Professor Katiraie Calculus I Spring 2008 Form A Test I (chapter 1)

Name: _____

Total Possible Points = 140
(Plus 10 pts Extra Credit)

(16 Points)

1) For the rational function: $f(x)=\frac{5x-1}{x-2}$ determine the following: (2 pts ea)

a) The x-intercept(s) of f(x)

$(\frac{1}{5}, 0)$ $\quad 0=\frac{5x-1}{x-2} \Rightarrow 5x-1=0 \quad x=\frac{1}{5}$

b) The y-intercept(s) of f(x)

$f(0)=\frac{-1}{-2}=\frac{1}{2}$ $\quad (0,\frac{1}{2})$

c) The equation of any vertical asymptote(s)

$x=2$

d) The equation of any horizontal asymptote(s)

$y=5$

e) The domain of f(x)

All Real Numbers except 2

f) The range of f(x)

All Reals except 5

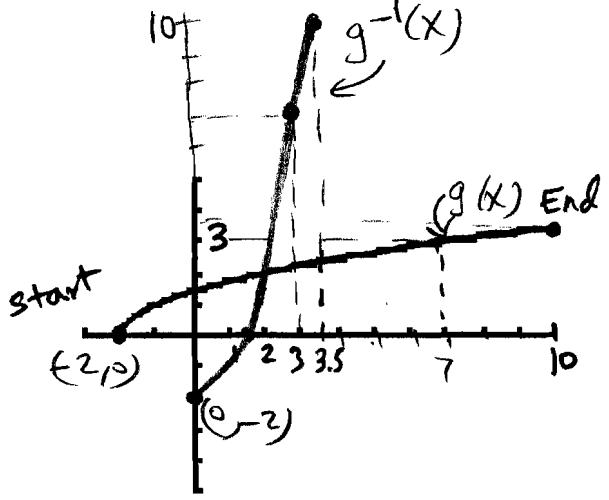
g) Complete the following: As $x\to\infty$, $f(x)\to$ 5

As $x\to-\infty$, $f(x)\to$ 5

2) Find a rational function with vertical asymptotes at $x=\pm5$, a horizontal asymptote at $y=3$ and a y-intercept at 4.

(8 pts)

$$f(x)=\frac{3x^2-100}{(x+5)(x-5)}$$



(10 Points)

3)The graph of g(x) is given above

a) State the value of g(7) $= 3$

b) Why is g one-to-one?

Because it passes the Horizontal line test.

c) Estimate the value of $g^{-1}(3)$?

$g^{-1}(3) = 7$

d)Estimate the domain of $g^{-1}(x)$

$[0, 3.5]$

d) Sketch the graph of $g^{-1}(x)$

See above

4) Determine whether f is even, odd, or neither even nor odd (10 Points)

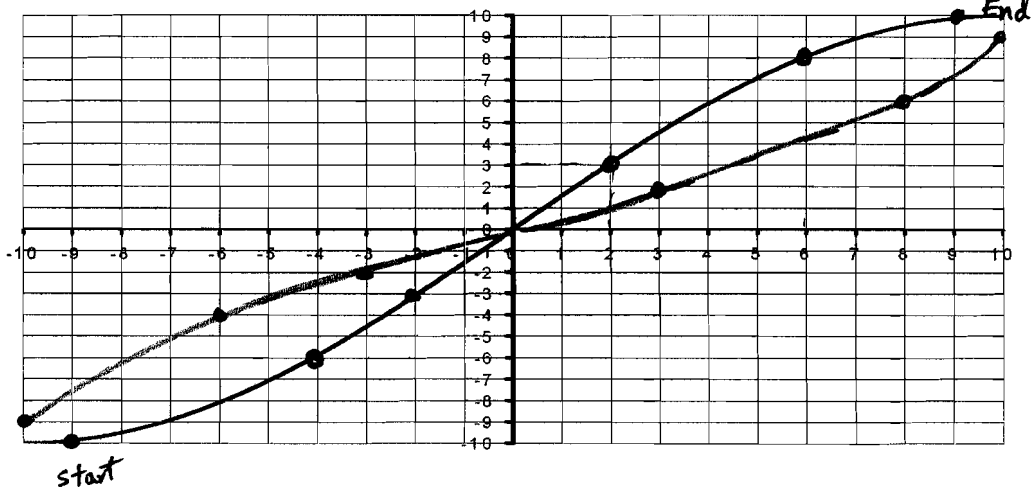
a) $f(x) = 3x^5 - 4x^3 + 3x + 1$

b) $f(x) = e^{x^2} + \cos(x)$

c) $f(x) = x + \sin(x)$

d) $f(x) = x^4 + 2x^2$

e) $f(x) = |x| + 4$



3) The graph of g(x) is given above (10 Points)

a) State the value of g(6) $= 8$

b) Why is g one-to-one?

Because it passes the Horizontal line test

c) Estimate the value of $g^{-1}(3)$?

$g^{-1}(3) = 2$

d) Estimate the domain of $g^{-1}(x)$

$[-10, 10]$

d) Sketch the graph of $g^{-1}(x)$

See above

4) Determine whether f is even, odd, or neither even nor odd (10 Points)

a) $f(x) = 3x^5 - 4x^3 + 3x + 1$

$f(-x) = 3(-x)^5 - 4(-x)^3 + 3(-x) + 1 = -3x^5 + 4x^3 - 3x + 1 \neq f(x)$

$\neq -f(x)$

Neither

b) $f(x) = e^{x^2} + \cos(x)$

$f(-x) = e^{(-x)^2} + \cos(-x) = e^{x^2} + \cos x$ Even

$= f(x)$

c) $f(x) = x + \sin(x)$

$f(-x) = -x + \sin(-x) = -x - \sin x$ odd

$= -f(x)$

d) $f(x) = x^4 + 2x^2$

$f(-x) = (-x)^4 + 2(-x)^2 = x^4 + 2x^2 = f(x)$ even

e) $f(x) = |x| + 4$

$f(-x) = |-x| + 4 = |x| + 4 = f(x)$ Even

(1000, $9000)
(1500, 12000)

$m = \frac{12000-9000}{1500-1000} = \frac{3000}{500} = 6$

5) A small-appliance manufacturer finds that it costs $9000 to produce 1000 toaster ovens a week and $12000 to produce 1500 toaster ovens a week. (10 Points)

a) Express the cost as a function of the number of the toaster ovens produced, assuming that it is linear.

$y - 9000 = 6(x - 1000)$

$y - 9000 = 6x - 6000$

$+9000 \quad +9000$

$y = 6x + 3000$ $\Rightarrow$ $C(x) = 6x + 3000$

b) What is the slope of the graph and what does it represent?

Slope is 6; Every extra toaster costs $6 to make.

c) What is the y-intercept of the graph and what does it represent?

y intercept = (0, 3000)

the fixed cost is $3000.

(10 Points)

6) If $f(x) = 5x + \ln(x+2)$

a) find $f^{-1}(-1)$ $\quad -1 = 5x + \ln(x+2)$

$x = -0.305$

b) find $f(5) = 5(5) + \ln(5+2) = 25 + \ln 7 = 26.95$

c) State the domain of f(x) $\quad x + 2 > 0$

$x > -2$

d) State the range of f(x)

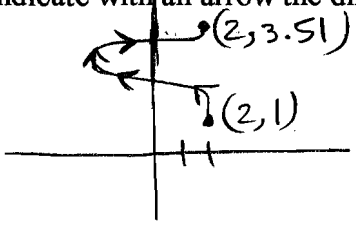
All Reals

(10 Points)

7a) Sketch the curve represented by the parametric equation

$x = 2\cos t \qquad y = \sqrt{t} + 1 \qquad 0 \le t \le 2\pi$

And indicate with an arrow the direction in which the curve is traced as t increases.



7b) Eliminate the parameter to find a Cartesian equation of the curve.

Method One

$$\begin{cases} y = \sqrt{t} + 1 \Rightarrow \sqrt{t} = y - 1 \\ \qquad\qquad\quad t = (y-1)^2 \\ x = 2\cos t \end{cases}$$

$$\boxed{x = 2\cos(y-1)^2}$$

Method Two

$$\begin{cases} x = 2\cos t \Rightarrow \cos t = \frac{x}{2} \Rightarrow t = \cos^{-1}\left(\frac{x}{2}\right) \\ y = \sqrt{t} + 1 \Rightarrow \boxed{y = \sqrt{\cos^{-1}\left(\frac{x}{2}\right)} + 1} \end{cases}$$

8) Let f be a one-to-one function whose inverse function is given by the formula:

$f^{-1}(x) = x^5 + 2x^3 + 3x + 1$ (10 Points)

a) Compute $f^{-1}(-1) = (-1)^5 + 2(-1)^3 + 3(-1) + 1 = \boxed{-5}$

b) Compute $f(1) \Rightarrow 1 = x^5 + 2x^3 + 3x + 1$

$$\boxed{x = 0}$$

c) Compute the value of x such that $f(x) = 1$

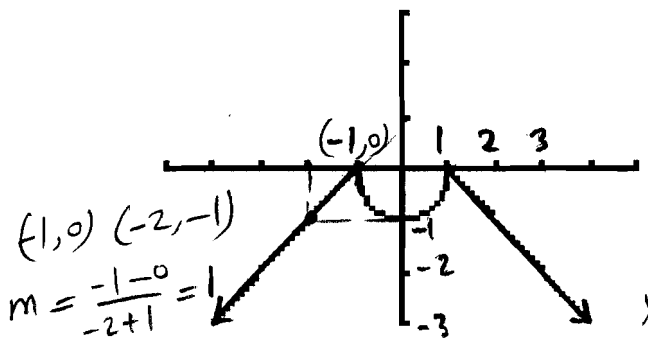
$$f^{-1}(1) = (1)^5 + 2(1)^3 + 3(1) + 1 = \boxed{7}$$

d) Compute the value of y such that $f^{-1}(y) = 1$

$$1 = y^5 + 2y^3 + 3y + 1$$

$$\boxed{y = 0}$$

9) Find a formula that describes the following function: (10 Points)



(1,0) (-2,-1)

$m = \frac{-1-0}{-2+1} = 1$

$y = x + 1$

$$y = \begin{cases} x+1 & x \le -1 \\ -\sqrt{1-x^2} & -1 < x \le 1 \\ -x+1 & 1 < x \end{cases}$$

$x^2 + y^2 = 1$

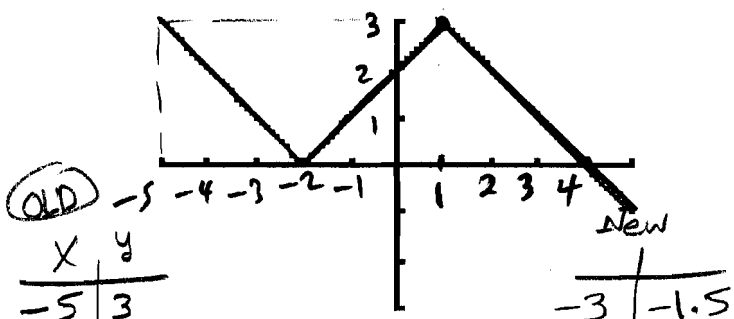
$y^2 = 1 - x^2$

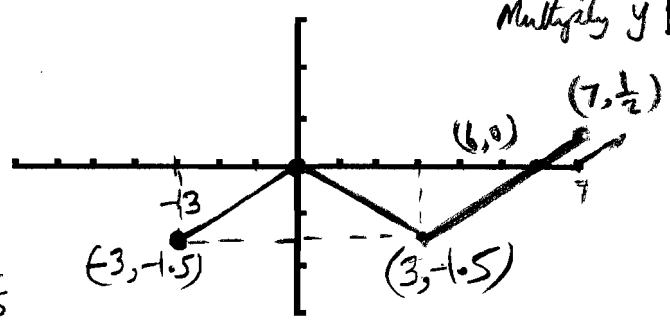
$y = \pm\sqrt{1-x^2}$

10) The graph of $y = f(x)$ is given below; Sketch a graph of $y = -\frac{1}{2}f(x-2)$ (10 points)

Shift Right 2 units

Multiply y by $-\frac{1}{2}$





OLD

x	y
-5	3
-2	0
1	3
4	0
5	-1

New

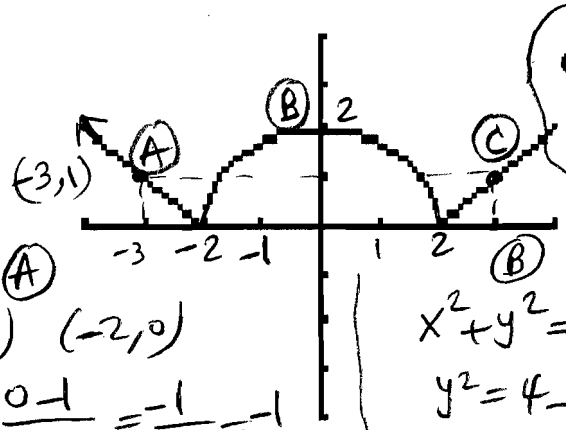
-3	-1.5
0	0
3	1.5
6	0
7	1/2

11) Given the function: $f(x) = 3x^2 + 5x - 8$ (10 pts)

Find the following $\frac{f(x+h) - f(x)}{h}$

(Clearly state each step of the process).

9) Find a formula that describes the following function: (10 Points)



$$y = \begin{cases} -x-2 & x \le -2 \\ \sqrt{4-x^2} & -2 < x \le 2 \\ x-2 & 2 < x \end{cases}$$

(A) $(-3,1)$ $(-2,0)$

$m = \frac{0-1}{-2-(-3)} = \frac{-1}{1} = -1$

$y-0 = -1(x-(-2))$

$y = -x-2$

$x^2+y^2=2^2$

$y^2 = 4-x^2$

$y = \pm\sqrt{4-x^2}$

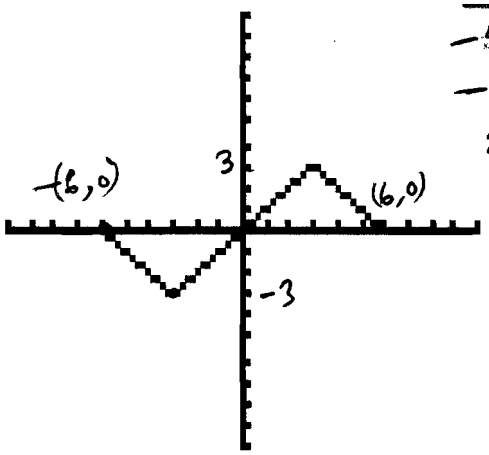
(C) $(2,0)$ $(3,1)$

$m = \frac{1-0}{3-2} = 1$

$y-0 = 1(x-2) = x-2$

The graph of $y = f(x)$ is given below;

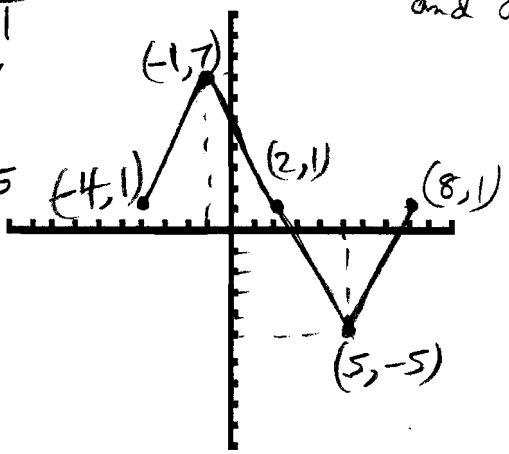
x	y
-6	0
-3	-3
0	0
3	3
6	0



Sketch a graph of $y = -2f(x-2)+1$ (10 points)

shift right 2 units
Multiply y by −2
and add 1 to y

x	y
-4	+1
-1	7
2	1
5	-5
8	1



10) Given the function: $f(x) = 3x^2+5x-8$ (10 pts)

Find the following $\frac{f(x+h)-f(x)}{h}$

(Clearly state each step of the process).

$f(x+h) = 3(x+h)^2+5(x+h)-8 = 3x^2+6xh+3h^2+5x+5h-8$

$f(x+h)-f(x) = 3x^2+6xh+3h^2+5x+5h-8-(3x^2+5x-8)$

$= h(6x+3h+5)$

$\frac{f(x+h)-f(x)}{h} = \frac{h(6x+3h+5)}{h} = 6x+3h+5$

11) Find the inverse of the following functions. (16 points)
(Must Show All the Appropriate Steps)

a) $y=(x+2)^3-5$

$X=(y+2)^3-5$; swap x and y

$X+5=(y+2)^3$; solve for y

$y+2=\sqrt[3]{X+5}$

$y=\sqrt[3]{X+5}-2$

b) $f(x)=\frac{1}{3}\log(5x)$

$y=\frac{1}{3}\log(5x)$

Swap x and y: $X=\frac{1}{3}\log(5y)$

Solve for y: $3X=\log_{10} 5y$

$10^{3X}=5y$

$y=\frac{10^{3X}}{5}$

$f^{-1}(x)=\frac{10^{3X}}{5}$

25) Solve the following algebraically: (10 points)

a) $\left(\frac{1}{5}\right)^{2-x}=25$

$(5^{-1})^{2-X}=5^2$

$5^{-2+X}=5^2 \Rightarrow -2+X=2$

$X=4$

b) $e^{x^2}\cdot\frac{1}{e^6}=(e^{5x})$

$e^{X^2-6}=e^{5X} \Rightarrow X^2-6=5X$

$X^2-5X-6=0$

$(X-6)(X+1)=0$

$X=6$ $X=-1$

c) If $3^x=\frac{1}{49}$, what does 3^{-2x} equal?

$3^{-2X}=(3^X)^{-2}=\left(\frac{1}{49}\right)^{-2}=2401$

EXTRA CREDITS

12) **SOLVE for X (ALGEBRAICALLY)** (10 Points)

(You must show work for full Credit)

Show work & don't forget to check your answers!!

a) $\log_3(2x+4) = -2$

$3^{-2} = 2x+4 \implies \frac{1}{9} = 2x+4 \implies \boxed{x = \frac{-35}{18}}$

b) $4^x - 9 = 15$

$4^x = 24 \implies x = \log_4 24 = \frac{\log 24}{\log 4} = \boxed{2.292}$

c) Solve by the quadratic formula: $x^2 + 11 = 7x$

$x^2 - 7x + 11 = 0 \qquad a=1 \quad b=-7 \quad c=11$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(11)}}{2(1)} = \boxed{\frac{7 \pm \sqrt{5}}{2}}$

d) Algebraically Solve the equation $e^{5-3x} = 10$

$5 - 3x = \ln 10$

$-3x = \ln 10 - 5 \implies x = \frac{\ln 10 - 5}{-3} = \boxed{0.899}$

e) Solve for x Algebraically $\sqrt{3x-3} - 4 = 2$

$\sqrt{3x-3} = 6$

$3x - 3 = 36$

$3x = 39 \implies \boxed{x = 13}$