Professor Katiraie Calculus I Spring 2008 Test III Form A (chapters 1 -- 3)

Name (☺ 1 Point)________________________________ Total Possible Points = 140
(Plus 10 pts Extra Credit ☺)

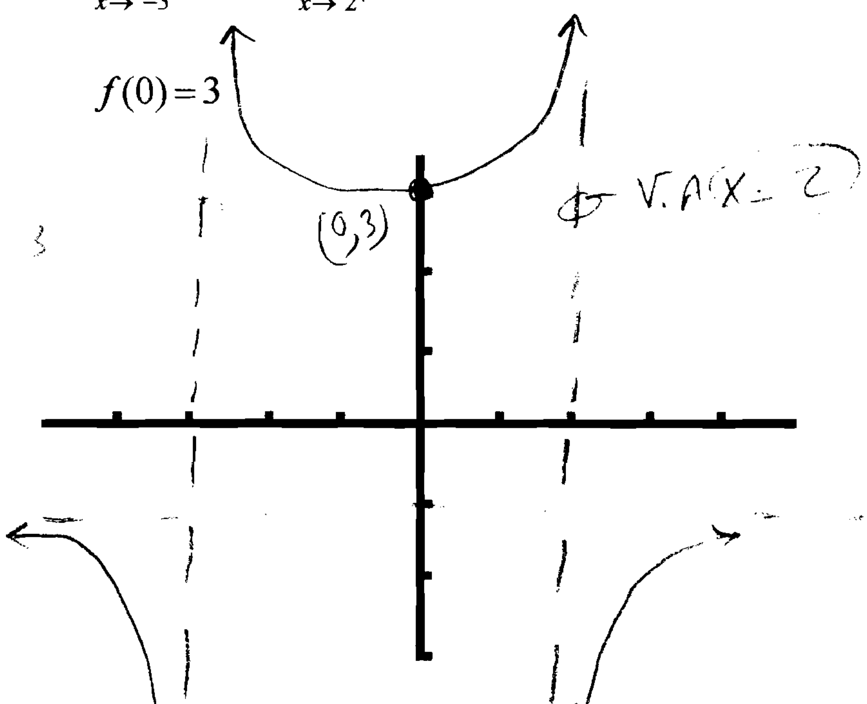
1) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (10 Points)

$$\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = -1$$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to 2^-} f(x) = \infty$$

$$\lim_{x\to -3^-} f(x) = \lim_{x\to 2^+} f(x) = -\infty$$

$f(0) = 3$



2) Given $f(x) = \begin{cases} 2x^3+7 & x \le -1 \\ x^2+bx+c & -1 < x < 1 \\ x^4-10 & x \ge 1 \end{cases}$ determine the values for b and c so that $f(x)$ is continuous everywhere. (8 Points)

$2(-1)^3+7 = (-1)^2+b(-1)+c \Rightarrow 5 = 1-b+c \Rightarrow \begin{cases} b-c = -4 \\ b+c = -1 \end{cases}$

$1^2+b+c = 1^4-10 \Rightarrow b+c = -10$

$2b = -14$

$b = -7$

$c = -3$

3) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (2 , 5) and (4 , -5). Find the following: (9 Points)

a) Find $f'(3)$ $= \frac{-5-5}{4-2} = \frac{-10}{2} = \boxed{-5}$

b) Find an equation of the line tangent to f at $x = 3$

$y - 5 = -5(x-2)$

$y = -5x + 10 + 5 = \boxed{-5x + 15}$

c) Find $f(3)$ $= -5(3) + 15 = -15 + 15 = \boxed{0}$

4) Find the following limits: (12 Points)

a) $\lim_{t\to 3} \frac{\sqrt{t+6}-3}{t-3} \cdot \frac{\sqrt{t+6}+3}{\sqrt{t+6}+3} = \lim_{t\to 3} \frac{t+6-9}{(t-3)(\sqrt{t+6}+3)}$

$= \boxed{\frac{1}{6}}$

b) $\lim_{x\to -7} \frac{7+x}{\frac{1}{7}+\frac{1}{x}}$

$= \lim_{x\to -7} \frac{7+x}{1} \cdot \frac{7x}{x+7} = -49$

c) $\lim_{t\to 0} \frac{\sqrt{t^2+25}-5}{t^2} \cdot \frac{\sqrt{t^2+25}+5}{\sqrt{t^2+25}+5} = \lim_{t\to 0} \frac{t^2+25-25}{t^2(\sqrt{t^2+25}+5)}$

$= \boxed{\frac{1}{10}}$

d) $\lim_{t\to 0} \frac{2}{t^2+t} - \frac{2}{t}$ $= \lim_{t\to 0} \frac{2-2(t+1)}{t(t+1)} = \lim_{t\to 0} \frac{-2t}{t(t+1)}$

$= \boxed{-2}$

5) Find the derivative of the following functions: **(4 Points each)**

(Do Not Simplify)

a) $y=\sqrt{x}+\frac{1}{\sqrt[5]{x^7}}$ $= x^{1/2} + x^{-7/5}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} \cdot \frac{7}{5}x^{-\frac{12}{5}}$

$= \frac{1}{2\sqrt{x}} - \frac{7}{5\sqrt[5]{x^{12}}}$

b) $y=e^{\tan(2\theta)}$

$y' = e^{\tan 2\theta} \sec^2(2\theta) \cdot 2$

c) $y=\cos^5(4x)$ $= (\cos 4x)^5$

$y' = 5(\cos 4x)^4$ [illegible] (4)

d) $y=\sin((5x)^3)$

$y' =$ [illegible]

e) $y=\sqrt{x+\sqrt{x}}$ $= (x + x^{\frac{1}{2}})^{\frac{1}{2}}$

$y' = \frac{1}{2}(x + x^{\frac{1}{2}})^{-\frac{1}{2}}$ [illegible]

f) $y=\left(\frac{3x-7}{x^2-1}\right)^7$

$y' = 7\left(\frac{3x-7}{x^2-1}\right)^6$ $\left(\frac{3(x^2-1)-2x(3x-7)}{(x^2-1)^2}\right)$

g) $y=10^{\sec\pi\theta}$

$y' = 10^{\sec\pi\theta}$ $(\sec\pi\theta \tan\pi\theta)$ [illegible]

h) $y=\sin(\cot\sqrt{1+x^2})$

$y' =$ [illegible] $\frac{1}{2}(1+x^2)$ [illegible]

i) $y=\sin^{-1}(x^2+2x+1)$

$y' = \frac{1}{\sqrt{1-(x^2+\ldots)^2}}$ [illegible]

j) $y=\ln(x^2+2x+1)$

$y' = \frac{1}{(x^2+2x+1)}$ $(2x+2)$

6) Find the equation of the tangent line to the curve $\sqrt{x} + \sqrt{y} = 7$, at the point (9,16). **(10 Points)**

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}y' = 0 \Longrightarrow$ [illegible] $\Longrightarrow y' = -$ [illegible]

$y' = -\frac{y^{1/2}}{x^{1/2}}$ [illegible] $\Longrightarrow y' = -\frac{16^{1/2}}{9^{1/2}} = -\frac{4}{3}$

$y - 16 = -\frac{4}{3}$ [illegible] $y = -\frac{4}{3}x + 12 + 16$

$y = -\frac{4}{3}x + 28$

7) Find the derivative of the function $y^x = (x)^{\cos x}$ **(10 points)**

Compute y' in terms of x, and y. **(Hint: Use Natural Logarithms)**

[illegible]

$\ln y + x \frac{1}{y} y' = -\sin x$ [illegible] $+ \cos x \frac{1}{x}$ $\Longrightarrow y' = \frac{y}{x}$ [illegible]

$\frac{x}{y}y' =$ [illegible]

8) Suppose that $h(x) = g(x)f(x)$, and $F(x) = g(f(x))$, where
$f(5) = 4$,
$g(5) = 2$,
$g'(5) = -1$,
$f'(5) = -2$,
$g'(4) = -5$

a) Find $h'(5)$. $=$ [illegible] **(5 Points)**

$= (-1)(4) + 2)(-2)$ [illegible]

[illegible]

b) Find $F'(5)$. $= g'(f(5)) \cdot f'(5)$ **(5 Points)**

[illegible]

$= \boxed{10}$

9) Find all values of x so that the graph of $f(x)=x-2\sin x$ will have a horizontal tangent?

(5 Points)

$f'(x) = 1 - 2\cos x = 0$

$-2\cos x = -1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

$x = \frac{5\pi}{3}$

10) Find the equation of the tangent line to the curve $y = x\cos x$, at the point $(\pi,-\pi)$.

(5 Points)

$y' = 1\cos x + x(\ldots)\Big|_{\text{at } x=\pi} = \ldots = -1$

$y - -\pi = -1(x - \pi$

$y = -x + \pi - \pi \ldots = -x$

11) A particle moves on a vertical line so that its coordinate at time t is

$$s(t)=t^3-12t^2+3 \quad t\geq 0$$

where S(t) is measured in meters and t is measured in seconds.

(10 Points)

a) When is the particle moving upward?

$a(t) = 6t - 24$

$3t^2 - 24t > 0$

$t > 8$

b) Find the distance that the particle travels in the time interval $7 \leq t \leq 9$ seconds.

c) When is the particle speeding up?

$0 < t < 4$

$8 < t$

12) Consider the circle $x^4 + y^4 = 1$. (5 Points)

At what point(s) is the slope of the tangent line equal to 1?

$y^3 = -x^3$

$y = -x$

$x^4 + (-x)^4 = 1$

13) Let $y = e^{\frac{x}{5}}$.

a) Find the differential dy. (3 Points)

b) Evaluate dy if $x = 0$, and $dx = 0.3$ (2 Points)

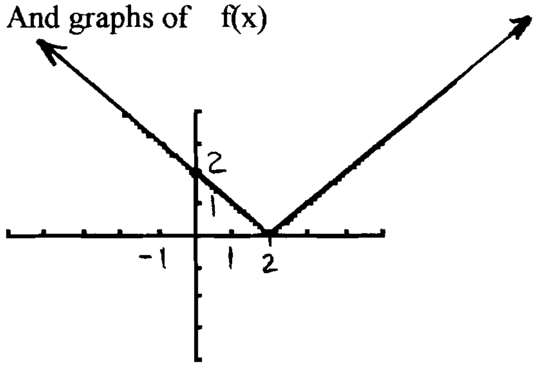
Extra Credits

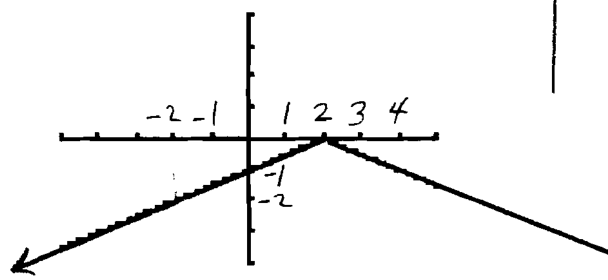
x	g(x)
0	-1
2	0
-2	-2
4	-1

14) Given that $v(x)=\dfrac{f(x)}{g(x)}$, and $w(x)=f(x)g(x)$ (5 Points)

And graphs of f(x) and g(x)

x	f(x)
0	2
-1	3
2	0
3	1





Find the following:

$v'(0)$	$w'(1)$

15a) Find the linearization of $f(x)=\sqrt[3]{1+x}$ at a = 0. (3 points)

$L(x) = f'(0)(x-0) + f(0)$

$= \frac{1}{3}(x-0) + 1$

$L(x) = \frac{1}{3}x + 1$

$f(0) = 1$

15b) Use the above to give an approximate value for $\sqrt[3]{0.95}$. (2 Points)

Professor Katiraie Calculus I Spring 2008 Test III Form B (chapters 1 -- 3)

Name (☺ 1 Point) SOLUTIONS Total Possible Points = 140
(Plus 10 pts Extra Credit ☺)

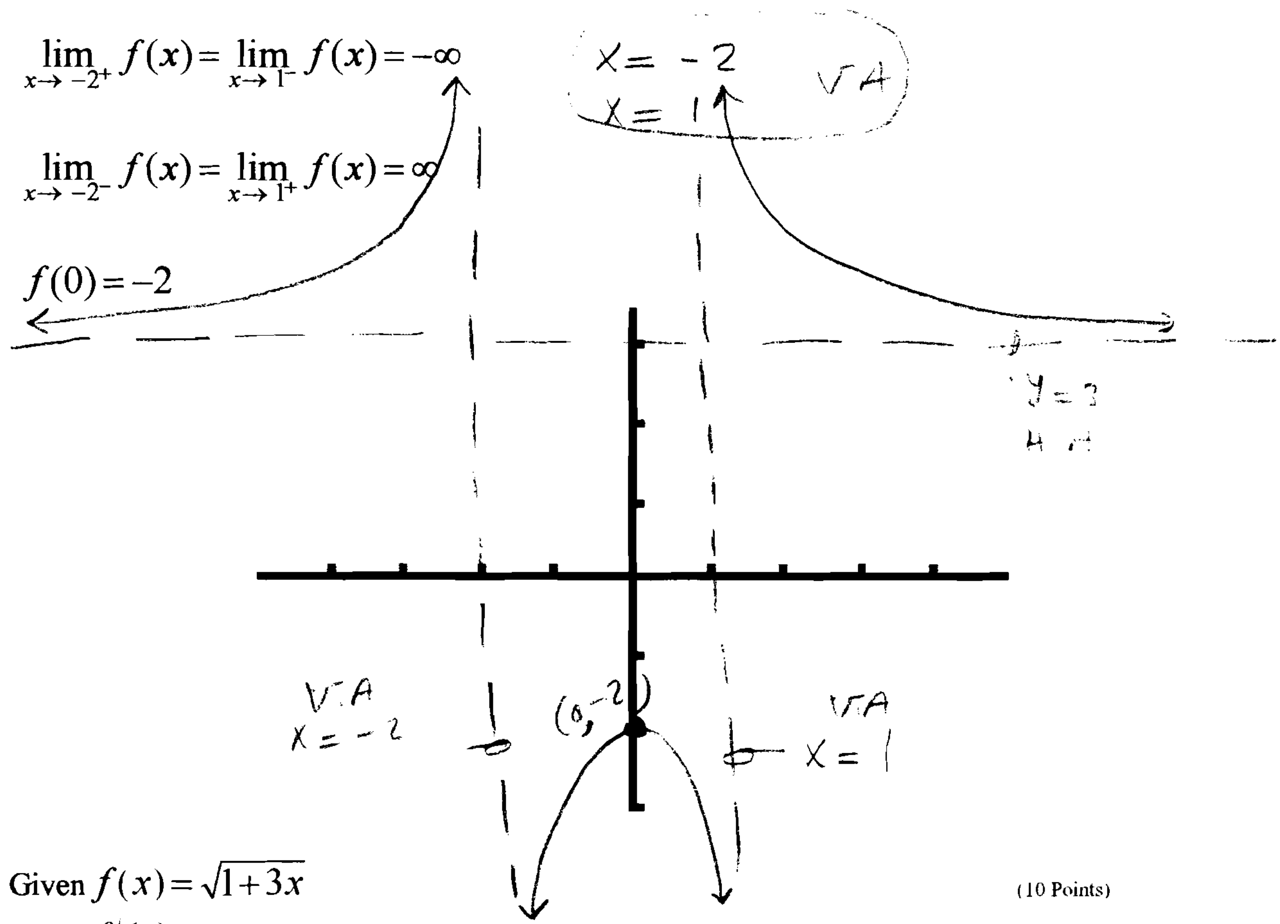
1) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (10 Points)

$$\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = 3$$ H.A $y = 3$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$$ $x = -2$, $x = 1$ VA

$$\lim_{x\to -2^-} f(x) = \lim_{x\to 1^+} f(x) = \infty$$

$$f(0) = -2$$



2) Given $f(x) = \sqrt{1+3x}$ (10 Points)

Find the $f'(x)$ using either of the two definitions discussed in class
(Must Use the Definition Of Derivative for Full Credits)

$$\lim_{h\to 0} \frac{\sqrt{1+3x+3h} - \sqrt{1+3x}}{h} \cdot \frac{\sqrt{1+3x+3h} + \sqrt{1+3x}}{\sqrt{1+3x+3h} + \sqrt{1+3x}}$$

$$= \lim_{h\to 0} \frac{1+3x+3h-(1+3x)}{h\left(\sqrt{1+3x+3h} + \sqrt{1+3x}\right)} = \lim_{h\to 0} \frac{3}{\sqrt{1+3x+3h} + \sqrt{1+3x}}$$

$$= \frac{3}{2\sqrt{1+3x}}$$

5) Find the derivative of the following functions: **(4 Points each)**

(Do Not Simplify)

a) $y=\sqrt[3]{x^7}+\dfrac{1}{\sqrt{x}} = x^{\frac{7}{3}} + x^{-\frac{1}{2}}$

$y' = \frac{7}{3}x^{\frac{4}{3}} - \frac{1}{2}x^{-\frac{3}{2}}$

b) $y=e^{\sec(2\theta)}$

$y' = e^{\sec(2\theta)} \cdot \sec(2\theta)\tan(2\theta)\cdot 2$

$= 2e^{\sec(2\theta)}\sec(2\theta)\tan(2\theta)$

c) $y=\sin^5(4x) = [\sin(4x)]^5$

$y' = 5(\sin(4x))^4\cos(4x)\cdot 4$

$= 20(\sin(4x))^4\cos(4x)$

d) $y=\tan((5x)^3)$

$y' = \sec^2((5x)^3)\cdot 3(5x)^2(5)$

$= 375x^2\sec^2(125x^3)$

$= 375x^2\sec^2((5x)^3)$

e) $y=\sqrt{2x+\sqrt{3x}} = (2x+(3x)^{1/2})^{1/2}$

$y' = \frac{1}{2}(2x+(3x)^{\frac{1}{2}})^{-\frac{1}{2}}\left(2+\frac{1}{2}(3x)^{-\frac{1}{2}}(3)\right)$

$= \frac{1}{2}(2x+(3x)^{\frac{1}{2}})^{-\frac{1}{2}}\left(2+\frac{3}{2}(3x)^{-\frac{1}{2}}\right)$

f) $y=\left(\dfrac{-3x-7}{2x^2-1}\right)^7$

$y' = 7\left(\frac{-3x-7}{2x^2-1}\right)^6\left(\frac{-3(2x^2-1)-4x(-3x-7)}{(2x^2-1)^2}\right)$

g) $y=10^{[\sin(3\theta)+\cos(2\theta)]}$

$y' = 10^{(\sin 3\theta+\cos 3\theta)}\cdot\ln 10\cdot(3\cos(3\theta)-2\sin(2\theta))$

h) $y=\sin(\sec(\sqrt{1+x^2}))$

$y' = \cos(\sec\sqrt{1+x^2})(\sec\sqrt{1+x^2})(\tan\sqrt{1+x^2})\cdot\frac{1}{2}(1+x^2)^{-\frac{1}{2}}(2x)$

i) $y=\sin^{-1}(x^2+2x+1)$

$y' = \dfrac{1}{\sqrt{1-(x^2+2x+1)^2}}\cdot(2x+2)$

j) $y=\ln(\frac{x^2}{2}+2x+1)$

$y' = \dfrac{1}{(\frac{x^2}{2}+2x+1)}\cdot(x+2)$

$2x^{\frac{1}{2}} + 4y^{\frac{1}{2}} = 14$

6) Find the equation of the tangent line to the curve $2\sqrt{x} + 4\sqrt{y} = 14$ at the point (9 , 4). (10 Points)

$y - 4 = \frac{-1}{3}(x - 9)$

$y = \frac{-1}{3}x + 3 + 4$

$\boxed{y = \frac{-1}{3}x + 7}$

Take Derivative

$x^{-\frac{1}{2}} + 2y^{-\frac{1}{2}}y' = 0$

$2y^{-\frac{1}{2}}y' = -x^{-\frac{1}{2}}$

$y' = \frac{-x^{-\frac{1}{2}}}{2y^{-\frac{1}{2}}} = \frac{-y^{\frac{1}{2}}}{2x^{\frac{1}{2}}}\Big|_{\text{at } x=9,\, y=4} = \frac{-4^{\frac{1}{2}}}{2(9)^{\frac{1}{2}}} = \frac{-2}{6} = \boxed{\frac{-1}{3}}$

7) Find the derivative of the function $y^{2x} = (3x)^{\cos x}$ (10 points)

Compute y' in terms of x, and y. **(Hint: Use Natural Logarithms)**

$2x \ln y = \cos x \ln(3x)$

$2\ln y + 2x\frac{1}{y}y' = -\sin x \ln 3x + \cos x \frac{1}{3x} \cdot 3$

$\boxed{y' = \left(-\sin x \ln 3x + \frac{\cos x}{x} - 2\ln y\right) \cdot \frac{y}{2x}}$

8) Suppose that $h(x) = g(x)f(x)$, and $F(x) = g(f(x))$, where
$f(-3) = 4$,
$g(-3) = 2$,
$g'(-3) = -1$,
$f'(-3) = -3$,
$g'(4) = -5$

$h'(-3) =$

a) Find $h'(-3) = g'(-3)f(-3) + g(-3)f'(-3)$ (5 Points)

$= (-1)(4) + (2)(-3)$

$= -4 - 6 = \boxed{-10}$

b) Find $F'(-3)$. $= g'(f(-3)) \cdot f'(-3)$ (5 Points)

$= g'(4) \cdot f'(-3)$

$= (-5)(-3) = \boxed{15}$

9) Find all values of x so that the graph of $f(x)=\sqrt{3}\ x+2\sin x$ will have a horizontal tangent?

(5 Points)

$f'(x) = \sqrt{3} + 2\cos x = 0$ $\qquad 120^\circ$ & 240°

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$$\boxed{\begin{aligned} x &= \frac{2\pi}{3} + 2n\pi \\ x &= \frac{4\pi}{3} + 2n\pi \end{aligned}}$$

10) Find the equation of the tangent line to the curve $y = x\cos x + x$, at the point $(\pi, 0)$.

(5 Points)

$y' = 1\cos x + x(-\sin x) + 1$

$y'\big|_{\text{at } x=\pi} = \cos\pi + \pi(-\sin\pi) + 1 = 0 \Rightarrow \boxed{m = 0}$

$y - y_1 = m(x - x_1)$

$y - 0 = 0(x - \pi)$

$\boxed{y = 0}$

11) A particle moves on a vertical line so that its coordinate at time t is

$s(t) = t^3 - 12t^2 + 3 \quad t \geq 0$

where S(t) is measured in meters and t is measured in seconds.

(10 Points)

$s'(t) = 3t^2 - 24t$

a) When is the particle moving backward?

when $s'(t) < 0$

$0 < t < 8$ seconds

b) Find the distance that the particle travels in the time interval $5 \leq t \leq 10$ seconds.

$s(5) = -172$ $\qquad s(8) = -253$

$s(8) = -253$ $\qquad s(10) = -197$

$|s_8 - s_5| = 81$ meters $\qquad |s_{10} - s_8| = 56$ meters

Total Distance $= 81 + 56 = 137$ meters

c) When is the particle slowing down?

When $s'(t)$ and $s''(t)$ Have Different Signs

$\boxed{4 < t < 8 \text{ Seconds}}$

12) Given $f(x) = -2e^x g(x) - 7x$
And $g(0) = 4$ and $f'(0) = -6$, find $g'(0)$.

(3 Points)

$f'(x) = -2e^x g(x) + -2e^x g'(x) - 7$

$-6 = -2e^0(4) - 2e^0 g'(0) - 7$

$-6 = -8 - 2e^0 g'(0) - 7$

$-6 = -15 - 2e^0 g'(0) \Longrightarrow 9 = -2g'(0)$ $\quad g'(0) = -\frac{9}{2}$

Prove that $\frac{d}{dx}(10\sec x) = 10\sec x \tan x$

(2 Points)

$\frac{d}{dx}\left(\frac{10}{\cos x}\right) = \frac{0\cos x + \sin x(10)}{\cos^2 x}$ $\quad \frac{10}{\cos x}\,\frac{\sin x}{\cos x}$

$= 10\sec x \tan x$

13) Find the linearization of $f(x) = \sqrt[3]{1+3x}$ at $a = 0$. (3 Points)

a) State the corresponding linear approximation.

$f'(x) = \frac{1}{3}(1+3x)^{-\frac{2}{3}}(3) = (1+3x)^{-\frac{2}{3}} \Longrightarrow f'(0) = 1$

$f(0) = 1$

$y = f'(0)(x-0) + f(0)$

$= 1(x-0) + 1 \Longrightarrow \boxed{y = x+1}$

i.e. $\sqrt[3]{1+3x} \approx x+1$

b) Use the above to give an approximate value for $\sqrt[3]{1.03}$ (2 Points)

$\sqrt[3]{1.03} = \sqrt[3]{1+3x}$

$1.03 = 1+3x$

$0.03 = 3x$

$0.01 = x$

$\sqrt[3]{1.03} \approx 0.01 + 1$

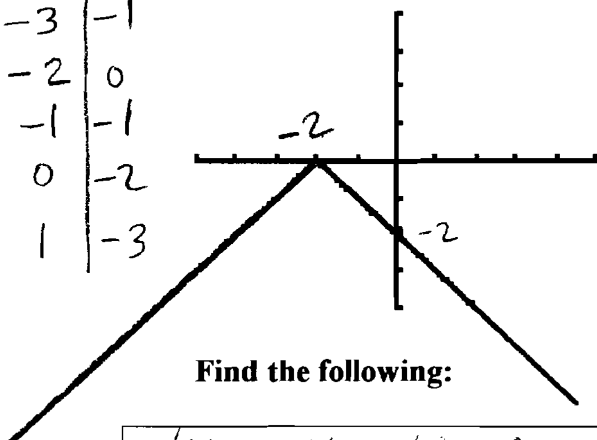
$\sqrt[3]{1.03} \approx 1.01$

Extra Credits

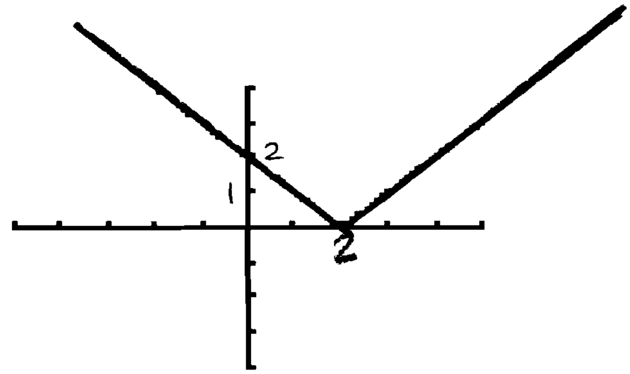
14) Given that $v(x) = \dfrac{f(x)}{g(x)}$, and $w(x) = f(x)g(x)$ (5 Points)

And graphs of f(x) and g(x)

x	$f(x)$
-3	-1
-2	0
-1	-1
0	-2
1	-3



x	$g(x)$
-1	3
0	2
2	0
3	1



Find the following:

$v'(0) = \dfrac{f'(0)g(0) - f(0)g'(0)}{g^2(0)} = \dfrac{(-1)(2) - (-2)(-1)}{2^2} = \dfrac{-2-2}{4} = \boxed{-1}$	$w'(-1) = f'(-1)g(-1) + f(-1)g'(-1) = (-1)(3) + (-1)(-1) = -3 + 1 = \boxed{-2}$

15) Given $x^3 + y^3 = 6xy^2$ **Find** y' **in terms of** x, and y.

(5 Points)

$$3x^2 + 3y^2y' = 6y^2 + 12xyy'$$

$$3y^2y' - 12xyy' = 6y^2 - 3x^2$$

$$y'(3y^2 - 12xy) = 6y^2 - 3x^2$$

$$y' = \boxed{\frac{6y^2 - 3x^2}{3y^2 - 12xy}} = \frac{\cancel{3}(2y^2 - x^2)}{\cancel{3}(y^2 - 4xy)} \text{ or } \frac{2y^2 - x^2}{y(y - 4x)}$$