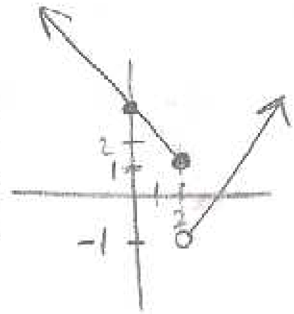
Professor Katiraie Calculus I Fall 2006 Test I

Name:______________________________ Total Possible Points = 140

(24 Points, Domain of each problem is worth 3 Points, and Range is worth 3 Points)

1) Find the **Domain** and **Range** of the following functions:



a) $f(x) = \sqrt{(4-4x^2)}$

$4 - 4x^2 \geq 0$

$4x^2 \leq 4$

$x^2 \leq 1$

Domain $-1 \leq x \leq 1$

Range $0 \leq y \leq 2$

b) $g(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x \leq 2 \\ 2x - 5 & \text{if } x > 2 \end{cases}$

Domain: $\mathbb{R}$

Range: Reals

c) $g(x) = \dfrac{1}{2\cos x}$

Domain: All Reals except $x \neq \frac{n\pi}{2}$ where $n = \pm 1, \pm 3, \pm 5,$

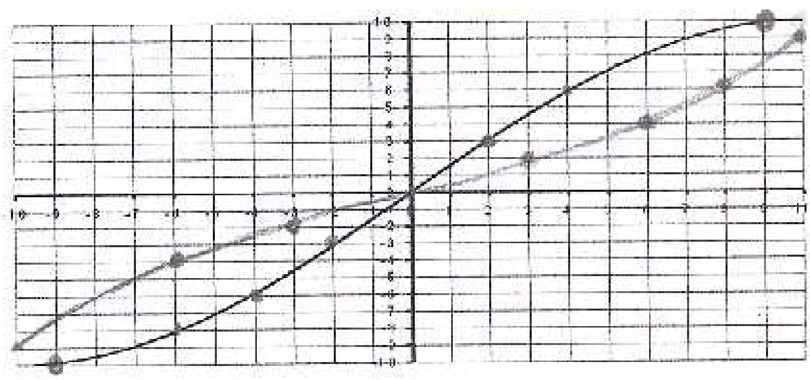
Range: $(-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$

d) $g(x) = \dfrac{5x+4}{x^2+3x+2} = \dfrac{5x+4}{(x+2)(x+1)}$

Domain: All Reals except $x \neq -2$, $x \neq -1$

Range $(-\infty, 2.1] \cup [11.91, \infty)$

(10 Points) The graph of g (x) is given.



a) State the value of g(2) = 3

b) Is g one-to-one? Yes

c) Estimate the value of $g^{-1}(-8)$? -6

d) Estimate the domain of $g^{-1}(x)$ $[-10, 10]$

d) Sketch the graph of $g^{-1}(x)$

(9 Points) 3) Given $x+(y-1)^2=0$

a) Find an expression for the function whose graph is the bottom half of the above parabola

$-x=(y-1)^2 \Rightarrow y-1=\pm\sqrt{-x}$

$y=1\pm\sqrt{-x}$

$\boxed{y=1-\sqrt{-x}}$

b) State the domain of the bottom half of the above parabola

$x\leq 0$

c) State the range of the bottom half of the above parabola

$y\leq 1$

(12 Points) 4) Determine whether the following function is even, odd, or neither
(You Must Use Formal Definition of Even, Odd Functions For Full Credit)

a) $f(x)=1+2x^5-3x^3$

Neither

$f(-x)=1+2(-x)^5-3(-x)^3=1-2x^5+3x^3\neq f(x)$
$\neq -f(x)$

b) $h(x)=\dfrac{x}{x+1}$

Neither

$h(-x)=\dfrac{-x}{-x+1}\neq h(x)$
$\neq -h(x)$

c) $g(x)=\sin(x)+x$

$g(-x)=\sin(-x)+(-x)$
$=-\sin x - x = -g(x)$

Odd

d) $f(x)=x^8+x^4+2x^2$

$f(-x)=(-x)^8+(-x)^4+2(-x)^2=x^8+x^4+2x^2=f(x)$

Even

$(0, 30)$

$(15, 345)$

(10 Points) 5) In 1980 a bus company had 30 busses; in 1995 the company had 345 busses. Let f(t) represents the number of busses t years after 1980. Assume f(t) is a linear function.

a. Find the slope of f(t), and state what the slope represents in terms of the story?

$$m = \frac{345-30}{15-0} = 21$$

Every year the No of Busses increases by 21

b. Use your slope and one ordered pair to write the equation for f(t).

$(0,30) \Rightarrow y = mx + b$

$$\boxed{f(t) = 21t + 30}$$

$30 = 21(0) + b \Rightarrow \boxed{b = 30}$

c. Predict the number of busses in the year 2007.

In the year 2007 then $t = 2007 - 1980 = 27$

$f(27) = 21(27) + 30 = 597$ Busses

d. Determine the year when number of busses was 400.

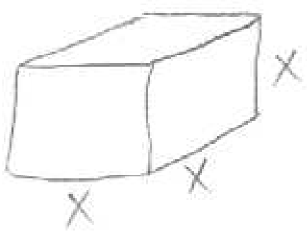
$$400 = 21t + 30$$
$$-30 \qquad\qquad -30$$
$$370 = 21t \Longrightarrow t = \frac{370}{21} = 17.62 \approx 18$$

$\boxed{\text{Year } 1998}$

(5 Points)

6) Express the surface area of a cube as a function of its volume.

$V = X^3 \rightarrow X = \sqrt[3]{V}$



$S = 6X^2 \Rightarrow$

$S = 6(\sqrt[3]{V})^2 = 6V^{2/3}$

$\therefore \boxed{S = 6V^{\frac{2}{3}}}$

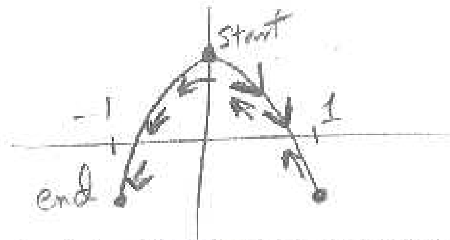
(10 Points)

7a) Sketch the curve represented by the parametric equation $y(t) = \cos 2t$, $x(t) = \sin t \quad 0 < t < \infty$

Indicate with an arrow the direction in which the curve is traced as t increases.

Make a table of points with the corresponding values of t.

t	x	y
0	0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	0
$\frac{\pi}{2}$	1	-1



7b) Eliminate the parameter to find a Cartesian equation of the curve
(Hint: $\cos(2\theta) = \cos^2\theta - \sin^2\theta$)

$y = \cos 2t = \cos^2 t - \sin^2 t = (1-\sin^2 t) - \sin^2 t = 1 - 2\sin^2 t = 1 - 2x^2$

$\boxed{y = 1 - 2x^2}$

7c) Indicate the Domain of the Cartesian equation

$\boxed{-1 \leq x \leq 1}$

7d) Indicate the Range of the Cartesian equation

$\boxed{-1 \leq y \leq 1}$

(12 Points)

8) Use the following table to evaluate the expressions.

X	1	2	3	4	5	6
g(x)	6	5	4	1	3	5
f(x)	6	3	5	1	2	3

a) f(f(g(2))) = $f(f(5)) = f(2) = 3$

b) g(g (g(6))) = $g(g(5)) = g(3) = 4$

c) (gogof)(5) = $g(g(2)) = g(5) = 3$

d) (fogof)(6) = $f(g(3)) = f(4) = 1$

-6	-1
-3	0
0	1
15	2
174	3
849	4

(12 Points) 9) Let f be a one-to-one function whose inverse function is given by the formula:

$$f^{-1}(z) = z^5 - 3z^3 + 5z - 3$$

a) Compute the value of x such that $f(x) = -1$ $\boxed{\text{then } x = -6}$ B/c $f^{-1}(-1) = -6$

b) Compute $f(-6)$ then $f^{-1}(x) = -6$

$$\boxed{f(-6) = -1}$$

c) Compute $f^{-1}(-2)$

$$f^{-1}(-2) = (-2)^5 - 3(-2)^3 + 5(-2) - 3 = \boxed{-21}$$

d) Compute the value of y such that $f^{-1}(y)=174$

$$174 = y^5 - 3y^3 + 5y - 3$$

then let $Y_1 = 174$ and $Y_2 = x^5 - 3x^3 + 5x - 3$ and use TI calc.

we get $\boxed{y = 3}$

(12 Points) 10) Given $f(x) = \dfrac{4x-1}{2x+3}$

a) State the domain of f(x) All Reals except $-\frac{3}{2} \neq x$

b) State the range of f(x) All Reals except $y \neq 2$

c) Find a formula for the inverse of f(x)

$$y = \frac{4x-1}{2x+3} \Rightarrow x = \frac{4y-1}{2y+3} \Rightarrow 2xy + 3x = 4y - 1$$

$$y(2x-4) = -3x-1$$

$$y = \frac{-3x-1}{2x-4} \Rightarrow \boxed{f^{-1}(x) = \frac{-3x-1}{2x-4}}$$

d) State the domain of $f^{-1}(x)$

All Reals except $x \neq 2$

e) State the range of $f^{-1}(x)$

All Reals except $y \neq \frac{-3}{2}$

(12 Points) 11a) If $f(x)=2x^2-7$, find and simplify $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$

$f(x+h) = 2(x+h)^2 - 7 = 2x^2 + 4xh + 2h^2 - 7$

$f(x+h) - f(x) = \cancel{2x^2} + 4xh + 2h^2 \cancel{-7} - (\cancel{2x^2} \cancel{-7}) = 4xh + 2h^2$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{4xh + 2h^2}{h} = \dfrac{\cancel{h}(4x + 2h)}{\cancel{h}} = \boxed{4x + 2h}$

11b) Given $f(x) = \dfrac{1}{x-1}$ find and simplify $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$

$f(x+h) = \dfrac{1}{x+h-1}$

$f(x) = \dfrac{1}{x-1}$

$f(x+h) - f(x) = \dfrac{1}{x+h-1} - \dfrac{1}{x-1} = \dfrac{x-1-(x+h-1)}{(x+h-1)(x-1)}$

$= \dfrac{\cancel{x}\cancel{-1} - \cancel{x} - h \cancel{+1}}{(x+h-1)(x-1)} = \dfrac{-h}{(x+h-1)(x-1)}$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{-h}{(x+h-1)(x-1)} \cdot \dfrac{1}{h} = \boxed{\dfrac{-1}{(x+h-1)(x-1)}}$

(12 Points) 12) **SOLVE for X (ALGEBRAICALLY)**

(You must show work for full Credit)

Show work & don't forget to check your answers!!

a) $\log_4(2x + 6) = \frac{1}{2}$

$4^{\frac{1}{2}} = 2x+6$

$2 = 2x + 6 \Rightarrow 2 - 6 = 2x \Rightarrow \boxed{x = \frac{-4}{2} = -2}$

b) $4^x - 9 = 15$

$4^x = 24 \Rightarrow \log 4^x = \log 24 \Rightarrow x \log 4 = \log 24$

$\boxed{x = \frac{\log 24}{\log 4} = 2.29}$

c) Solve by the quadratic formula: $x^2 + 11 = 7x$

$x^2 - 7x + 11 = 0$ $\quad a = 1 \quad b = -7 \quad c = 11$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(11)}}{2(1)} = \frac{7 \pm \sqrt{49-44}}{2}$

$= \boxed{\frac{7 \pm \sqrt{5}}{2}}$

d) Solve by factoring: $2x^2 - x = 15$

$2x^2 - x - 15 = 0$

$(2x + 5)(x - 3) = 0$

$x = \frac{-5}{2}$ $\quad x = 3$

e) Solve for x Algebraically $\sqrt{3x - 3} - 4 = 2$

$\quad +4 \quad +4$

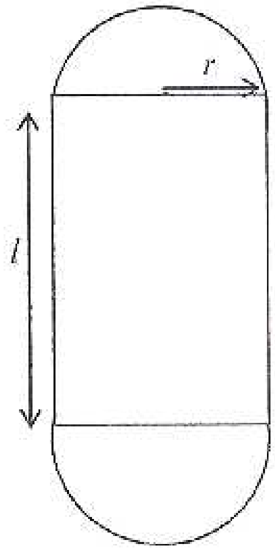
$\sqrt{3x-3} = 6$

$3x - 3 = 36$

$3x = 39$

$\boxed{x = 13}$

Extra Credits:
(6 points)



A field has the shape of a rectangle with a semicircle at each end. The length of the rectangular portion of the field is l, and the radius of each semicircle is r. If the outside perimeter of the field is 250 meters, express the **area** of the field as a function of r, and simplify your answer.

$$250 = 2L + 2\pi r \Rightarrow 125 = L + \pi r$$
$$\therefore L = 125 - \pi r$$

$$A = \pi r^2 + (2r)(L)$$
$$= \pi r^2 + 2r(125 - \pi r)$$
$$= \pi r^2 + 250r - 2\pi r^2$$
$$A = -\pi r^2 + 250r \ (m^2)$$

Extra Credits:
(4 points each)

14a) Solve the equation $e^{5-3x} = 10$ Algebraically.

$$\cancel{\ln e}^{5-3X} = \ln 10$$
$$5 - 3X = \ln 10 \rightarrow -3X = (\ln 10) - 5$$
$$X = \frac{(\ln 10) - 5}{-3} = \boxed{0.899}$$

14b) Express $\ln a + \frac{1}{2}\ln b$ as a single logarithm

$$= \ln a + \ln b^{1/2}$$
$$= \ln a + \ln\sqrt{b} = \boxed{\ln(a\sqrt{b})}$$