Dr. Katiraie Calculus I Fall 2008 Test I (chapter 1)

Name: Solution Total Possible Points = 140 Points
(Plus 10 pts Extra Credit ☺)

1a) Given $f(x) = \dfrac{7}{x+1}$ find $\dfrac{f(x+h)-f(x)}{h}$ $h \neq 0$ (10 Points)

step I) $f(x+h) = \dfrac{7}{x+h+1}$

step II) $f(x+h) - f(x) = \dfrac{7}{x+h+1} - \dfrac{7}{x+1} = \dfrac{7x+7-7(x+h+1)}{(x+h+1)(x+1)} = \dfrac{-7h}{(x+h+1)(x+1)}$

step III) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{-7h}{(x+h+1)(x+1)}}{\frac{h}{1}} = \dfrac{-7h}{(x+h+1)(x+1)} \cdot \dfrac{1}{h} = \boxed{\dfrac{-7}{(x+h+1)(x+1)}}$

1b) If $f(x) = 2x^2 - 7$, find and simplify $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$ (10 Points)

step I) $f(x+h) = 2(x+h)^2 - 7 = 2(x^2+2xh+h^2) - 7 = 2x^2+4xh+2h^2-7$

step II) $f(x+h) - f(x) = 2x^2+4xh+2h^2-7-(2x^2-7) = 4xh+2h^2$

step III) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4xh+2h^2}{h} = \dfrac{h(4x+2h)}{h} = \boxed{4x+2h}$

2) Find the Domain and Range of the following functions: (8 Points)

a) $f(x) = \sqrt{(16-x^2)}$

Domain $-4 \leq x \leq 4$

Range $0 \leq y \leq 4$

b) $g(x) = \ln(\ln(x+6))$

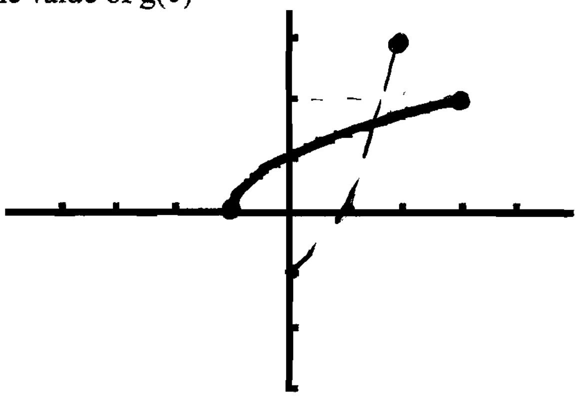
Domain
$\begin{cases} \ln(x+6) > 0 \\ x+6 > e^0 \\ x+6 > 1 \\ x > -5 \end{cases} \Rightarrow \boxed{\text{Domain } x > -5}$

Range

All Real Numbers

3) The graph of g is given. (10 Points)

a) State the value of g(0) $=1$



b) Why is g one-to-one?
Because it passes the Horizontal line test

c) Estimate the value of $g^{-1}(2)$? $g^{-1}(2)=3$

d) Estimate the domain of $g^{-1}(x)$

$[0,2]$

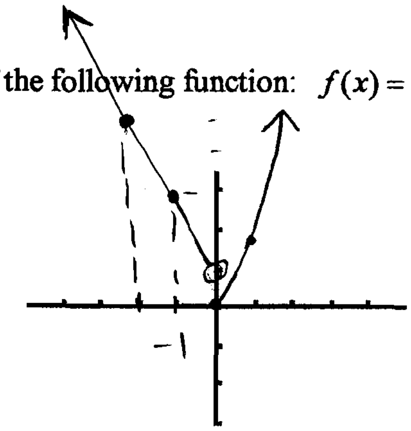
e) Sketch the graph of $g^{-1}(x)$

See Above

4) Sketch the graph of the following function: $f(x)=\begin{cases}1-2x & x<0\\ e^x-1 & x\geq 0\end{cases}$ (6 Points)

$y=1-2x$

x	y	
0	1 open circle	
-1	$1-2(-1)=3$	$(-1,3)$
-2	$1-2(-2)=5$	



$y=e^x-1$

x	y
0	$1-1=0$
1	$e^1-1=1.72$

5) Find the equation of the exponential function of the form $y=Ca^x$ that passes through the points (2, 5) and (1, 15). (10 Points)

$(2,5) \Rightarrow 5=Ca^2$

$(1,15) \Rightarrow 15=Ca^1$

$\Rightarrow \frac{5}{15}=\frac{Ca^2}{Ca^1} \Rightarrow \boxed{a=\frac{1}{3}}$

$15=C\left(\frac{1}{3}\right)^1 \Rightarrow \boxed{C=45}$

$\boxed{y=45\left(\frac{1}{3}\right)^x}$

6) Determine (**algebraically**) whether f is even, odd, or neither even nor odd (12 Points)

a) $f(x)=3x^5-4x^2+3$

$f(-x)=3(-x)^5-4(-x)^2+3=-3x^5-4x^2+3$ Neither

b) $f(x)=e^{-x}$

$f(-x)=e^{-(-x)}=e^{x}$ Neither

c) $f(x)=x^3+\sin(x)$

$f(-x)=(-x)^3+\sin(-x)=-x^3-\sin x$
$=-f(x)$ odd

d) $f(x)=x^4+2x^2$

$f(-x)=(-x)^4+2(-x)^2=x^4+2x^2$ even
$=f(x)$

7) Solve the following equations algebraically. (10 points)
(Must Show All the Appropriate Steps)

a) $\log x+\log(x+3)=1$

$\log(x(x+3))=1 \implies x^2+3x=10$
$x^2+3x-10=0$
$(x+5)(x-2)=0$ $\cancel{x=-5}$ $\boxed{x=2}$

b) $\ln(3-x)-\ln(x+4)=\ln(2)$

$\ln\left(\frac{3-x}{x+4}\right)=\ln 2 \implies \frac{3-x}{x+4}=\frac{2}{1} \implies 2x+8=3-x$
$3x=-5$
$\boxed{x=\frac{-5}{3}}$

8) If $f(x)=\ln(x+2)$, find $f^{-1}(-2)$ (5 Points)

$-2=\ln(x+2) \implies x+2=e^{-2}$

$x=-2+e^{-2}$
$x\cong-1.865$

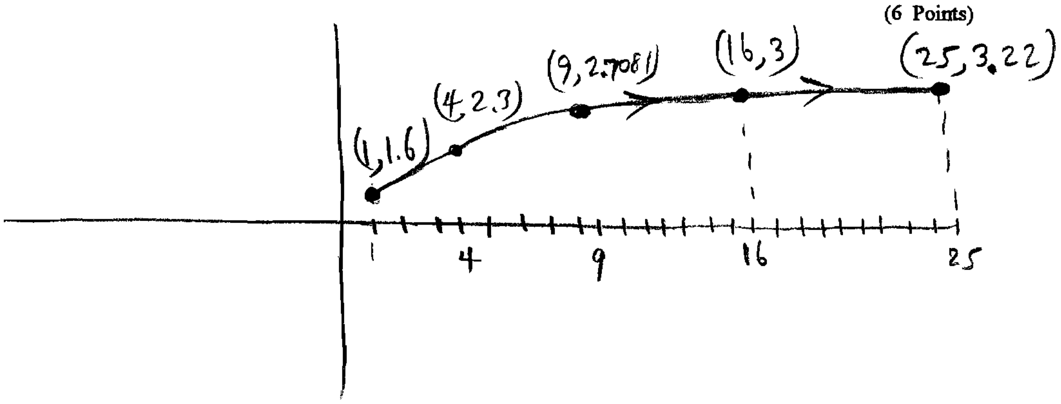
9a) Sketch the curve represented by the parametric equation

$x = t^2 \quad y = \ln(5t) \quad 1 \le t \le 5$

And indicate with an arrow the direction in which the curve is traced as t increases.

(6 Points)

t	X	y
1	1	1.6094
2	4	2.3
3	9	2.7081
4	16	2.9957
5	25	3.22



9b) Eliminate the parameter to find a Cartesian equation of the curve. (3 Points)

$X = t^2 \Rightarrow t = \sqrt{X}$

$y = \ln(5t) = \ln(5\sqrt{X}) \qquad \boxed{y = \ln(5\sqrt{X})}$

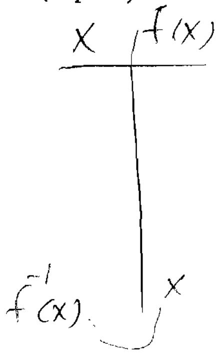
9c) State the domain and range of the above graph. (3 Points)

$1 \le X \le 25 \qquad 1.6094 \le y \le 3.22$

10) Let f be a one-to-one function whose inverse function is given by the formula:

(12 points)

$$f^{-1}(x) = x^5 + 3x^3 + 2x$$



a) Compute the value of y such that $f^{-1}(y) = 6$

$6 = y^5 + 3y^3 + 2y \quad \Rightarrow \boxed{y = 1}$

b) Compute $f^{-1}(-2) = (-2)^5 + 3(-2)^3 + 2(-2) \; \boxed{= -60}$

c) Compute $f(326)$ $\quad 326 = X^5 + 3X^3 + 2X$

$X \cong 2.99$

d) Compute the value of x such that $f(x) = 1$

$f^{-1}(1) = 1^5 + 3(1)^3 + 2(1) \boxed{= 6}$

11) **SOLVE for X** **(ALGEBRAICALLY)** (20 Points)

(You must show work for full Credit)

Show work & don't forget to check your answers!!

a) **$\log_4 (2x + 6) = ½$**

$4^{\frac{1}{2}} = 2x+6 \implies 2 = 2x+6$

$2x = -4$

$\boxed{x = -2}$

b) $4^x - 9 = 15$

$+9 \quad +9$

$4^x = 24$

$x = \log_4 24 = \frac{\log 24}{\log 4} = \boxed{2.29}$

c) Solve by the quadratic formula: $x^2 + 11 = 7x$

$x^2 - 7x + 11 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 - 4(1)(11)}}{2} = \boxed{\frac{7 \pm \sqrt{5}}{2}}$

d) Solve by factoring: $2x^2 - x = 15$

$2x^2 - x - 15 = 0$

$(2x+5)(x-3) = 0$ $\quad x = \frac{-5}{2}, \quad x = 3$

e) Solve for x Algebraically $\sqrt{3x-3} - 4 = 2$

$+4 \quad +4$

$\sqrt{3x-3} = 6$

$3x - 3 = 36$

$3x = 39$

$x = 13$

12) Given $x+(y-1)^2=0$

a) Find an expression for the function whose graph is the bottom half of the above (7 Points)

$(y-1)^2 = -x$

$y-1 = \pm\sqrt{-x}$ top half $\Rightarrow y = +1+\sqrt{-x}$

$y = +1 \pm \sqrt{-x}$ $\Longrightarrow$ Bottom half $\Rightarrow y = +1-\sqrt{-x}$

b) State the domain of the bottom half of the above parabola (3 Points)

$x \leqslant 0$

c) State the range of the bottom half of the above parabola (3 Points)

$y \leqslant +1$

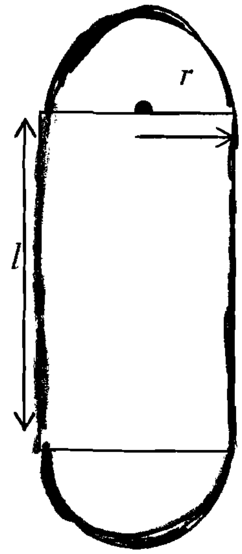
d) Is the following a function? $x+(y-1)^2=0$ (2 Points)

No, Because it fails the vertical line test

i.e for every x, there are two different y values

(Extra Credit 5 Points)

17) A field has the shape of a rectangle with a semicircle at each end. The length of the rectangular portion of the field is l, and the radius of each semicircle is r. If the <u>outside perimeter</u> of the field is 250 meters, express the **area** of the field as a function of r and simplify your answer.



$$2l + 2\pi r = 250$$

$$l + \pi r = 125$$

$$\boxed{l = 125 - \pi r}$$

$$A = \pi r^2 + l * 2r = \pi r^2 + (125 - \pi r) \cdot 2r$$

$$A = \pi r^2 + 250r - 2\pi r^2 \ (\text{meters}^2)$$

$$\boxed{A = -\pi r^2 + 250r} \ (m^2)$$

(Extra Credits: **5 pts)**

14a) Solve the following equation algebraically.

$e^{5-3x} = 10$

$$\cancel{\ln e}^{5-3x} = \ln 10$$

$$5 - 3x = \ln 10 \implies -3x = \ln 10 - 5$$

$$x = \frac{\ln 10 - 5}{-3} \cong 0.899$$

14b) Express $\ln a + \frac{1}{2}\ln b - \ln c$ as a single logarithm

$$= \ln a + \ln b^{\frac{1}{2}} - \ln c$$

$$= \ln\left(\frac{a b^{\frac{1}{2}}}{c}\right) \overset{\text{OR}}{=} \ln\left(\frac{a\sqrt{b}}{c}\right)$$