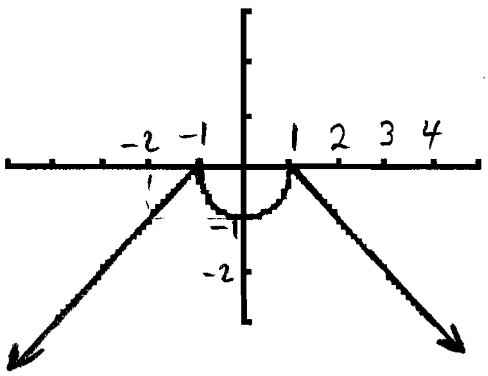
Professor Katiraie Calculus I Spring 2008 Test II Form A (chapters 1 and 2)

Name:____________________ Total Possible Points = 140
(Plus 10 pts Extra Credit ☺)

1) Find a formula that describes the following function: (8 Points)



$$\boxed{f(x) = \begin{cases} x+1 & x \le -1 \\ -\sqrt{1-x^2} & -1 < x \le 1 \\ -x+1 & x > 1 \end{cases}}$$

$(2,-1)$
$(-1,0)$

$m = \frac{0--1}{-1--2} = \frac{1}{1} = 1$

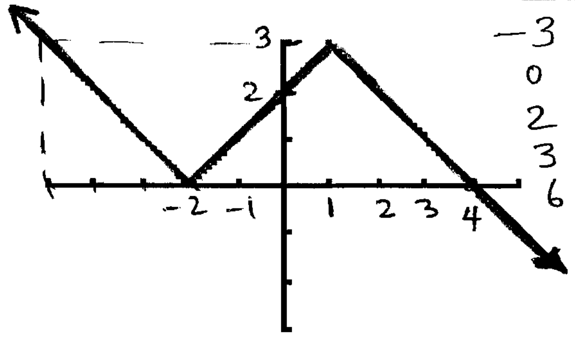
$x^2 + y^2 = 1$
$y^2 = 1 - x^2$
$y = -\sqrt{1-x^2}$

2) The graph of $y = f(x)$ is given below. Sketch a graph of $y = -\frac{1}{2} f(x-2)$ (8 points)

shift 2 units Right and Multiply the y values by $-\frac{1}{2}$

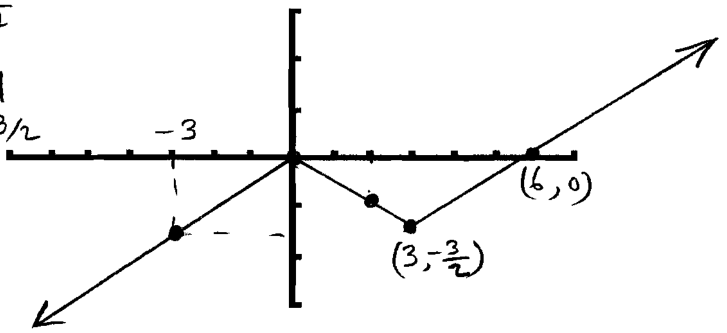
old

x	y
-5	3
-2	0
0	2
1	3
4	0



New

x	y
-3	$\frac{-3}{2}$
0	0
2	-1
3	-3/2
6	0



3) Given the function: $f(x) = \frac{2}{x-1}$ (8 pts)

Find the following $\frac{f(x+h) - f(x)}{h}$

(Clearly state each step of the process).

$f(x+h) = \frac{2}{x+h-1}$; $f(x+h) - f(x) = \frac{2}{x+h-1} - \frac{2}{x-1} = \frac{\cancel{2x} - \cancel{2} - \cancel{2x} - 2h + \cancel{2}}{(x+h-1)(x-1)}$

$\frac{f(x+h) - f(x)}{\frac{h}{1}} = \frac{-2\cancel{h}}{(x+h-1)(x-1)} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{-2}{(x+h-1)(x-1)}}$

4) Find the inverse of the following functions. (10 points)
(Must Show All the Appropriate Steps)

a) $y=\sqrt[3]{x-2}-5$	b) $f(x)=\frac{1}{4}\log(7x)$
Swap x and y	$y=\frac{1}{4}\log(7x)$
$x=\sqrt[3]{y-2}-5$	Swap x and y
Solve for y	$x=\frac{1}{4}\log(7y)$
$x+5=\sqrt[3]{y-2}$	$4x=\log(7y)$
$(x+5)^3=y-2$	$10^{4x}=7y$
$y=(x+5)^3+2$	$f^{-1}(x)=\frac{10^{4x}}{7}$
$f^{-1}(x)=(x+5)^3+2$	

5) Solve the following algebraically: (6 points)

a) $\left(\frac{1}{5}\right)^{2-x}=25$

$5^{-1(2-x)}=5^2$

$-2+x=2$

$x=4$

b) $e^{x^2}\bullet\frac{1}{e^6}=\left(e^{5x}\right)$

$e^{x^2-6}=e^{5x}$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6$ $x=-1$

c) If $3^x=\frac{1}{49}$, what does 3^{-2x} equal?

$3^{-2x}=(3^x)^{-2}=\left(\frac{1}{49}\right)^{-2}=2401$

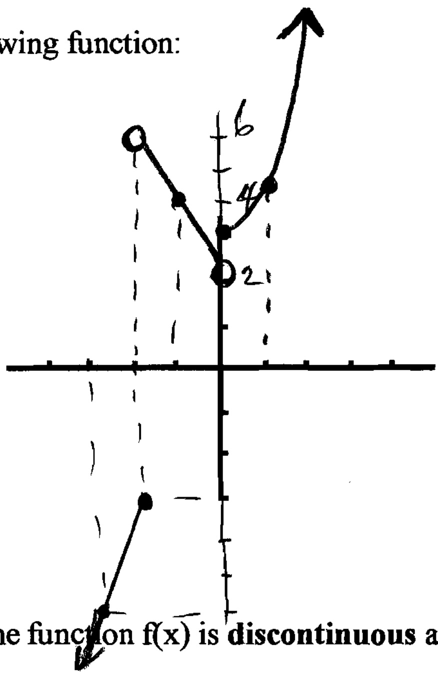
6a) Sketch the graph of the following function:

$$f(x)=\begin{cases}2x & x\le -2\\ 2-2x & -2<x<0\\ e^{x}+2 & x\ge 0\end{cases}$$

(5 Points)

X	y
-3	-6
-2	-4

X	y = 2-2X
-2	6
-1	4
0	2



X	$y=e^{X}+2$
0	3
1	e+2 ≅ 4.718

6b) Discuss **(with reasons)** where the function f(x) is **discontinuous** and why. (5 Points)

f(x) is discontinuous at X = -2 and X = 0

$\lim_{X\to -2} f(x) = \text{DNE}$ and f(-2) = -4

$\lim_{X\to -2} f(x) \ne f(-2)$

$\lim_{X\to 0} f(X) = \text{DNE}$

and f(0) = 3

$\lim_{X\to 0} f(x) \ne f(0)$

7) If an arrow is shot upward on the planet X with a velocity of 70 m/s, its height in meters after t seconds is given by $h(t)=70t-2t^2$ (10 Points)

a) Find the average velocity over the given time intervals:

t	h(t)
2	132
2.5	162.5
2.1	138.18
2.01	132.6198
2.001	132.062

i) [2, 2.5] $\bar{V} = \dfrac{162.5-132}{2.5-2} = \boxed{61 \text{ m/sec}}$

j) [2, 2.1] $\bar{V} = \dfrac{138.18-132}{2.1-2} = \boxed{61.8 \text{ m/sec}}$

k) [2, 2.01] $\bar{V} = \dfrac{132.6198-132}{2.01-2} = \boxed{61.98 \text{ m/sec}}$

l) [2, 2.001] $\bar{V} = \dfrac{132.062-132}{2.001-2} = \boxed{62 \text{ m/sec}}$

b) Find the instantaneous velocity after two seconds.

$\boxed{62 \text{ m/sec}}$

8) $f(x)=\begin{cases} x^3+2 & x\le -2 \\ x^2+x+1 & -2<x<1 \\ x^4+3 & x\ge 1 \end{cases}$ (10 Points)

Find the following limits (give reasons, if the limit does not exist)

a) $\lim_{x\to -2} f(x) = \text{DNE}$

$\lim_{x\to -2^-} f(x) = (-2)^3+2 = -6$

$\lim_{x\to -2^+} f(x) = (-2)^2+(-2)+1 = 3$

b) $\lim_{x\to -1} f(x) = (-1)^2+\cancel{(-1)+1}$

$= 1+0 = \boxed{1}$

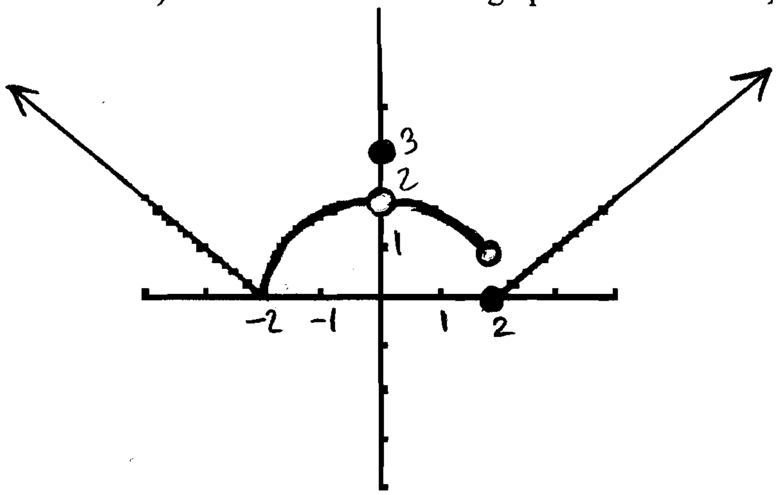
c) $\lim_{x\to 1^+} f(x)$

$= (1)^4+3 = \boxed{4}$

d) $\lim_{x\to 4} f(x)$

$= 4^4+3 = \boxed{259}$

9) For the function whose graph is shown below, answer the following equations: (9 Points)



a) At what number "a" does $\lim_{x\to a} f(x)$ **not** exist? $\boxed{x=2}$

Because $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

b) At what numbers "a" does $\lim_{x\to a} f(x)$ exists, yet $f(x)$ is **not continuous**? $\boxed{x=0}$

Because $\lim_{x\to 0} f(x) = 2$ But $f(0)=3$

c) At what numbers "a" $f(x)$ is continuous, but is **not differentiable**? $\boxed{x=-2}$

Because there is a sharp edge at $x=-2$

10) Given $f(x)=\begin{cases} 2x^3+16 & x\le -1 \\ x^2+bx+c & -1<x<1 \\ 3x^4-47 & x\ge 1 \end{cases}$ determine the values for b and c so that $f(x)$ is continuous everywhere. (10 Points)

$2(-1)^3+16 = (-1)^2+b(-1)+c \Rightarrow \begin{cases} 14 = 1 \cancel{-b} + c \\ -44 = 1 \cancel{+b} + c \end{cases}$

$3(1)^4-47 = (1)^2+b(1)+c$

ADD

$-30 = 2 + 2c$ $\qquad \boxed{b=-29}$

$-32 = 2c \Rightarrow \boxed{c=-16}$

11) Given the following information about the limits, sketch a graph which could be the graph of y = $f(x)$. **Label all horizontal and vertical asymptote(s).** (9 Points)

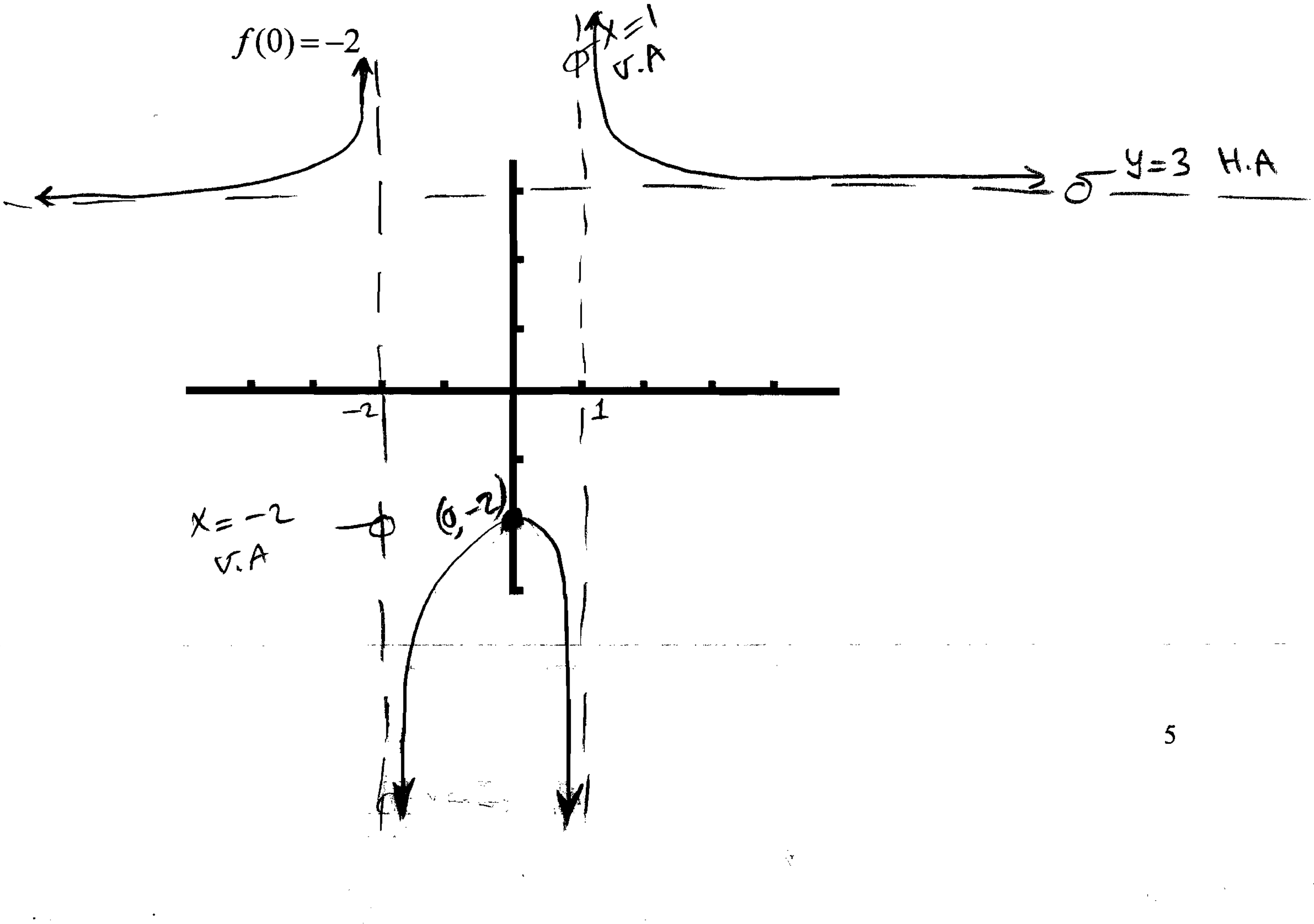
$\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(x) = 3$

$\lim_{x\to -2^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$ $\qquad$ V.A at $x=-2$

$x=1$

$\lim_{x\to -2^-} f(x) = \lim_{x\to 1^+} f(x) = \infty$

$f(0)=-2$



12) Find the following limits: (12 Points)

a) $\lim_{t\to 13} \frac{\sqrt{t+3}-4}{t-13} \bullet \frac{\sqrt{t+3}+4}{\sqrt{t+3}+4} = \lim_{t\to 13} \frac{\cancel{t+3-16}}{\cancel{(t-13)}(\sqrt{t+3}+4)} = \boxed{\frac{1}{8}}$

b) $\lim_{x\to -8} \frac{8+x}{\frac{1}{8}+\frac{1}{x}} = \lim_{x\to -8} \frac{8+x}{\frac{x+8}{8x}}$

$= \lim_{x\to -8} \frac{\cancel{8+x}}{1} \bullet \frac{8x}{\cancel{x+8}} = \lim_{x\to -8} 8x = 8(-8) = \boxed{-64}$

c) $\lim_{x\to\infty} (\sqrt{x^2+2x}-x) \bullet \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} = \lim_{x\to\infty} \frac{\cancel{x^2}+2x-\cancel{x^2}}{\sqrt{x^2+2x}+x}$

$= \lim_{x\to\infty} \frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}+\frac{x}{x}} = \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = \frac{2}{2} = \boxed{1}$

d) $\lim_{t\to 0} \frac{1}{t} - \frac{1}{t^2+t} = \lim_{t\to 0} \frac{t+1-1}{t(t+1)} = \lim_{t\to 0} \frac{\cancel{t}}{\cancel{t}(t+1)}$

$= \lim_{t\to 0} \frac{1}{t+1} = \boxed{1}$

13) Suppose that the line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points (2 , 3) and (4 , -5). Find the following: (12 Points)

a) Find $f'(3) = \frac{-5-3}{4-2} = \frac{-8}{2} = -4$

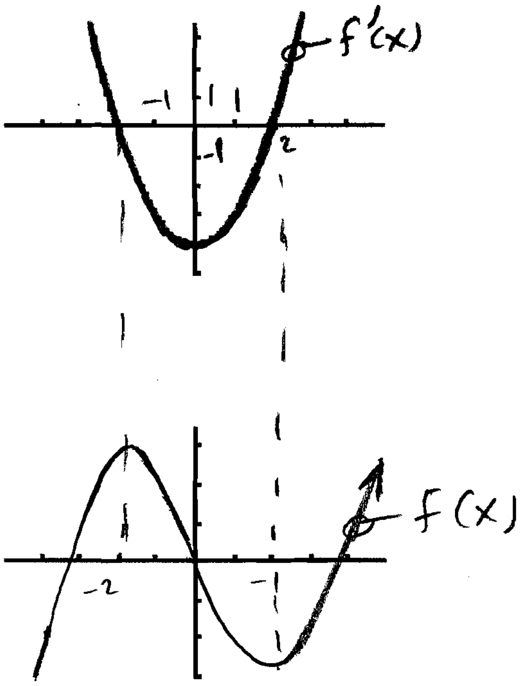
b) Find $f(3) = -4(3)+11 = -12+11 = \boxed{-1}$

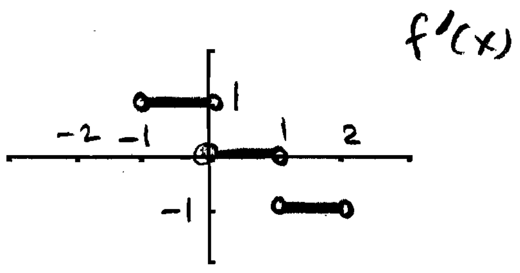
c) Find an equation of the line tangent to f at $x = 3$ $\boxed{y = -4x + 11}$

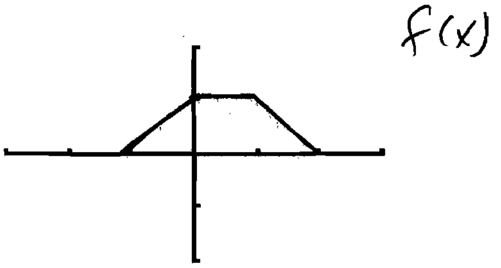
$3 = -4(2) + b$

$3 = -8 + b \Rightarrow \boxed{b = +11}$

14) Given the graph of $y = f'(x)$, sketch the graph of $y = f(x)$ (12 Points)







15) Find the following limit $\lim_{x \to \infty} \frac{\cos x}{x^4}$ (6 Points)

(Hint: Use the Squeeze Theorem)

Observe that $-1 \leq \cos x \leq 1$

Multiplying by $\frac{1}{x^4}$

$$\lim_{x \to \infty} \frac{-1}{x^4} \leq \lim_{x \to \infty} \frac{\cos x}{x^4} \leq \lim_{x \to \infty} \frac{1}{x^4}$$

$\lim_{x \to \infty} \frac{-1}{x^4} = 0$ and $\lim_{x \to \infty} \frac{1}{x^4} = 0$

Then by Squeeze theorem $\boxed{\lim_{x \to \infty} \frac{\cos x}{x^4} = 0}$

16) Given $f(x)=\dfrac{1}{\sqrt{x-3}}$ (Extra Credits 5 Points)

Find the $f'(x)$ using either of the two definitions discussed in class.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h-3}}-\frac{1}{\sqrt{x-3}}}{h}$$

$$\frac{1}{\sqrt{x+h-3}}-\frac{1}{\sqrt{x-3}} = \frac{\sqrt{x-3}-\sqrt{x+h-3}}{\sqrt{x-3}\sqrt{x+h-3}}\cdot\frac{\sqrt{x-3}+\sqrt{x+h-3}}{\sqrt{x-3}+\sqrt{x+h-3}} = \frac{\cancel{x}-\cancel{3}-(\cancel{x}+h-\cancel{3})}{(\sqrt{x-3})(\sqrt{x+h-3})(\sqrt{x-3}+\sqrt{x+h+3})}$$

$$\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h-3}}-\frac{1}{\sqrt{x-3}}}{h} = \lim_{h\to 0}\frac{-\cancel{h}}{(\sqrt{x-3})(\sqrt{x+h-3})(\sqrt{x-3}+\sqrt{x+h-3})}\cdot\frac{1}{\cancel{h}}$$

$$= \boxed{\frac{-1}{2(x-3)^{3/2}}}$$

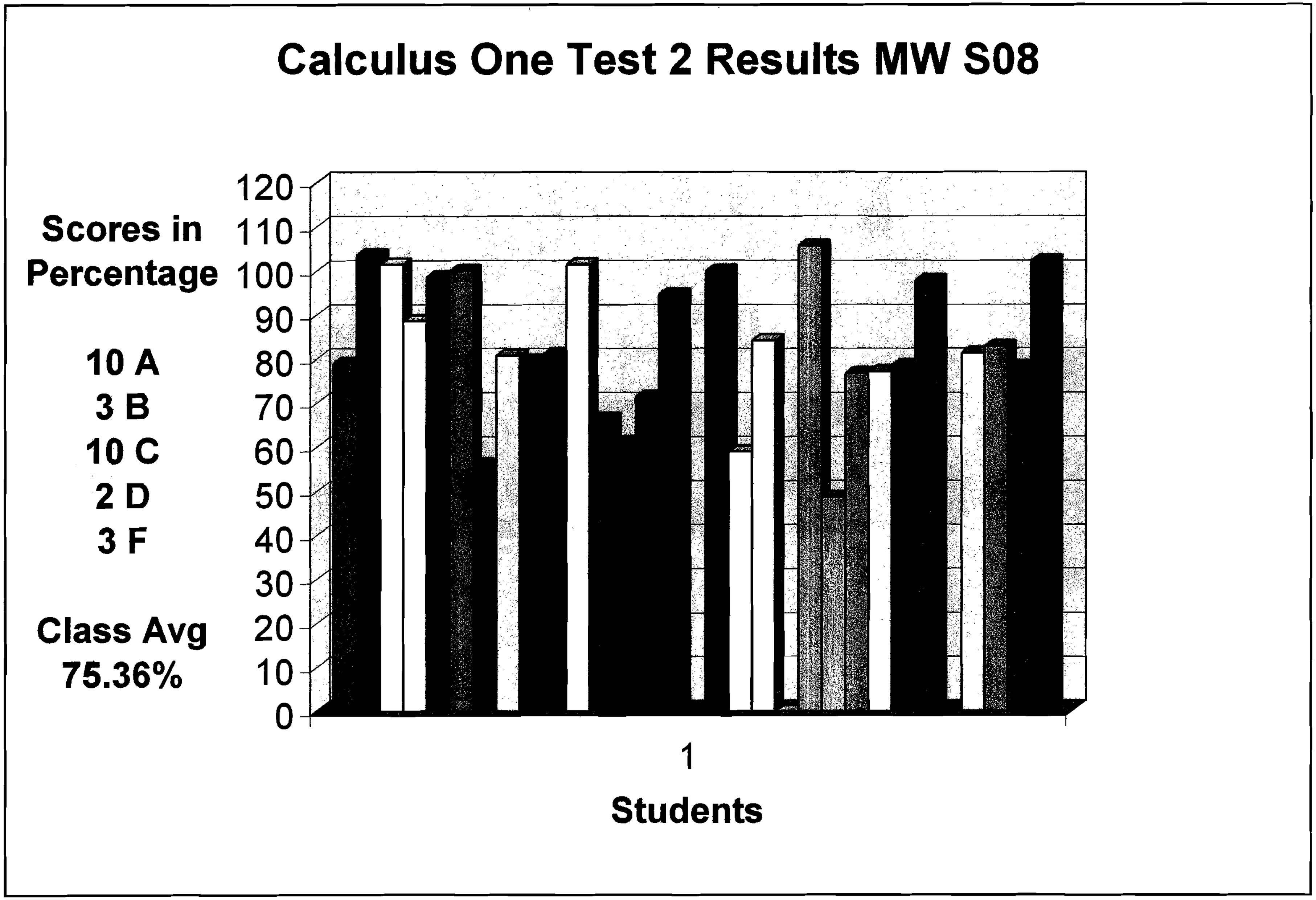
17) Given $f(x)=\sqrt{1+2x}$ (Extra Credits 5 Points)

Find the $f'(x)$ using either of the two definitions discussed in class

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h}\cdot\frac{\sqrt{1+2(x+h)}+\sqrt{1+2x}}{\sqrt{1+2(x+h)}+\sqrt{1+2x}}$$

$$\lim_{h\to 0}\frac{\cancel{1}+2(\cancel{x}+h)-(\cancel{1}+\cancel{2x})}{h(\sqrt{1+2(x+h)}+\sqrt{1+2x})} = \lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}(\sqrt{1+2(x+h)}+\sqrt{1+2x})}$$

$$= \frac{2}{2\sqrt{1+2x}} = \boxed{\frac{1}{\sqrt{1+2x}}}$$



Calculus One Test 2 Results MW S08
Scores in Percentage
10 A
3 B
10 C
2 D
3 F
Class Avg
75.36%
120
110
100
90
80
70
60
50
40
30
20
10
0
1
Students