Professor Katiraie Calculus I Spring 2008 Test IV Form A (chapters 1 -- 4)

Name Solution

Total Possible Points = 140

☺☺☺ **Plus 10 Points Extra Credit** ☺☺☺

1) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (6 Points)

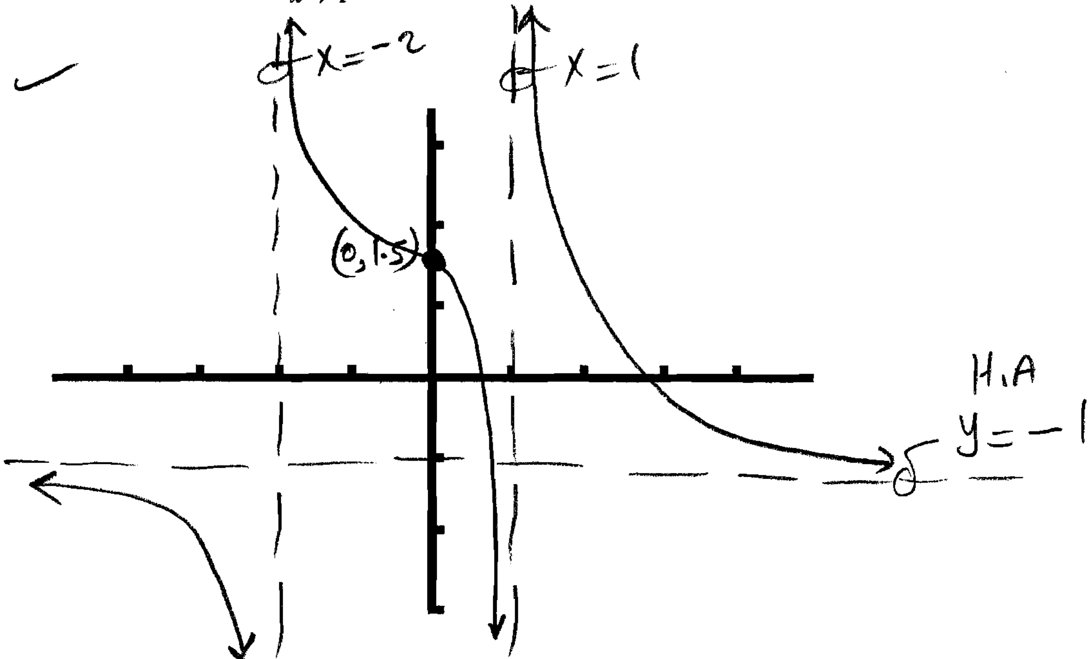
$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to-\infty} f(x) = -1$ $\Longrightarrow$ $y = -1$ H.A

$\lim\limits_{x\to-2^+} f(x) = \infty$ ✓ $\lim\limits_{x\to1^-} f(x) = -\infty$ ✓

$\left.\begin{array}{l} x = -2 \\ x = 1 \end{array}\right\}$ V.A

$\lim\limits_{x\to-2^-} f(x) = -\infty$ ✓ $\lim\limits_{x\to1^+} f(x) = \infty$ ✓

$f(0) = 1.5$ ✓



2) Find the equation of the tangent line to the curve $y = x\cos x + x\sin x$, at the point $(\pi, -\pi)$. (10 Points)

$y' = 1\cos x - x\sin x + 1\sin x + x\cos x$

at $x = \pi$ $y' = m = \cos\pi + \pi\cos(\pi) = (-1-\pi)$

$y - -\pi = (-1-\pi)(x-\pi)$

$y = (-1-\pi)x - \pi(-1-\pi) - \pi$

$= (-1-\pi)x \cancel{+\pi} + \pi^2 \cancel{-\pi}$

$= (-1-\pi)x + \pi^2$ or $y = -4.14x + 9.8696$

3) Find the derivative of the following functions:

(4 Points each)
(Do Not Simplify)

a) $y=\sqrt{2x+\sqrt{3x}}$

$y' = \frac{1}{2}\left(2x+\sqrt{3x}\right)^{-\frac{1}{2}}\left(2+\frac{1}{2}(3x)^{-\frac{1}{2}}(3)\right)$

$y' = \frac{1}{2}\left(2x+\sqrt{3x}\right)^{-\frac{1}{2}}\left(2+\frac{3}{2}(3x)^{-\frac{1}{2}}\right)$

b) $y=e^{\sec(2\theta)}$

$y' = e^{\sec(2\theta)} \cdot (\sec 2\theta)\tan(2\theta) \cdot 2$

c) $y=\sin^5(4x)$

$y' = 5\sin^4(4x)\cos(4x)\cdot 4$

d) $y=\sin^{-1}(x^2+2x+1)$

$y' = \frac{1}{\sqrt{1-(x^2+2x+1)^2}} \cdot (2x+2)$

e) $y=10^{[\sin(3\theta)+\cos(2\theta)]}$

$y' = 10^{[\sin(3\theta)+\cos(2\theta)]} \cdot \ln 10 \cdot \left(3\cos(3\theta) - 2\sin(2\theta)\right)$

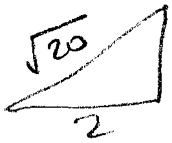
f) $y=\left(\frac{-3x-7}{2x^2-1}\right)^7$

$y' = 7\left(\frac{-3x-7}{2x^2-1}\right)^6\left(\frac{-3(2x^2-1)-4x(-3x-7)}{(2x^2-1)^2}\right)$

For Problems 4 - 6: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.

4) A particle starts at the origin and moves along the parabola $y = x^2$ such that its distance from the origin increases at 3 units per second. How fast is its x-coordinate changing as it passes through the point (2 , 4)? (10 points)

$$D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \Rightarrow D^2 = (x-0)^2 + (y-0)^2$$



$$D^2 = x^2 + y^2 \Rightarrow D^2 = x^2 + (x^2)^2 = x^2 + x^4$$

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 4x^3\frac{dx}{dt}$$

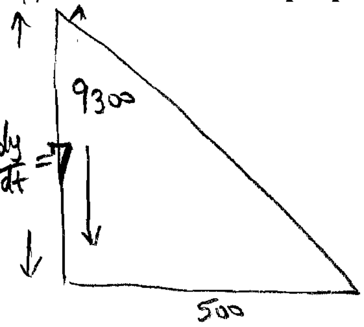
$$D\frac{dD}{dt} = x\frac{dx}{dt} + 2x^3\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{D\frac{dD}{dt}}{x+2x^3} = \frac{\sqrt{20}(3)}{2+2(2)^3} = \boxed{\frac{3\sqrt{20}}{18}\ \frac{\text{units}}{\text{sec}}}$$

$$= \frac{\sqrt{20}}{6} = \boxed{\frac{2\sqrt{5}}{6}}$$

$3\frac{ft}{sec} * \frac{60\ sec}{1\ min} * 25\ min = 4500$

$4 * 60 * 20 = 4800$

5) A man starts walking north at 3 ft/s from a point P. Five minutes later a woman starts walking south at 4 ft/s from a point 500 ft due east of P. At what rate are the people moving apart 20 minutes after the woman starts walking? (10 points)



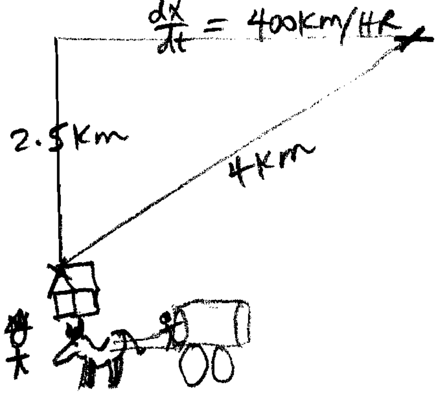
$$c^2 = x^2 + y^2$$

$$\cancel{2}c\frac{dc}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$$

0

$$\frac{dc}{dt} = \frac{(9300)(7)}{\sqrt{500^2+9300^2}} = \boxed{6.99\ \text{ft/sec}}$$

6) A plane flying horizontally at an altitude of 2.5 km and a speed of 400 km/h passed directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 4 km away from the station (10 points)



$$x^2 + y^2 = c^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}c\frac{dc}{dt}$$

$$(3.122)(400) + 0 = 4\frac{dc}{dt}$$

$x^2 + 2.5^2 = 4^2$

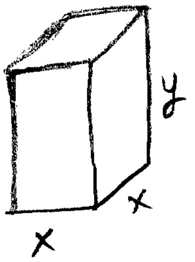
$x = 3.122$

$$\frac{dc}{dt} = (400)(3.122) \div 4$$

$$\boxed{= 312.2\ Km/HR}$$

For problems 7 - 9: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.

7) If 1400 sq. cm of material is available to make a box with a square base and an open top, find the largest possible volume of the box. (10 points)



$$x^2 + 4xy = 1400$$

$$y = \frac{1400 - x^2}{4x}$$

$$\text{Volume} = x^2 y$$

$$= x^2 \left(\frac{1400 - x^2}{4x}\right)$$

$$\text{Volume} = (0.25x)(1400 - x^2) = 350x - 0.25x^3$$

$$\frac{d\,\text{Volume}}{dx} = 350 - 0.75x^2 = 0 \qquad 0.75x^2 = 350$$

$$x^2 = 466.67 \Rightarrow x \cong 21.6 \text{ cm}$$

Max Volume $\cong$ 5040.58 Cm3

8) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point (2,0) (10 points)

$$y^2 = 4 - 4x^2$$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D^2 = \sqrt{(x-2)^2 + (y-0)^2}$$

$$D^2 = (x-2)^2 + 4 - 4x^2$$

$$\frac{dD}{dx} = 2(x-2)' - 8x$$

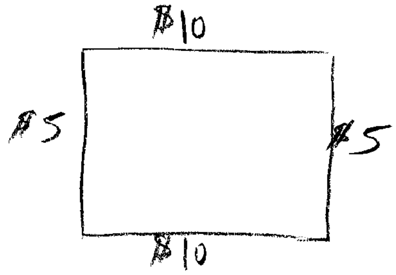
$$2x - 4 - 8x = 0$$

$$-6x = 4$$

$$\boxed{x = \frac{4}{-6} = \frac{-2}{3} \;;\; y = \pm 1.491}$$

$$\left(-\frac{2}{3}, 1.491\right)$$

9) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$10 per foot for two opposite sites, and \$5 per foot for the other two sides. Find the dimensions of the field of area 730 sq. ft. that would be the cheapest to enclose. (10 points)



$$\text{Cost} = 10x + 10x + 5y + 5y$$

$$x \cdot y = 730$$

$$y = \frac{730}{x}$$

$$\text{Cost} = 20x + 10\left(\frac{730}{x}\right)$$

$$\text{Cost} = 20x + 7300x^{-1}$$

$$\text{Cost}' = 20 - 7300x^{-2} = 0$$

$$\frac{7300}{x^2} = 20 \Rightarrow 20x^2 = 7300$$

x = 19.10 feet

y = 38.21 feet

10) Analytically find the exact value of all critical numbers of the following functions. (In other words, find the x-coordinates of the critical points.) (12 points)

a) $y = x^{\frac{4}{5}}(x-4)^2$	b) $y = x^{\frac{2}{3}}(x^2-4)$
$y' = \frac{4}{5}x^{-\frac{1}{5}}(x-4)^2 + x^{\frac{4}{5}}(2)(x-4)$	$y' = \frac{2}{3}x^{-\frac{1}{3}}(x^2-4) + x^{\frac{2}{3}}(2x)$
$= \frac{4(x-4)^2}{5x^{\frac{1}{5}}} + (2x-8)(x)^{\frac{4}{5}}$	$= \frac{2(x^2-4)}{3x^{\frac{1}{3}}} + (2x)(x^{\frac{2}{3}})$
$= \frac{4(x^2-8x+16)+5x(2x-8)}{5x^{\frac{1}{5}}}$	$= \frac{2x^2-8+6x^2}{3x^{\frac{1}{3}}}$
$= \frac{4x^2-32x+64+10x^2-40x}{5x^{1/2}}$	$f'(x) = \frac{8x^2-8}{3x^{\frac{1}{3}}}$
$= \frac{14x^2-72x+64}{5x^{1/2}}$ $x=0$, $x=1.143$, $x=4$	$x=0$, $x=\pm 1$

$x = \frac{72 \pm \sqrt{72^2-4(14)(64)}}{28} = \frac{72 \pm 40}{28} =$

11) Find all value(s) of c (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{1+x}$ on the interval $[0,1]$ (5 points)

$f(x) = (1+x)^{-1}$

$f'(x) = -1(1+x)^{-2}$

$\frac{-1}{(1+x)^2} = \frac{f(1)-f(0)}{1-0} \Rightarrow \frac{-1}{(1+x)^2} = \frac{0.5-1}{1-0} \Rightarrow \frac{-1}{(1+x)^2} = \frac{-0.5}{1}$

$\frac{1}{(x+1)^2} = \frac{1}{2} \Rightarrow 2 = (x+1)^2 \quad x+1 = \pm\sqrt{2}$

$x = -1 + \sqrt{2}$

Answer $c = -1 + \sqrt{2}$

12) Given that the function $f(x) = x^3 + ax^2 + bx$ has critical numbers at $x = 1$, and $x = -2$, find a and b. (5 points)

$f'(x) = 3x^2 + 2ax + b$

$f'(1) = 3 + 2a + b = 0$

$f'(-2) = 3(-2)^2 + 2a(-2) + b = 0$

$\xrightarrow{+2} \begin{cases} 3 + 2a + b = 0 \\ 12 - 4a + b = 0 \end{cases}$ $\quad \begin{cases} +6 + 4a + 2b = 0 \\ 12 - 4a + b = 0 \end{cases}$

$18 + 3b = 0$

$b = -6$

$3 + 2a - 6 = 0$

$2a = 3$

$a = \frac{3}{2}$

$f'(x) = 5x^4 e^{-x} - e^{-x} x^5$

13) Find all the points of inflection of $f(x) = x^5 e^{-x}$ (5 points)
(Must Justify Your Answer)

$f''(x) = 20x^3 e^{-x} - 5x^4 e^{-x} + e^{-x} x^5 - 5e^{-x} x^4 = 0$

$\boxed{\begin{array}{l} x=0 \\ x = 5 \pm \sqrt{5} \end{array}}$

$+e^{-x} [-10x^4 + 20x^3 + x^5) = 0$

$e^{-x} x^3 [x^2 - 10x + 20] = 0$

$x = 0$

$x = \frac{10 \pm \sqrt{100 - 4(1)(20)}}{2} = \frac{10 \pm \sqrt{20}}{2}$

$\frac{10 \pm 2\sqrt{5}}{2} = 5 \pm \sqrt{5}$

14) A company has cost function $C(x) = 84 + 1.26x - 0.01x^2 + 0.00007x^3$ and demand function $p(x) = 3.5 - 0.01x$, where x is the number of staplers and $p(x)$ is in dollars.

a. How many units should the company make to maximize its profit? (5 points)

$R(x) = x(3.5 - 0.01x) = 3.5x - 0.01x^2$

$\text{Profit} = R - C = \cancel{(3.5x - 0.01x^2)} - (84 + 1.26x - \cancel{0.01x^2} + 0.00007x^3)$

$\text{Profit} = -0.00007x^3 + 2.24x - 84$

$\text{Profit}' = -0.00021x^2 + 2.24 = 0$

$x = 103.28 \approx 104$ units

b. How much is the maximum profit? (3 points)

Max Profit $\approx \$70.22$

c. What price would produce maximum profit? (2 points)

price $= 3.5 - 0.01(104) \approx \$2.46$

15) Given $f'(x) = 2\sqrt{x} \bullet (6 - 5x)$ and $f(1) = 10$; Find $f(x)$ (5 points)

$f'(x) = 12x^{1/2} - 10x^{\frac{3}{2}}$

$f(x) = \frac{12x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{10x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$f(x) = 8x^{\frac{3}{2}} - 4x^{\frac{5}{2}} + c$

$f(1) = 10 \Rightarrow 10 = 8 - 4 + c$

$6 = c$

$f(x) = 8x^{\frac{3}{2}} - 4x^{\frac{5}{2}} + 6$

16) Given $f''(x) = 2x^{-2}$, $x > 0$, $f(1) = 1$, $f(2) = 0$

Find $f(x)$ (8 points)

$f'(x) = \frac{2x^{-1}}{-1} + C = \frac{-2}{x} + C$

$f(x) = -2\ln x + Cx + D \Longrightarrow f(x) = 2\ln x + (-1 - 2\ln 2)x + (2 + 2\ln 2)$

$f(1) = 1 \Rightarrow 1 = 2\ln 1 + C + D \Rightarrow -2\begin{cases} C + D = 1 \\ 2C + D = +2\ln 2 \end{cases}$

$f(2) = 0 \Rightarrow 0 = 2\ln 2 + 2C + D \Rightarrow \begin{cases} \cancel{-2C} - 2D = -2 \\ \cancel{2C} + D = +2\ln 2 \end{cases}$

$f(x) = 2\ln x - 0.386x + 0.614$

$-D = -2 + 2\ln 2$

$D = 2 - 2\ln 2 = 0.614$

$C = 1 - D = 1 - (2 - 2\ln 2)$

$C = -1 + 2\ln 2$

17) A pumpkin pie is thrown upward with a speed of 25 ft/sec from the edge of a cliff 200 feet above the ground. (Assume gravity of earth is -32)

a) Find the pie's height above the ground t seconds later. (4 points)

$a = -32$

$v = -32t + 25$

$s(t) = -16t^2 + 25t + 200$

b) When does the pie reach its maximum height? (3 points)

Velocity $= 0$

$-32t + 25 = 0$

$t = 0.78$ sec

c) When does the pie hit the ground? (3 points)

$-16t^2 + 25t + 200 = 0$

$t = \frac{-25 \pm \sqrt{(25)^2 - 4(-16)(200)}}{2(-16)} = 4.40$ seconds

Bonus Question:

18) A particle moves along a path described by

$$4y = 4 - 3x^2.$$

At what point(s) along the curve are x and y changing at the same rate?

(5 points)

$$4\frac{dy}{dt} = -6x\frac{dx}{dt}$$

But $\frac{dy}{dt} = \frac{dx}{dt}$

$$4\frac{dy}{dt} + 6x\frac{dy}{dt} = 0$$

$$\frac{dy}{dt}(4+6x) = 0$$

$$6x = -4$$

$$x = -\frac{2}{3}$$

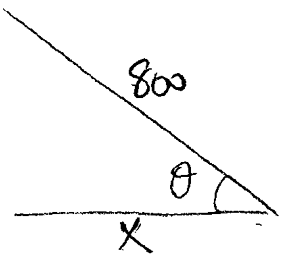
$$y = \frac{4-3x^2}{4}$$

$$y = +\frac{2}{3}$$

19) The angle of elevation of the Sun is decreasing at a rate of 0.25 rad/hour. How fast is the shadow cast by a 400–foot–tall building increasing when the angle of elevation of the Sun is $\frac{\pi}{6}$

(5 points)

$$\frac{d\theta}{dt} = -0.25\ \frac{\text{radians}}{\text{HR}}$$



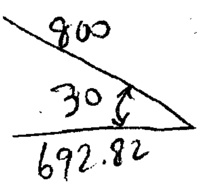
$$\tan\theta = \frac{400}{x}$$

$$\frac{\pi}{6} = \frac{400}{x}$$

$$= 692.82$$

$$\tan\theta = 400x^{-1}$$, now take derivative of both sides of the equation

$$\sec^2\theta\,\frac{d\theta}{dt} = -400x^{-2}\frac{dx}{dt}$$



$$\left(\frac{800}{692.82}\right)^2(-0.25) = \frac{-400}{(692.82)^2}\left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} = 400\ \text{ft/sec}$$

Professor Katiraie Calculus I Spring 2008 Test IV Form B (chapters 1 -- 4)

Name Solution Total Possible Points = 140

☺☺☺ **Plus 10 Points Extra Credit** ☺☺☺

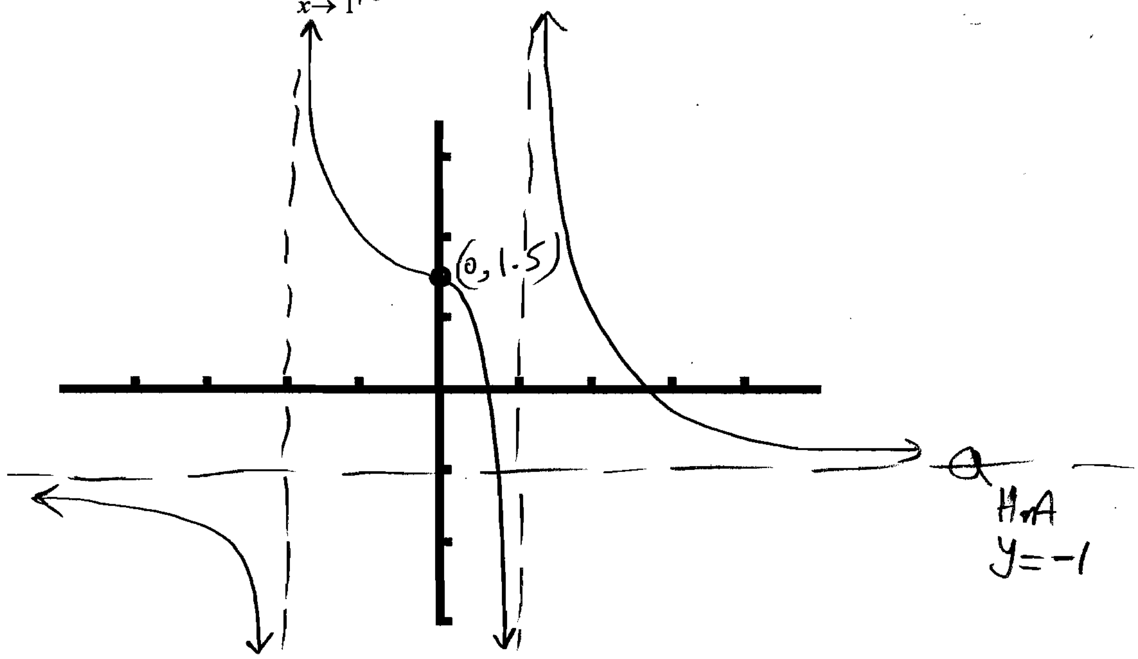
1) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (6 Points)

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = -1$

$\lim_{x\to-2^+} f(x) = \infty$ $\lim_{x\to1^-} f(x) = -\infty$

$\lim_{x\to-2^-} f(x) = -\infty$ $\lim_{x\to1^+} f(x) = \infty$

$f(0) = 1.5$



2) Find the equation of the tangent line to the curve $y = x\cos x + x\sin x$, at the point $(\pi, -\pi)$. (10 Points)

$y' = 1\cos x - x\sin x + 1\sin x + x\cos x$

at $x = \pi$ $y' = m = \cos\pi - \cancel{\pi\sin\pi} + \cancel{\sin\pi} + \pi\cos\pi$

$m = -1 - \pi$

$y - -\pi = (-1-\pi)(x-\pi)$

$y + \pi = (-1-\pi)x - \pi(-1-\pi)$

$y + \pi = (-1-\pi)x + \pi + \pi^2$

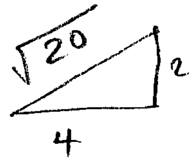
$y = (-1-\pi)x + \pi^2$

3) Find the derivative of the following functions: **(4 Points each)**

(Do Not Simplify)

a) $y=\sqrt{2x+\sqrt{3x}}$ $y'=\frac{1}{2}(2x+\sqrt{3x})^{-\frac{1}{2}}(2+\frac{1}{2}(3x)^{-\frac{1}{2}}(3))$ $y'=\frac{1}{2}(2x+\sqrt{3x})^{-\frac{1}{2}}(2+\frac{3}{2}(3x)^{-\frac{1}{2}})$	b) $y=e^{\sec(2\theta)}$ $y'=e^{\sec(2\theta)}\sec(2\theta)\tan(2\theta)\cdot 2$
c) $y=\sin^5(4x)$ $y'=5\sin^4(4x)\cos(4x)\cdot 4$	d) $y=\sin^{-1}(x^2+2x+1)$ $y'=\frac{1}{\sqrt{1-(x^2+2x+1)^2}}(2x+2)$
e) $y=10^{[\sin(3\theta)+\cos(2\theta)]}$ $y'=10^{[\sin 3\theta+\cos 2\theta]}\cdot\ln 10\cdot(3\cos(3\theta)-2\sin(2\theta))$	f) $y=\left(\frac{-3x-7}{2x^2-1}\right)^7$ $y'=7\left(\frac{-3x-7}{2x^2-1}\right)^6\left(\frac{-3(2x^2-1)-4x(-3x-7)}{(2x^2-1)^2}\right)$

For Problems 4 - 6: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.



4) A particle is moving along the curve $y=\sqrt{x}$. As the particle passes through the point (4 , 2), its x-coordinate increases at a rate of 3 cm/sec. How fast is the distance from the particle to the origin changing at this instant? (10 points)

$$\text{Distance} = \sqrt{(x_2-0)^2+(y-0)^2}$$

$$D^2 = (x_2-0)^2+(y-0)^2$$

$$D^2 = x^2+x$$

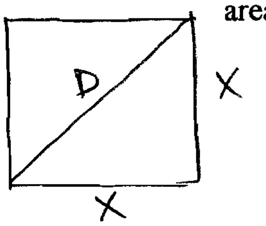
$$2D\frac{dD}{dt} = 2x\frac{dx}{dt}+\frac{dx}{dt}$$

$$2\sqrt{20}\,\frac{dD}{dt} = 2(4)(3)+3$$

$$\frac{dD}{dt} = \frac{24+3}{2\sqrt{20}} = \frac{27}{4\sqrt{5}} \text{ cm/sec}$$

$$= \boxed{\frac{27\sqrt{5}}{20}}$$

5) If the diagonal of a square decreases at the rate of 2 inch/second, how fast is the area changing when the side of the square is 15 inches? (10 points)



$$D^2 = x^2+x^2$$

$$D^2 = 2x^2$$

$$x^2 = \frac{D^2}{2}$$

$$A = x\,x$$

$$A = x^2$$

$$A = \frac{D^2}{2}$$

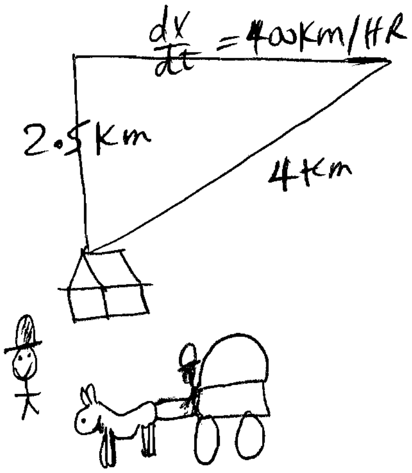
$$\frac{dA}{dt} = \frac{1}{2}\cdot 2D\frac{dD}{dt}$$

$$\frac{dA}{dt} = \sqrt{15^2+15^2}\,(-2) = -2(15\sqrt{2})$$

$$= -30\sqrt{2}\left(\frac{inch^2}{sec}\right)$$

$$= \boxed{-42.4\,\frac{in^2}{sec}}$$

6) A plane flying horizontally at an altitude of 2.5 km and a speed of 400 km/h passed directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 4 km away from the station (10 points)



$$x^2+y^2=c^2$$

$$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt} = \cancel{2}c\frac{dc}{dt}$$

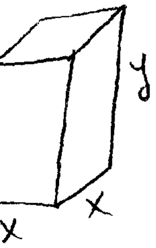
$$3.122(400)+2.5\frac{dy}{dt} = 4\frac{dc}{dt}$$

0

$$x^2+2.5^2=4^2$$

$$X=3.122$$

$$\frac{dc}{dt} = \frac{400(3.122)}{4} = 312.2 \text{ Km/HR}$$



For problems 7 - 9: Pick two of the following three problems. Clearly indicate the two problems you want to be graded.

7) If 1400 sq. cm of material is available to make a box with a square base and an open top, find the largest possible volume of the box. (10 points)

$= x^2 + 4xy$

$1400 = x^2 + 4xy$

$y = \frac{1400 - x^2}{4x}$

$\text{Volume} = x^2 y$

$V = x^2\left(\frac{1400 - x^2}{4x}\right) = \frac{1400x^2}{4x} - \frac{x^4}{4x}$

$V = 350x - 0.25x^3$

$\frac{dV}{dx} = 350 - 0.75x^2 = 0$

$x^2 = 466.666$

$X = 21.6 \text{ cm}$ $V_{max} = 5040. \text{ cm}$

8) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point (2,0) (10 points)

$\text{Distance} = \sqrt{(x-2)^2 + (y-0)^2}$

$D^2 = (x-2)^2 + y^2$

$D^2 = (x-2)^2 + 4 - 4x^2$

$2D\frac{dD}{dx} = 2(x-2) - 8x$

$\frac{dD}{dt} = \frac{2x - 4 - 8x}{2D} = 0$

$-6x - 4 = 0$

$\left(x = \frac{-4}{6} = -\frac{2}{3} \; ; \; y = \pm 1.49\right)$

9) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$10 per foot for two opposite sites, and \$5 per foot for the other two sides. Find the dimensions of the field of area 730 sq. ft. that would be the cheapest to enclose. (10 points)

$\text{Cost} = 10x + 10x + 5y + 5y$ and $xy = 730$

$y = \frac{730}{x}$

$\text{Cost} = 20x + 10\left(\frac{730}{x}\right)$

$\text{Cost} = 20x + 7300x^{-1}$

$\text{Cost}' = 20 - \frac{7300}{x^2} = 0 \implies \frac{7300}{x^2} = 20$

$20x^2 = 7300 \implies x^2 = \frac{7300}{20}$

$x = 19.10 \text{ feet} \; ; \; y = 38.21 \text{ feet}$

10) Analytically find the exact value of all critical numbers of the following functions.
(In other words, find the x-coordinates of the critical points.) **(12 points)**

$y = x^{\frac{4}{5}}(x-4)^2$

$y' = \frac{4}{5}x^{-\frac{1}{5}}(x-4)^2 + x^{\frac{4}{5}}(2)(x-4)$

$= \frac{4(x-4)^2}{5x^{\frac{1}{5}}} + (2x-8)x^{\frac{4}{5}}$

$= \frac{4(x^2-8x+16) + 5x(2x-8)}{5x^{\frac{1}{5}}}$

$= \frac{4x^2-32x+64+10x^2-40x}{5x^{1/2}}$

$= \frac{14x^2-72x+64}{5x^{1/2}}$

$x = \frac{72 \pm \sqrt{72^2-4(14)(64)}}{28} = \frac{72 \pm 40}{28}$

$x = 0$
$x = 1.143$
$x = 4$

11) Find all value(s) of c (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = (x-3)^3$ on the interval $[0,1]$ **(5 points)**

$f'(x) = 3(x-3)^2$

$3(c-3)^2 = \frac{f(1)-f(0)}{1-0}$

$3(c-3)^2 = \frac{-8 - -27}{1}$

$(c-3)^2 = \frac{19}{3}$

$c-3 = \pm\sqrt{\frac{19}{3}}$

$c = 3 \pm \sqrt{\frac{19}{3}}$

$\cancel{3+\sqrt{\frac{19}{3}}}$ not in $[0,1]$

$3-\sqrt{\frac{19}{3}}$ only solution

12) Given that the function $f(x) = x^3 + ax^2 + bx$ has critical numbers at $x = 1$, and $x = -2$, find a and b. **(5 points)**

$f'(x) = 3x^2 + 2ax + b$

$f'(1) = 3 + 2a + b = 0$

$f'(-2) = 3(-2)^2 + 2a(-2) + b = 0$

$2\begin{cases} 3 + 2a + b = 0 \\ 12 - 4a + b = 0 \end{cases}$

$\begin{cases} 6 + 4a + 2b = 0 \\ 12 - 4a + b = 0 \end{cases}$

$18 + 3b = 0 \Rightarrow b = -6$

$3 + 2a - 6 = 0$

$2a = 3$

$a = 3/2$

$f'(x) = 5x^4 e^{-x} - x^5 e^{-x}$

13) Find all the points of inflection of $f(x) = x^5 e^{-x}$ (Must Justify Your Answer)

$(0,0)$

$(5-\sqrt{5}, 10.170)$

$(5+\sqrt{5}, 14.287)$

$f''(x) = 20x^3 e^{-x} - 5x^4 e^{-x} - 5x^4 e^{-x} + x^5 e^{-x}$

$e^{-x}(-10x^4 + 20x^3 + x^5) = 0$

$x = 0$

$x = 5 \pm \sqrt{5}$

$e^{-x} x^3 (x^2 - 10x + 20) = 0$

$x = \frac{10 \pm \sqrt{100 - 4(1)(20)}}{2} = \frac{10 \pm \sqrt{100-80}}{2} = \frac{10 \pm \sqrt{20}}{2} = \frac{10 \pm 2\sqrt{5}}{2}$

$= 5 \pm \sqrt{5}$

14) A company has cost function $C(x) = 100 - 14x + x^2$ and demand function $p(x) = 18 - x$, where x is the number of staplers and $p(x)$ is in dollars.

a. How many units should the company make to maximize its profit? (5 points)

$R(x) = xp = x(18-x) = 18x - x^2$

Profit $=$ Revenue $-$ Cost $= 18x - x^2 - (100 - 14x + x^2) = -2x^2 + 32x - 100$

Profit$' = -4x + 32 = 0 \Rightarrow x = 8$ units

b. How much is the maximum profit? (3 points)

Profit $= -2(8)^2 + 32(8) - 100 =$ 28 dollars

c. What price would produce maximum profit? (3 points)

Price $= 18 - x = 18 - 8 =$ 10 dollars

15) Given $f''(x) = 2x^{-2}$, $x > 0$, $f(1) = 1$, $f(2) = 0$ Find $f(x)$ (12 points)

$f'(x) = +\frac{2x^{-1}}{-1} + c = \frac{-2}{x} + c$

$f(x) = 2\ln x + cx + D$

$f(1) = 1 \Rightarrow 1 = 2\ln 1 + c + D \Rightarrow c + D = 1$

$f(2) = 0 \Rightarrow 0 = 2\ln 2 + c(2) + D$

$\boxed{f(x) = -2\ln x + 0.386x + 0.614}$

$-2\begin{cases} c + D = 1 \\ 2c + D = +2\ln 2 \end{cases}$

$\begin{cases} \cancel{-2c} - 2D = -2 \\ \cancel{2c} + D = +2\ln 2 \end{cases}$

$-D = -2 + 2\ln 2$

$D = 2 - 2\ln 2$

$\boxed{D = 0.614}$ $\boxed{c = \pm 0.386}$

16) A pumpkin pie is thrown upward with a speed of 20 ft/sec from the edge of a cliff 150 feet above the ground. (Assume gravity of earth is -32)

$a = -32$

$v = -32t + 20$

$s(t) = -\frac{32t^2}{2} + 20t + 150$

a) Find the pie's height above the ground t seconds later. (4 points)

$s(t) = -\frac{32t^2}{2} + 20t + 150$

$s(t) = -16t^2 + 20t + 150$

b) When does the pie reach its maximum height? (3 points)

When Velocity is Zero

$-32t + 20 = 0$

$-32t = -20$

$t = 0.625$ sec

c) When does the pie hit the ground? (3 points)

$-16t^2 + 20t + 150 = 0$

$t = \frac{-20 \pm \sqrt{(20)^2 - 4(-16)(150)}}{-32}$

~~-2.5 seconds~~

$\boxed{3.75 \text{ seconds}}$

Bonus Question:

18) Find the **equation of the tangent line** to the parametric curve $x = t^2 + 3,\quad y = 2t^3 - t$ at the point corresponding to $t = 2$.

(5 points)

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \left.\frac{6t^2 - 1}{2t}\right|_{t=2} = \frac{23}{4}$$

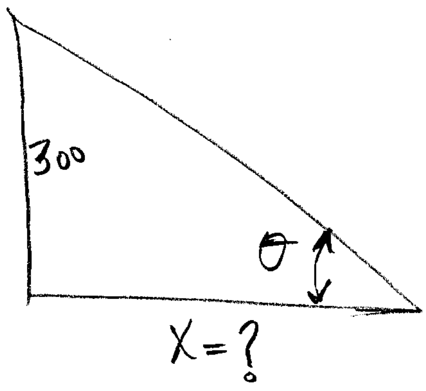
$$X = 7\ ;\quad y = 14$$

$$y - 14 = \frac{23}{4}(x - 7)$$

$$y = \frac{23}{4}X - 26.25$$

19) The angle of elevation of the Sun is decreasing at a rate of 0.25 radians/hour. How fast is the shadow cast by a 300–foot–tall building increasing when the angle of elevation of the Sun is $\frac{\pi}{3}$

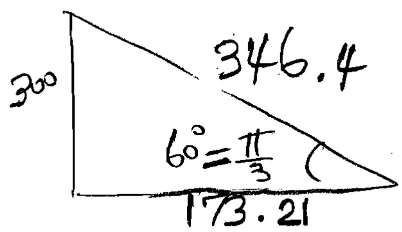
(5 points)



$$\frac{d\theta}{dt} = -0.25\ \frac{\text{radians}}{\text{sec}}$$

$$\tan\theta = \frac{300}{X} = 300X^{-1}$$

$$\sec^2\theta\,\frac{d\theta}{dt} = -300X^{-2}\frac{dX}{dt}$$



$$\tan 60° = \frac{300}{X}$$

$$X = 173.21$$

$$\frac{dx}{dt} = \frac{\sec^2\theta\,\frac{d\theta}{dt}}{-300X^{-2}}$$

$$\frac{dx}{dt} = \frac{(2)^2(-0.25)}{(-300)(173.21)^{-2}} = 100\ \frac{ft}{sec}$$