50x - x^2 - 200 + 50x - x^2$

$= -2x^2 + 100x - 200$

$P'(x) = -4x + 100 = 0 \quad 4x = 100,\ x = 25$

units

#60 $C(x) = 25 + 4x + 18x^{-1}$

marginal cost $= C'(x) = 4 - 18x^{-2}$

$C'(15) = 4 - 18(15)^{-2} \approx \$3.92$

#61 $x_1 = 2$ $\quad f(x) = x^5 - 34$

$f'(x) = 5x^4$

$$x_2 = 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \frac{(2^5 - 34)}{5(2)^4} \quad [illegible] \; 80 \; [illegible]$$

$$= \frac{81}{40}$$

#62 $2x^3 + 2x + 1 = 0$ $\quad x_1 = -1$

$f(x) = 2x^3 + 2x + 1$

$f'(x) = 6x^2 + 2$

$$x_2 = -1 - \frac{2(-1)^3 + 2(-1) + 1}{6(-1)^2 + 2} = -\frac{5}{8} = -0.625$$

$$6x^3 - x^2 - 19x + 6 = 0$$

#1621 $\quad f(x) = 6x^3 - x^2 - 19x + 6$

$$f'(x) = 18x^2 - 2x - 19$$

Let $x_1 = 0$

$$x_2 = 0 - \frac{f(0)}{f'(0)}$$

$\vdots$

$x_n = 0.32077985 = 0.321$

#1624 $\quad x^3 + 2x = 3.1$

$Y_1 = f(x) = x^3 + 2x - 3.1$

$Y_2 = f'(x) = 3x^2 + 2$

$x_1 = \text{guess } 1$

$$x_2 = 1 - \frac{f(1)}{f'(1)} = 1.02$$

$$x_3 = 1.02 - \frac{f(1.02)}{f'(1.02)} = 1.0197618$$

$x_4 =$

$\vdots$

$x_n = 1.0197604$